

## USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

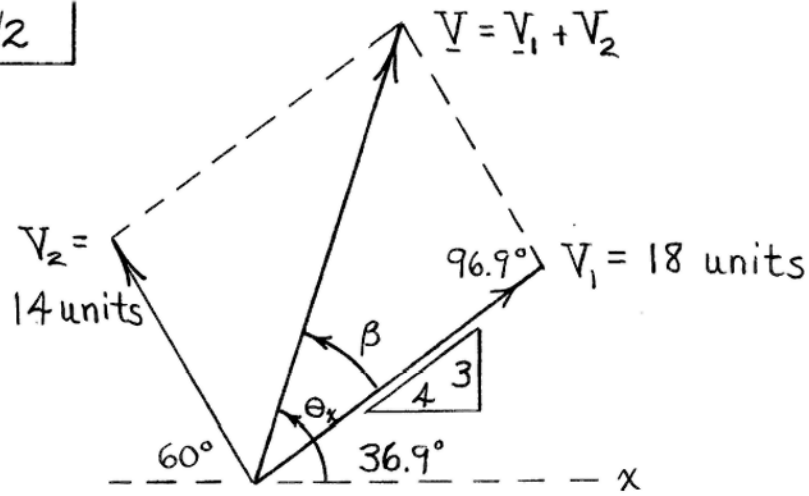
In the transparency master section there are approximately 40 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

$$\frac{1}{1} \quad V = \sqrt{10^2 + 24^2} = 26$$

$$\cos \theta_x = \frac{-10}{26} \quad \theta_x = 112.6^\circ$$

$$\underline{\underline{n}} = \frac{\underline{V}}{V} = \frac{-10\underline{i} + 24\underline{j}}{26} = \underline{-0.385\underline{i} + 0.923\underline{j}}$$

1/2



Graphically,  $V = 24$  units,  $\theta_x = 72^\circ$

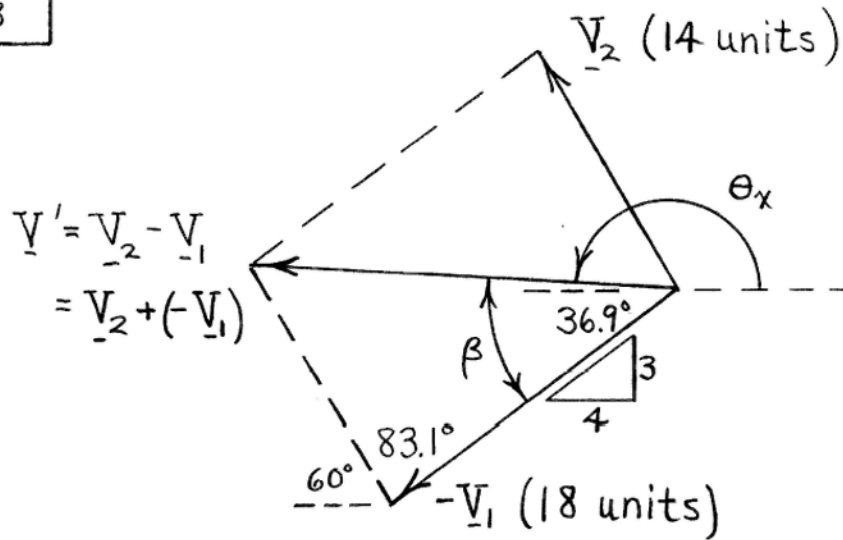
Algebraically,  $V^2 = 18^2 + 14^2 - 2(18)(14)\cos 96.9^\circ$

$$V = 24.1 \text{ units}$$

$$\frac{\sin \beta}{14} = \frac{\sin 96.9^\circ}{24.1}, \quad \beta = 35.2^\circ$$

$$\theta_x = \beta + 36.9^\circ = \underline{72.1^\circ}$$

1/3



Graphically,  $\underline{V}' = 21$  units,  $\theta_x = 176^\circ$

Algebraically,  $V'^2 = 18^2 + 14^2 - 2(18)(14) \cos 83.1^\circ$

$$\underline{V}' = 21.4 \text{ units}$$

$$\frac{\sin \beta}{14} = \frac{\sin 83.1^\circ}{21.4}, \quad \beta = 40.4^\circ$$

$$\theta_x + \beta = 217^\circ, \quad \theta_x = 217^\circ - \beta = 217^\circ - 40.4^\circ = \underline{176.5^\circ}$$

$$\frac{1}{4} \quad F = \sqrt{80^2 + 40^2 + 60^2} = 107.7 \text{ lb}$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{80}{107.7} = 0.743, \quad \underline{\theta_x = 42.0^\circ}$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-40}{107.7} = -0.371, \quad \underline{\theta_y = 111.8^\circ}$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{60}{107.7} = 0.557, \quad \underline{\theta_z = 56.1^\circ}$$

$$\frac{1}{5} \quad W = mg = 75(9.81) = \underline{736 \text{ N}}$$
$$W = 736 \text{ N} \left( \frac{1 \text{ lb}}{4.4482 \text{ N}} \right) = \underline{165.4 \text{ lb}}$$

$$\frac{1}{6} \quad F = W = \frac{G m_1 m_2}{r^2}$$

where  $G = 6.673 (10^{-11}) \text{ m}^3 / (\text{kg} \cdot \text{s}^2)$

$$m_1 = 80 \text{ kg}$$

$$m_2 = 5.976 (10^{24}) \text{ kg}$$

and  $r = (6371 + 250) (10^3) \text{ m}$

Substitute these numbers & obtain  $W = 728 \text{ N}$

U.S. units :  $W = 728 \text{ N} \left( \frac{1 \text{ lb}}{4.4482 \text{ N}} \right) = \underline{163.6 \text{ lb}}$

$$\frac{1}{7} \downarrow W = (130 \text{ lb}) \left( \frac{4.4482 \text{ N}}{1 \text{ lb}} \right) = \underline{578 \text{ N}}$$

$$m = \frac{W}{g} = \frac{130}{32.2} = \underline{4.04 \text{ slugs}}$$

$$m = \frac{W}{g} = \frac{578}{9.81} = \underline{58.9 \text{ kg}}$$



$$\frac{1}{8} \quad | \quad A = 8.69, \quad B = 1.427$$

$$(A+B) = 8.69 + 1.427 = \underline{10.12}$$

$$(A-B) = 8.69 - 1.427 = \underline{7.26}$$

$$(AB) = (8.69)(1.427) = \underline{12.40}$$

$$(A/B) = 8.69/1.427 = \underline{6.09}$$

1/9

$$F = \frac{G m_e m_m}{d^2} = \frac{6.673(10^{-11})(5.976 \cdot 10^{24})^2(1)(0.0123)}{(384398 \cdot 10^3)^2}$$

$$= \underline{1.984(10^{20}) \text{ N}}$$

$$F = 1.984(10^{20}) \text{ N} \left( \frac{1 \text{ lb}}{4.4482 \text{ N}} \right) = \underline{4.46(10^{19}) \text{ lb}}$$

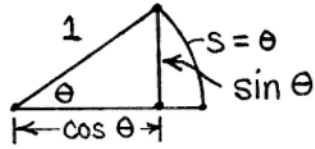
$$\frac{1}{10} \left| 20^\circ \left( \frac{\pi \text{ rad}}{180^\circ} \right) = 0.3491 \text{ rad} \right.$$

$$\sin 20^\circ = 0.3420$$

$$\text{Percent error is } \frac{0.3420 - 0.3491}{0.3420} (100) = \underline{2.06\%}$$

$$\tan 20^\circ = 0.3640$$

$$\text{Percent error is } \frac{0.3640 - 0.3491}{0.3640} (100) = \underline{4.09\%}$$



The approximation  $\sin \theta \cong \theta$  involves the approximation that the arclength  $s = \theta$  is the vertical side of the triangle. The approximation that  $\tan \theta \cong \theta$  involves, in addition, the approximation that 1 is the horizontal side of the triangle.

$$\underline{2/1} \quad \begin{cases} F_x = 500 \cos 40^\circ = \underline{383 \text{ N}} \\ F_y = -500 \sin 40^\circ = \underline{-321 \text{ N}} \end{cases}$$
$$\underline{F} = 383 \underline{i} - 321 \underline{j} \text{ N}$$

$$\frac{2}{2} \quad \underline{F} = 400 (-\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j}) \\ = -346 \underline{i} + 200 \underline{j} \text{ lb}$$

$$\text{Scalar components: } \begin{cases} F_x = -346 \text{ lb} \\ F_y = 200 \text{ lb} \end{cases}$$

$$\text{Vector components: } \begin{cases} \underline{F}_x = -346 \underline{i} \text{ lb} \\ \underline{F}_y = 200 \underline{j} \text{ lb} \end{cases}$$

$$\frac{2}{3} \quad \underline{F} = 5.2 \left( -\frac{12}{13} \underline{i} - \frac{5}{13} \underline{j} \right) \\ = -4.8 \underline{i} - 2 \underline{j} \text{ kN}$$

(Note: Writing 2, rather than 2.00, indicates an exact result.)

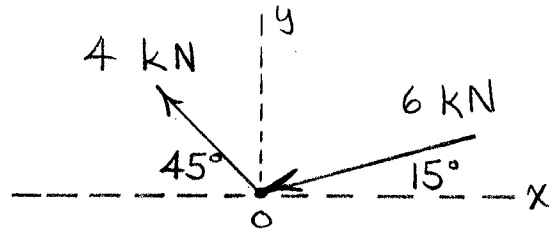
$$\underline{2/4} \quad \underline{F} = F_{n_{AB}} = 3000 \left[ \frac{15\underline{i} + 8\underline{j}}{\sqrt{15^2 + 8^2}} \right]$$
$$= 2650\underline{i} + 1412\underline{j} \text{ lb}$$

Scalar components :  $\begin{cases} F_x = 2650 \text{ lb} \\ F_y = 1412 \text{ lb} \end{cases}$

$$\frac{2}{5} \underline{F} = 1800 \left( -\frac{3}{5} \underline{i} - \frac{4}{5} \underline{j} \right) = \underline{-1080i - 1440j \text{ N}}$$



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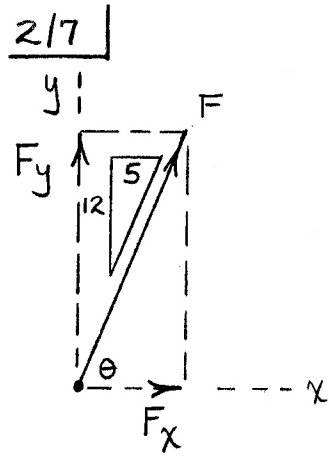


$$R_x = \sum F_x = -4 \cos 45^\circ - 6 \cos 15^\circ = -8.62 \text{ kN}$$

$$R_y = \sum F_y = 4 \sin 45^\circ - 6 \sin 15^\circ = 1.276 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = 8.72 \text{ kN}$$

$$\theta = \tan^{-1} \left( \frac{R_y}{R_x} \right) = \tan^{-1} \left( \frac{1.276}{-8.62} \right) = 171.6^\circ$$



$$\cos \theta = \frac{5}{13}, \quad \sin \theta = \frac{12}{13}$$

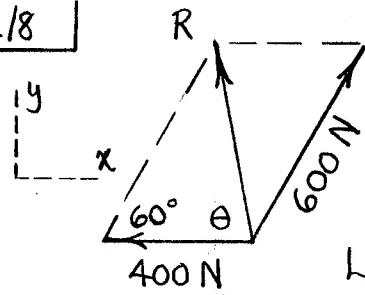
$$F_y = F \sin \theta = F \frac{12}{13} = 70$$

$$F = 75.8 \text{ lb}$$

$$F_x = F \cos \theta = 75.8 \left( \frac{5}{13} \right)$$

$$= 29.2 \text{ lb}$$

2/8



Law of cosines:

$$R^2 = 600^2 + 400^2 - 2(600)(400)\cos 60^\circ$$

$$R = 529 \text{ N}$$

Law of sines:

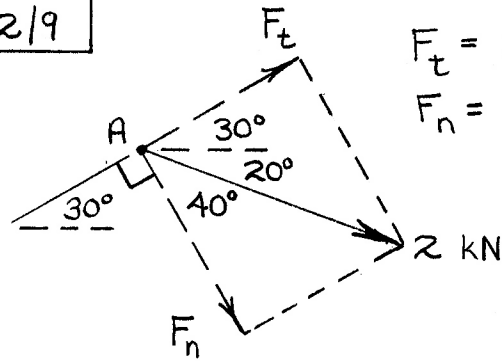
$$\frac{529}{\sin 60^\circ} = \frac{600}{\sin \theta} \quad \theta = 79.1^\circ$$

$$(b) R_x = \sum F_x = 600 \cos 60^\circ - 400 = -100 \text{ N}$$

$$R_y = \sum F_y = 600 \sin 60^\circ + 0 = 520 \text{ N}$$

$$\text{So } \underline{R = -100\hat{i} + 520\hat{j} \text{ N}}$$

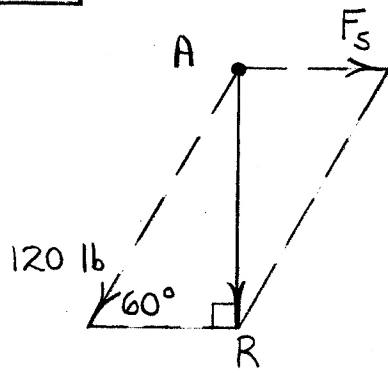
2/9



$$F_t = 2 \cos 50^\circ = 1.286 \text{ kN}$$

$$F_n = 2 \sin 50^\circ = 1.532 \text{ kN}$$

2/10



$$\cos 60^\circ = \frac{F_s}{120}$$

$$F_s = 60 \text{ lb}$$

$$\sin 60^\circ = \frac{R}{120}$$

$$R = 103.9 \text{ lb}$$

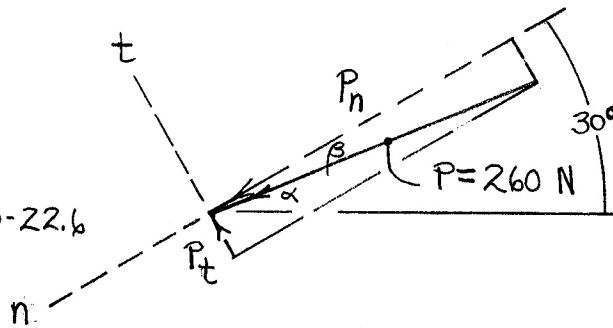
$$\underline{2/11} \quad P_x = -260 \left( \frac{12}{13} \right) = -240 \text{ N}$$

$$P_y = -260 \left( \frac{5}{13} \right) = \underline{-100 \text{ N}}$$

2/12

$$\alpha = \tan^{-1} \frac{5}{12}$$
$$= 22.6^\circ$$

$$\beta = 30 - \alpha = 30 - 22.6$$
$$= 7.38^\circ$$



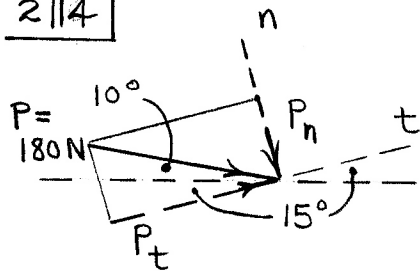
$$P_n = P \cos \beta = 260 \cos 7.38^\circ = \underline{258 \text{ N}}$$

$$P_t = P \sin \beta = 260 \sin 7.38^\circ = \underline{33.4 \text{ N}}$$

$$\begin{aligned} \underline{2/13} \quad R_x &= \sum F_x = 400 + 400 \cos 60^\circ = 600 \text{ N} \\ R_y &= \sum F_y = 400 \sin 60^\circ = 346 \text{ N} \\ \Rightarrow \underline{R} &= \underline{600i + 346j \text{ N}} \\ R &= \sqrt{600^2 + 346^2} = \underline{693 \text{ N}} \end{aligned}$$



2/14



$$P_t = 180 \cos 25^\circ = \underline{163.1 \text{ N}}$$

$$P_n = -180 \sin 25^\circ = \underline{-76.1 \text{ N}}$$

$$\underline{2/15} \quad (b) \quad \underline{\underline{R = 400 \underline{e}_t + 900 \underline{e}_n \quad lb}}$$

$$(a) \quad \underline{\underline{R = [400 \cos 15^\circ - 900 \sin 15^\circ] \underline{i} + [400 \sin 15^\circ + 900 \cos 15^\circ] \underline{j}}}$$
$$= \underline{\underline{153.4 \underline{i} + 973 \underline{j} \quad lb}}$$

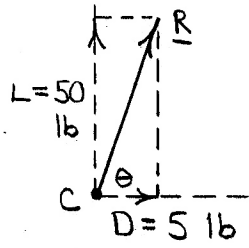
2/16 | Using the coordinates of the problem figure :

$$R_x = \sum F_x = 200 \cos 35^\circ - 150 \sin 30^\circ \\ = 88.8 \text{ N}$$

$$R_y = \sum F_y = 200 \sin 35^\circ + 150 \cos 30^\circ \\ = 245 \text{ N}$$

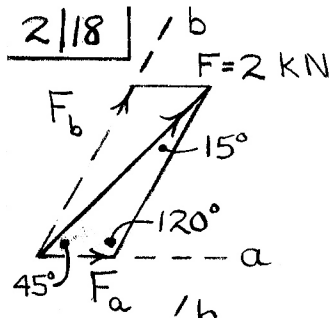
$$\therefore \underline{R} = 88.8 \underline{i} + 245 \underline{j} \text{ N}$$

$$\frac{2}{17} \quad \frac{L}{D} = \frac{50}{5} = 10 ; \quad D = 5 \text{ lb}$$



$$R = \sqrt{L^2 + D^2} = \sqrt{50^2 + 5^2}$$
$$= \underline{50.2 \text{ lb}}$$

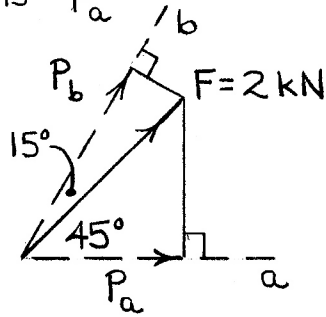
$$\theta = \tan^{-1} \left( \frac{L}{D} \right) = \tan^{-1} \left( \frac{50}{5} \right)$$
$$= \underline{84.3^\circ}$$



### Components

$$\frac{\sin 120^\circ}{2} = \frac{\sin 15^\circ}{F_a}, \quad \underline{F_a = 0.598 \text{ kN}}$$

$$\frac{\sin 120^\circ}{2} = \frac{\sin 45^\circ}{F_b}, \quad \underline{F_b = 1.633 \text{ kN}}$$

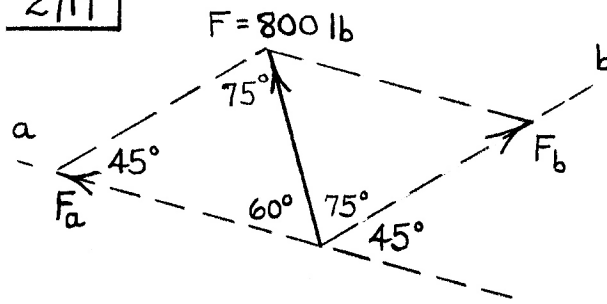


### Projections

$$P_a = 2 \cos 45^\circ = \underline{1.414 \text{ kN}}$$

$$P_b = 2 \cos 15^\circ = \underline{1.932 \text{ kN}}$$

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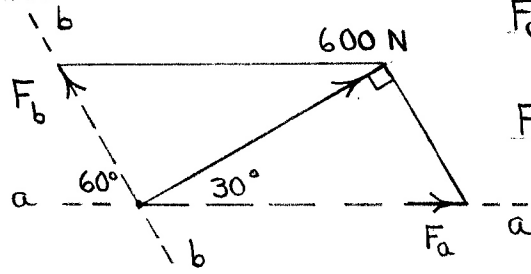


$$\frac{\sin 45^\circ}{800} = \frac{\sin 75^\circ}{F_a} = \frac{\sin 60^\circ}{F_b}$$

$$\text{Components: } \begin{cases} F_a = 1093 \text{ lb} \\ F_b = 980 \text{ lb} \end{cases}$$

$$\text{Projections: } \begin{cases} F_{p_a} = 800 \cos 60^\circ = 400 \text{ lb} \\ F_{p_b} = 800 \cos 75^\circ = 207 \text{ lb} \end{cases}$$

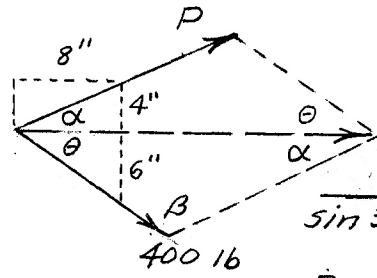
2/20



$$F_a = \frac{600}{\cos 30^\circ} = \underline{693 \text{ N}}$$

$$F_b = 600 \tan 30^\circ = \underline{346 \text{ N}}$$

2/21



$$\alpha = \tan^{-1} \frac{4}{8} = 26.57^\circ$$

$$\theta = \tan^{-1} \frac{6}{8} = 36.87^\circ$$

$$\beta = 180 - (\alpha + \theta) = 116.57^\circ$$

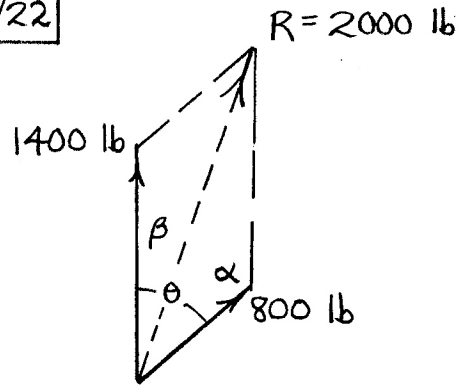
$$\frac{P}{\sin 36.87^\circ} = \frac{400}{\sin 26.57^\circ}$$

$$P = 400 \frac{0.6}{0.4472} = 537 \text{ lb}$$

$$\frac{T}{\sin 116.56^\circ} = \frac{400}{\sin 26.57^\circ}, T = 400 \frac{0.8944}{0.4472} = 800 \text{ lb}$$



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$$\text{Law of Cosines: } 2000^2 = 1400^2 + 800^2 - 2(1400)(800)\cos\alpha$$

With  $\alpha = 180 - \theta$  and  $\cos(180 - \theta) = -\cos\theta$ :

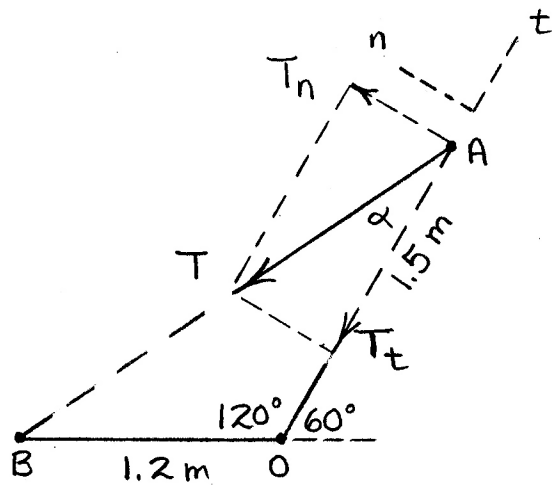
$$2000^2 = 1400^2 + 800^2 + 2(1400)(800)\cos\theta$$

$$\theta = 51.3^\circ$$

$$\text{Law of Sines: } \frac{800}{\sin\beta} = \frac{2000}{\sin(180^\circ - 51.3^\circ)}$$

$$\beta = 18.19^\circ$$

2/23



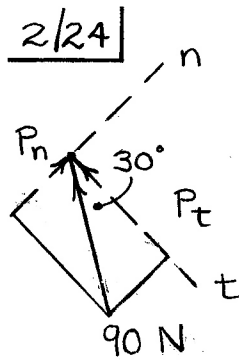
$$\overline{AB}^2 = 1.2^2 + 1.5^2 - 2(1.2)(1.5) \cos 120^\circ$$

$$\overline{AB} = 2.34 \text{ m}$$

$$\frac{\sin \alpha}{1.2} = \frac{\sin 120^\circ}{2.34} \quad \alpha = 26.3^\circ$$

$$T_n = T \sin \alpha = 750 \sin 26.3^\circ = \underline{333 \text{ N}}$$

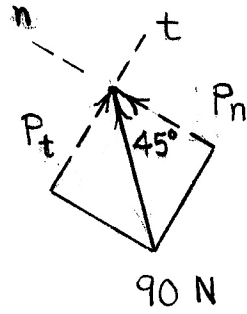
$$T_t = -T \cos \alpha = -750 \cos 26.3^\circ = \underline{-672 \text{ N}}$$



BC

$$P_t = -90 \cos 30^\circ = \underline{-77.9 \text{ N}}$$

$$P_n = 90 \sin 30^\circ = \underline{45.0 \text{ N}}$$

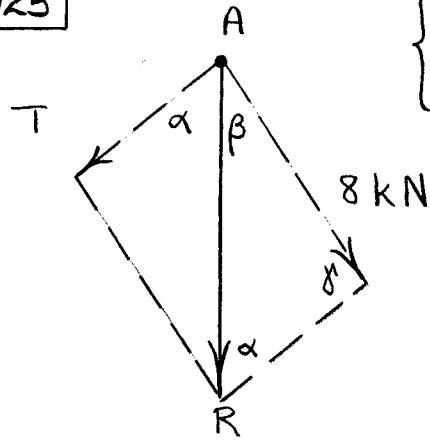


AB

$$P_t = 90 \sin 45^\circ = \underline{63.6 \text{ N}}$$

$$P_n = 90 \cos 45^\circ = \underline{63.6 \text{ N}}$$

2/25



$$\begin{cases} \alpha = \tan^{-1} \frac{50}{40} = 51.3^\circ \\ \beta = \tan^{-1} \frac{40}{60} = 33.7^\circ \end{cases}$$

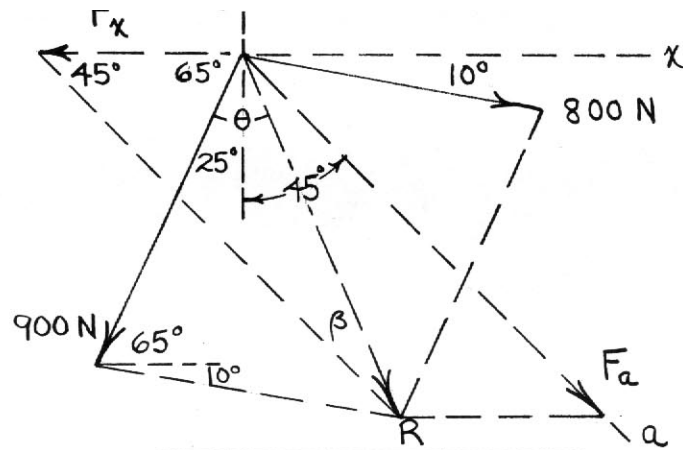
$$\gamma = 180 - \alpha - \beta = 95.0^\circ$$

$$\frac{\sin \beta}{T} = \frac{\sin \alpha}{8}$$

$$\underline{T = 5.68 \text{ kN}}$$

$$\frac{\sin \gamma}{R} = \frac{\sin \alpha}{8}$$

$$\underline{R = 10.21 \text{ kN}}$$



$$\text{Law of cosines: } R = \sqrt{900^2 + 800^2 - 2(900)(800)\cos 75^\circ} = 1038 \text{ N}$$

$$\text{Law of sines: } \frac{1038}{\sin 75^\circ} = \frac{800}{\sin \theta}, \quad \theta = 48.1^\circ$$

$$\beta = 180^\circ - 45^\circ - (65^\circ + 48.1^\circ) = 21.9^\circ$$

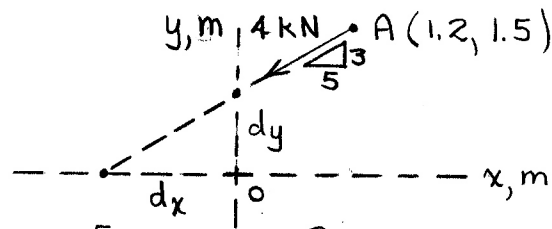
$$\frac{1038}{\sin 45^\circ} = \frac{F_x}{\sin 21.9^\circ}$$

$$\underline{F_x = 547 \text{ N}}$$

$$\frac{F_a}{\sin(65^\circ + 48.1^\circ)} = \frac{1038}{\sin 45^\circ}$$

$$\underline{F_a = 1350 \text{ N}}$$

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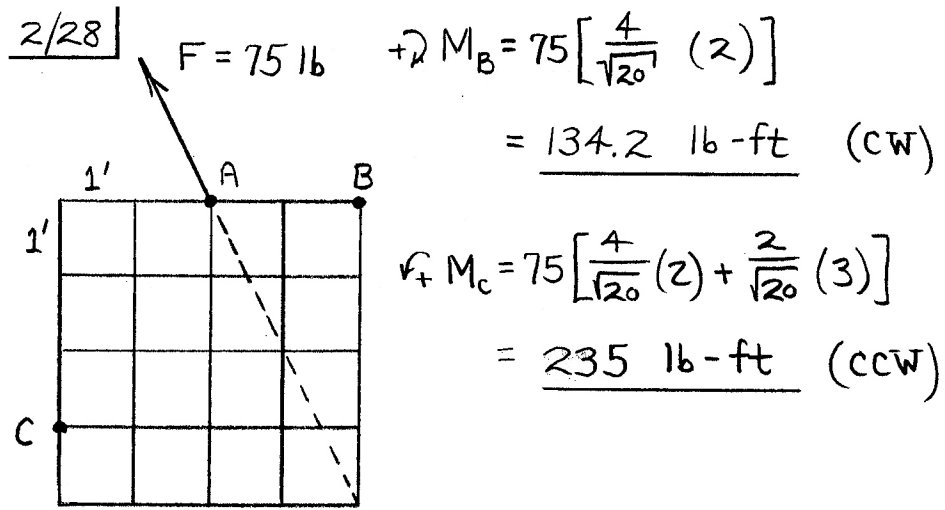
$$\sqrt{+} M_o = 4 \left[ \frac{5}{\sqrt{34}} (1.5) - \frac{3}{\sqrt{34}} (1.2) \right] = \underline{2.68 \text{ kN}\cdot\text{m}}$$

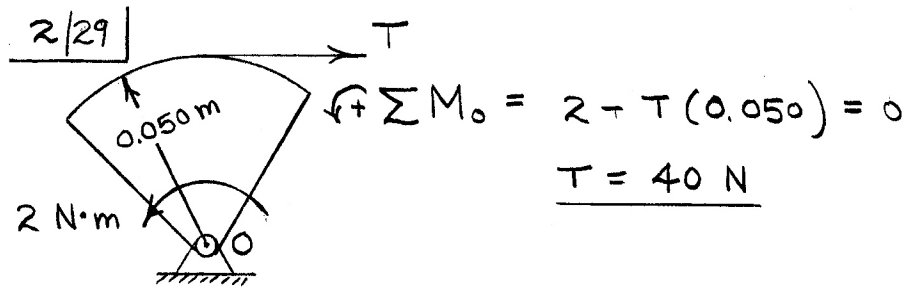
As a vector,  $\underline{M_o} = 2.68\mathbf{k} \text{ kN}\cdot\text{m}$

$$\frac{1.5}{d_x + 1.2} = \frac{3}{5}, \quad d_x = 1.3 \text{ m}$$

$$\frac{d_y}{1.3} = \frac{3}{5}, \quad d_y = 0.78 \text{ m}$$

Coordinates of intercepts:  $(-1.3, 0), (0, 0.78)$   
(in m)

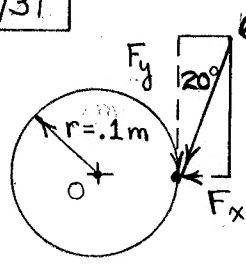






$$\frac{2/30}{+2} M_o = 180(9.81)(2.4) = \underline{4240 \text{ N}\cdot\text{m}}$$

2/31



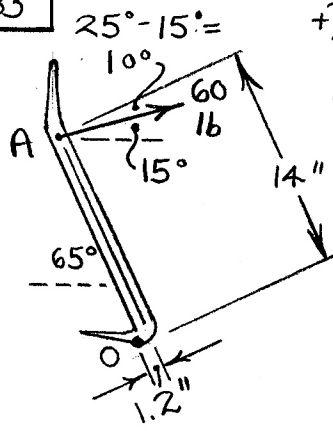
$$\begin{aligned} 60\text{ N} + 2 M_o &= r F_y \\ &= (0.1) (60 \cos 20^\circ) \\ &= \underline{5.64 \text{ N}\cdot\text{m}} \end{aligned}$$

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A diagram showing a force vector of 250 N acting at an angle of 15° to a horizontal line. The horizontal distance from the origin to the point of application is 200 mm, and the vertical distance is 30 mm. The origin is labeled 'O'. The force vector is labeled '250 N' and the angle is '15°'. The horizontal distance is labeled '200 mm' and the vertical distance is labeled '30 mm'. A curved arrow indicates a counter-clockwise moment.

$$M_o = 250 \cos 15^\circ (0.200) - 250 \sin 15^\circ (0.030)$$
$$= 48.30 - 1.941 = \underline{46.4 \text{ N}\cdot\text{m}}$$

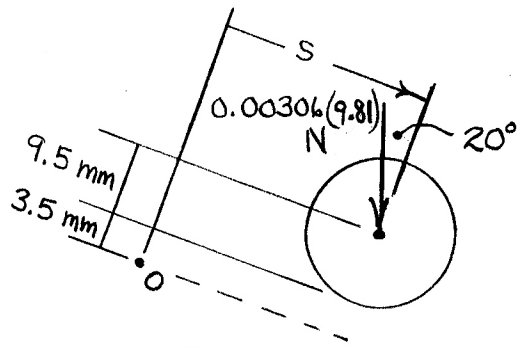
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$$+2 M_o = (60 \cos 10^\circ)(14) + (60 \sin 10^\circ)(1.2) = 840 \text{ lb-in.}$$

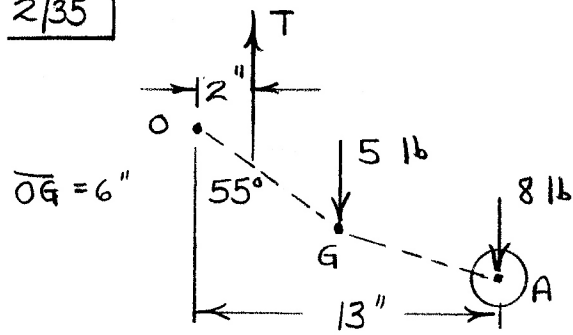
or  $M_o = 70.0 \text{ lb-ft CW}$

2/34



$$\begin{aligned} \text{+2 } M_o &= 0.00306(9.81) [s \cos 20^\circ + (9.5+3.5) \sin 20^\circ] \\ &= 0.1335 + 0.0282s \text{ N}\cdot\text{mm} \quad (s \text{ in mm}) \end{aligned}$$

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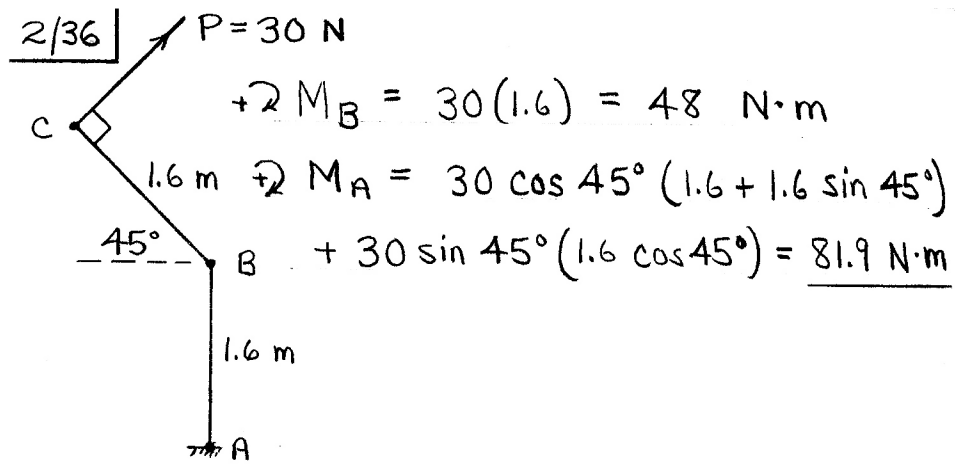


The combined moment about O of the 5-lb and 8-lb weights is

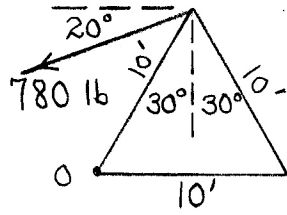
$$\curvearrowright M_o = 5(6 \sin 55^\circ) + 8(13) = 128.6 \text{ lb-in. (CW)}$$

$$\curvearrowright \sum M_o = 0 : -T(2) + 128.6 = 0$$

$$\underline{T = 64.3 \text{ lb}}$$

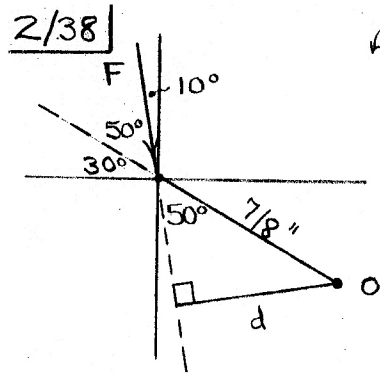


2/37



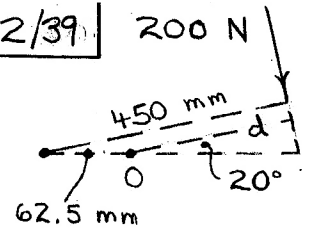
$$\begin{aligned} \curvearrowright M_o &= 780 \cos 20^\circ (10 \cos 30^\circ) \\ &\quad - 780 \sin 20^\circ (5) = \underline{5010 \text{ lb-ft}} \end{aligned}$$





$$\begin{aligned}
 \curvearrowright M_o &= Fd \\
 &= 0.4 \left( \frac{7}{8} \sin 50^\circ \right) \\
 &= \underline{0.268 \text{ lb-in.}}
 \end{aligned}$$

2/39 | 200 N



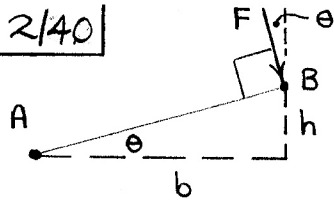
$$d = 450 - 62.5 \cos 20^\circ$$

$$= 391 \text{ mm}$$

$$\rightarrow M = Fd = 200(0.391)$$

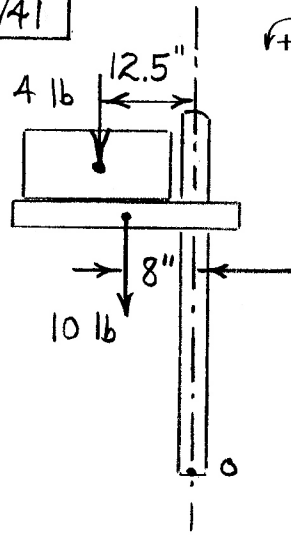
$$= \underline{78.3 \text{ N}\cdot\text{m}}$$

2/40



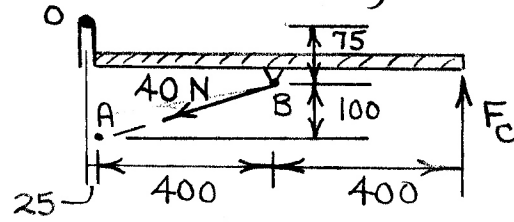
$M_A$  is maximum when  $F$  is perpendicular to  $AB$ .  
Thus  $\theta = \tan^{-1}(h/b)$

2/41



$$\begin{aligned} \curvearrowright M_o &= 4(12.5) + 10(8) \\ &= \underline{130 \text{ lb-in.}} \end{aligned}$$

2/42 (Dim. in mm)



$$AB = \sqrt{400^2 + 100^2} \\ = 412 \text{ mm}$$

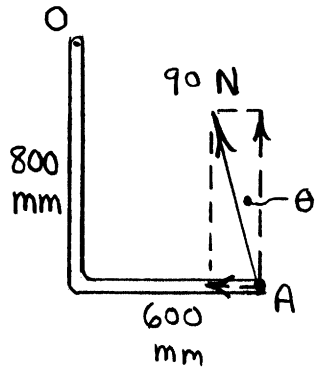
$$\begin{aligned} \sum M_o &= \left( \frac{400}{412} \cdot 40 \right) (75) + \left( \frac{100}{412} \cdot 40 \right) (425) \\ &= 7030 \text{ N}\cdot\text{mm} \quad \text{or} \quad \underline{7.03 \text{ N}\cdot\text{m}} \end{aligned}$$

$$\sum M_o = 0 : -F_c (825) + 7030 = 0$$

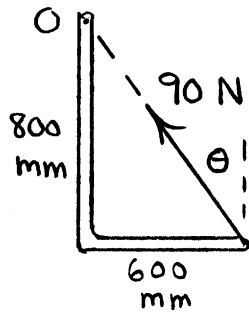
$$\underline{F_c = 8.53 \text{ N}}$$

2/43

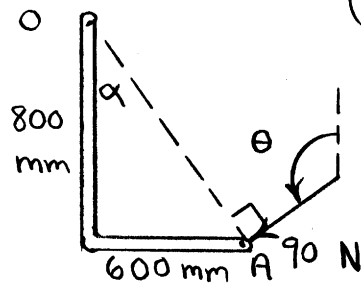
(a)  $\theta = 15^\circ$



$$\begin{aligned} \curvearrow M_o &= 90 \cos 15^\circ (0.6) - 90 \sin 15^\circ (0.8) \\ &= \underline{33.5 \text{ N}\cdot\text{m}} \end{aligned}$$



(b)  $\theta = \tan^{-1}\left(\frac{600}{800}\right) = \underline{36.9^\circ}$   
(or  $217^\circ$ )

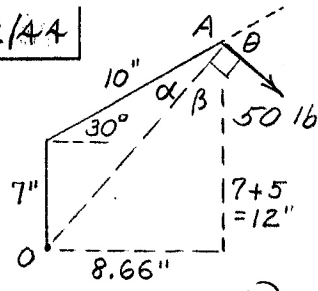


(c)  $F \perp OA$ , so

$$\theta = 90^\circ + \alpha, \quad \alpha = \tan^{-1} \frac{600}{800} = 36.9^\circ$$

$$\text{So } \theta = \underline{126.9^\circ} \text{ (or } \underline{307^\circ})$$

2/44



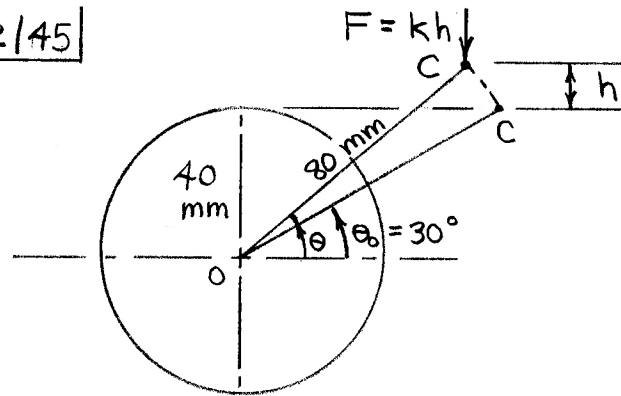
$$\bar{AO} = \sqrt{12^2 + 8.66^2} = 14.80 \text{ in.}$$

$$\frac{7}{\sin \alpha} = \frac{14.80}{\sin 120^\circ}, \sin \alpha = \frac{7}{14.80} \cdot 0.866$$
$$= 0.410$$
$$\alpha = 24.18^\circ$$

$$\theta = 180 - (24.18 + 90) = \underline{65.8^\circ}$$

$$+2) M_0 = 50(14.80) = \underline{740 \text{ lb-in.}}$$

2/45



$$\begin{aligned}
 M_o &= F(80 \cos \theta) = kh(80 \cos \theta) \\
 &= k(80 \sin \theta - 40)(80 \cos \theta) \\
 &= 3200k(2 \sin \theta \cos \theta - \cos \theta) = 3200k(\sin 2\theta - \cos \theta)
 \end{aligned}$$

For maximum  $M_o$ ,  $\frac{dM_o}{d\theta} = 0$  :

$$3200k(2 \cos 2\theta + \sin \theta) = 0$$

$$\Rightarrow 2(1 - 2 \sin^2 \theta) + \sin \theta = 0$$

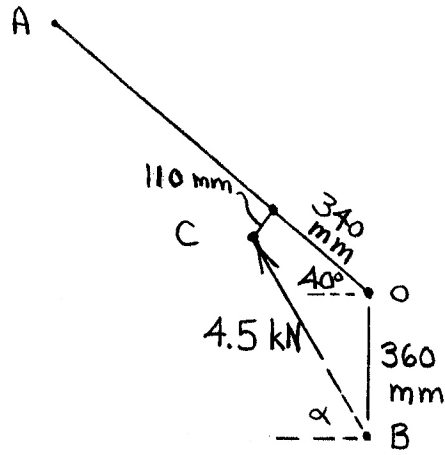
$$4 \sin^2 \theta - \sin \theta - 2 = 0$$

$$\sin \theta = \frac{1 \pm \sqrt{1+32}}{8} = 0.843 \text{ or } -0.593$$

$$\theta = 57.5^\circ \text{ (or } -36.4^\circ \text{ - on dwell part of cam)}$$



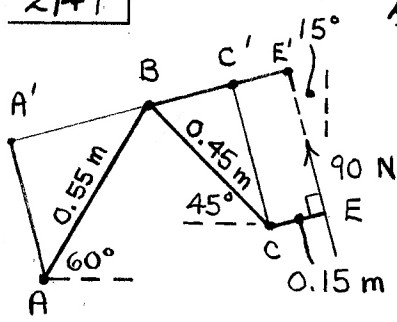
2/46



$$\alpha = \tan^{-1} \left[ \frac{360 + 340 \sin 40^\circ - 110 \sin 50^\circ}{340 \cos 40^\circ + 110 \cos 50^\circ} \right]$$
$$= 56.2^\circ$$

$$+2 M_o = 4.5 (0.360 \cos 56.2^\circ) = \underline{0.902 \text{ kN}\cdot\text{m CW}}$$

2/47



$$\curvearrowright M_C = F(\overline{CE}) = 90(0.15) = \underline{13.50 \text{ N}\cdot\text{m}}$$

$$\begin{aligned} M_B &= F(\overline{BE'}) \\ &= 90(0.15 + 0.45 \sin 30^\circ) \\ &= \underline{33.8 \text{ N}\cdot\text{m}} \end{aligned}$$

$$\begin{aligned} M_A &= F(\overline{A'E'}) \\ &= 90(0.15 + 0.45 \sin 30^\circ + 0.55 \sin 45^\circ) \\ &= \underline{68.8 \text{ N}\cdot\text{m}} \end{aligned}$$

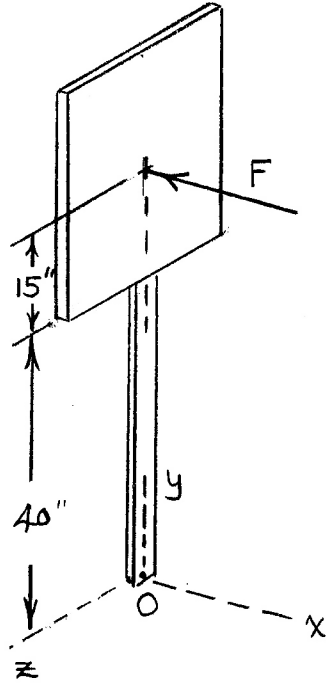
2/48

$$F = pA = 3.5 \frac{(30)(24)}{144}$$

$$= 17.5 \text{ lb}$$

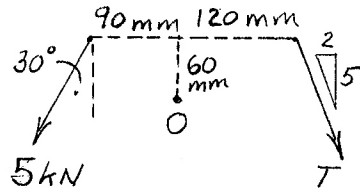
$$\underline{M}_o = \underline{r} \times \underline{F} = \frac{55}{12} \underline{j} \times (-17.5 \underline{i})$$

$$= \underline{80.2 \text{ k}} \text{ lb-ft}$$



$$2/49 \quad M_o = 5[(\cos 30^\circ)90 + (\sin 30^\circ)60]$$

$$- T \left[ \frac{5}{\sqrt{29}}(120) + \frac{2}{\sqrt{29}}(60) \right] = 0$$

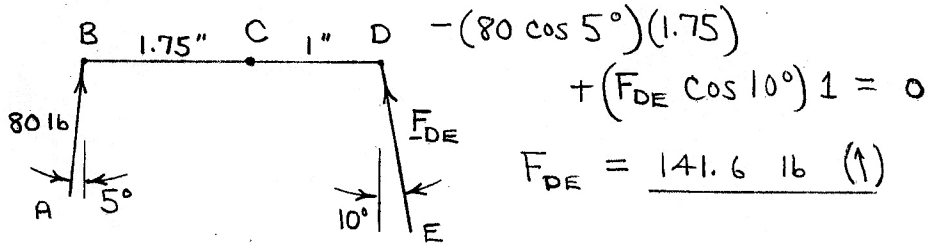


$$539.7 - 133.7T = 0, \quad \underline{T = 4.04 \text{ kN}}$$

$$\sqrt{2^2 + 5^2} = \sqrt{29}$$

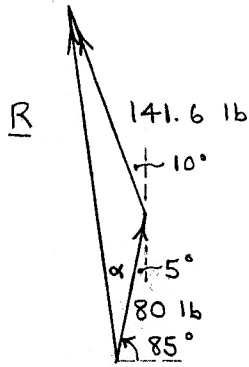
2/50

$$\uparrow \sum M_C = 0 :$$



$$-(80 \cos 5^\circ)(1.75) + (F_{DE} \cos 10^\circ) 1 = 0$$

$$F_{DE} = \underline{141.6 \text{ lb}} \quad (\uparrow)$$



Law of cosines :

$$R^2 = 80^2 + 141.6^2 - 2(80)(141.6) \cos 165^\circ$$

$$R = \underline{220 \text{ lb}}$$

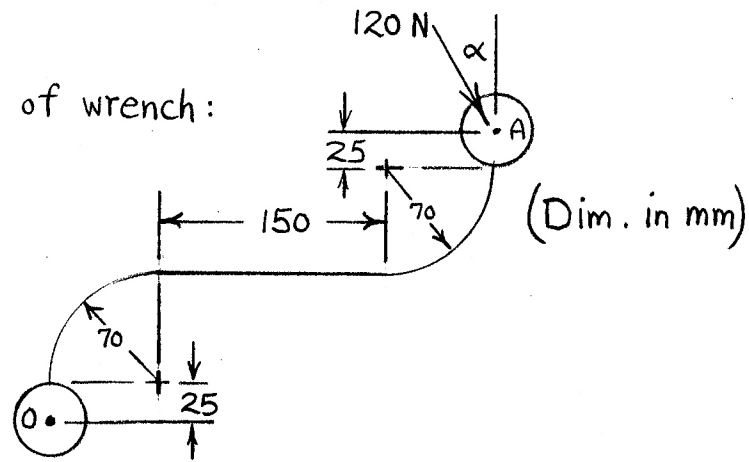
Law of sines :

$$\frac{\sin \alpha}{141.6} = \frac{\sin 165^\circ}{219.9}$$

$$\alpha = 9.60^\circ, \quad \theta = 85^\circ + \alpha = \underline{94.6^\circ}$$

2/51

Elements of wrench:



$$\alpha = 30^\circ:$$

$$\begin{aligned} \rightarrow M_o &= 120 \cos 30^\circ [70 + 150 + 70] \\ &\quad + 120 \sin 30^\circ [25 + 70 + 70 + 25] = 41\,500 \text{ N}\cdot\text{mm} \end{aligned}$$

$$\text{or } \underline{M_o = 41.5 \text{ N}\cdot\text{m CW}}$$

For maximum  $M_o$ :

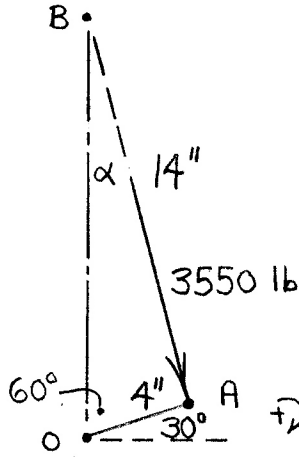
$$\alpha = \tan^{-1} \left[ \frac{25 + 70 + 25 + 70}{70 + 150 + 70} \right] = \underline{33.2^\circ}$$

$$\begin{aligned} (M_o)_{\max} &= 120 \sqrt{(25 + 70 + 25 + 70)^2 + (70 + 150 + 70)^2} \\ &= 41\,600 \text{ N}\cdot\text{mm} \text{ or } \underline{41.6 \text{ N}\cdot\text{m CW}} \end{aligned}$$

2/52

Law of sines:  $\frac{4}{\sin \alpha} = \frac{14}{\sin 60^\circ}$

$$\alpha = 14.33^\circ$$



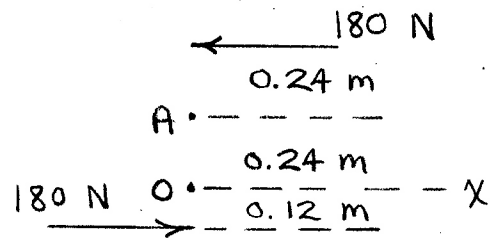
$$\begin{aligned} \overline{BO} &= 14 \cos 14.33^\circ + 4 \cos 60^\circ \\ &= 15.56 \text{ in.} \end{aligned}$$

Consider 3550 lb acting at B:

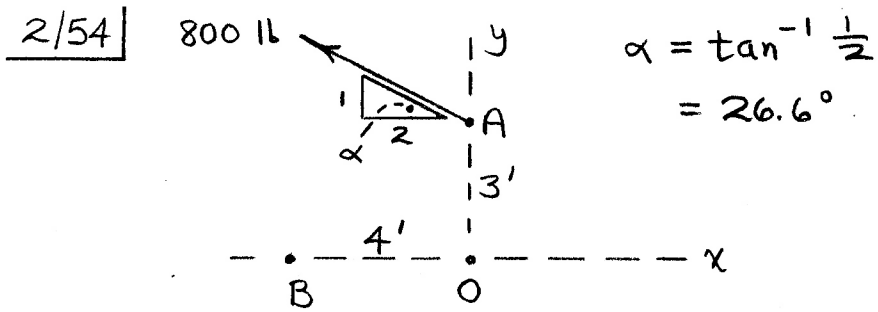
$$\begin{aligned} +\curvearrowright M_o &= (3550 \sin 14.33^\circ)(15.56) \\ &= \underline{13,670 \text{ lb-in.}} \end{aligned}$$

$$\text{(or } M_o = \underline{1139 \text{ lb-ft)}} \text{)$$

$$\begin{aligned}
 \underline{2/53} \quad \curvearrowright M &= M_o = M_A = Fd \\
 &= 180 (0.24 + 0.24 + 0.12) \\
 &= \underline{108 \text{ N}\cdot\text{m CCW}}
 \end{aligned}$$







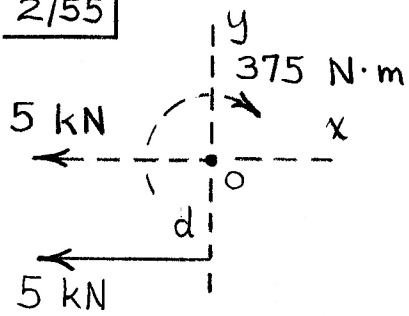
At O :  $F = 800 \text{ lb}$   $26.6^\circ$

$\checkmark + M_o = 800 \frac{2}{\sqrt{5}} (3) = \underline{2150 \text{ lb-ft CCW}}$

At B :  $F = 800 \text{ lb}$   $26.6^\circ$

$\checkmark + M_B = 800 \frac{2}{\sqrt{5}} (3) + 800 \frac{1}{\sqrt{5}} (4) = \underline{3580 \text{ lb-ft CCW}}$

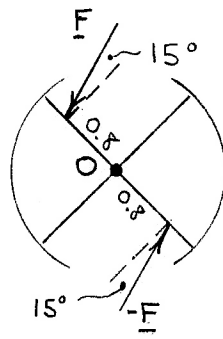
2/55



$$\begin{aligned} \oplus M = Fd: 375 &= 5000d \\ d &= 0.075 \text{ m} \end{aligned}$$

$$\therefore \underline{y = -75 \text{ mm}}$$

2/56

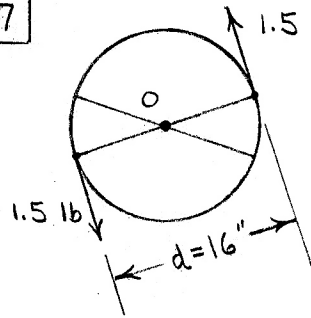


$$\curvearrowright M_o = \sum Fd$$

$$25 = 2 F(\cos 15^\circ)(0.8)$$

$$F = \underline{16.18 \text{ N}}$$

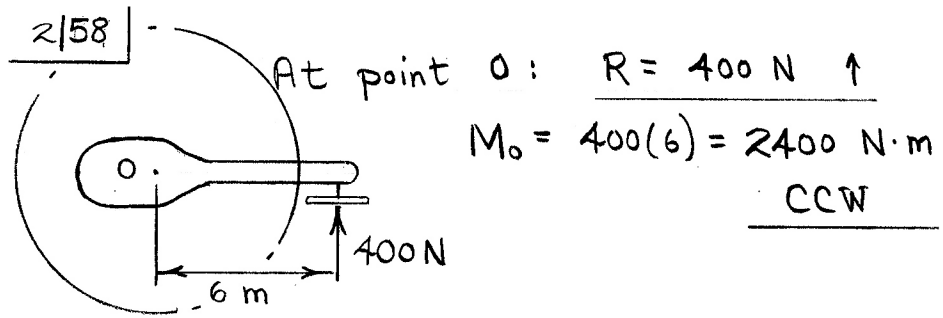
2/57



$$\begin{aligned} \Rightarrow M &= Fd = (1.5)(16) \\ &= \underline{24 \text{ lb-in.}} \end{aligned}$$

For constant forces:  
Increasing  $d$  increases  
 $M$  and increases

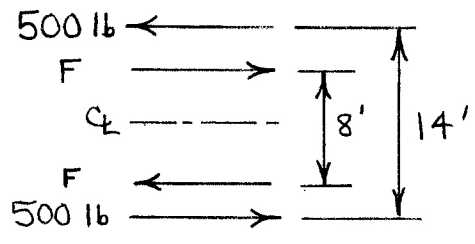
circumferential hand movement. Decreasing  
 $d$  decreases  $M$  and decreases hand motion.



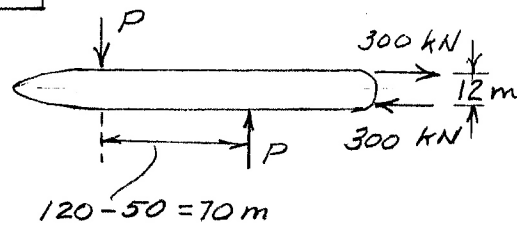
2/59 |

$$\sum M = 500(14) - F(8) = 0$$

$$F = 875 \text{ lb}$$



2/60



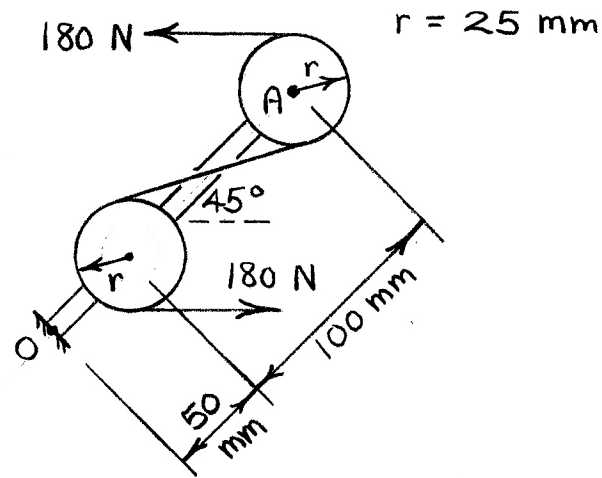
$$70P = 300(12)$$

$$P = 51.4 \text{ kN}$$

$$\underline{2/61} \quad M = Fd, \quad F = \frac{4000 \times 12}{4} = \underline{12,000 \text{ lb}}$$



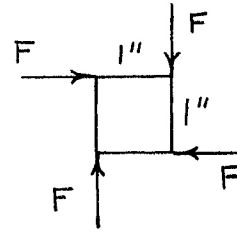
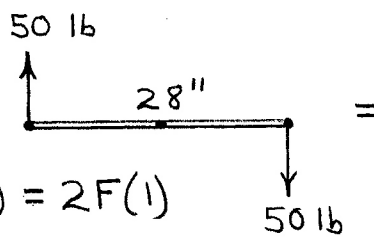
2/62



The system at O is a couple.

$$\begin{aligned}\sum M &= Fd = 180(100 \sin 45^\circ + 25 + 25) \\ &= 21700 \text{ N}\cdot\text{mm} \text{ or } \underline{21.7 \text{ N}\cdot\text{m} \text{ CCW}}\end{aligned}$$

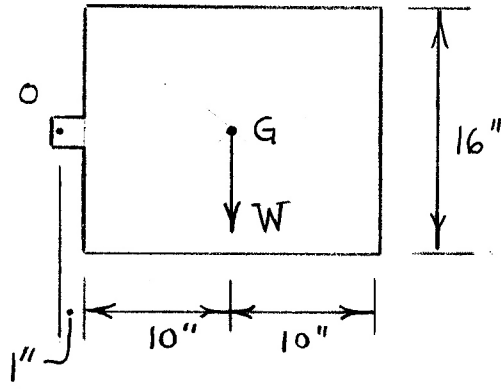
2/63



$$M = 50(28) = 2F(1)$$

$$\underline{F = 700 \text{ lb}}$$

2/64



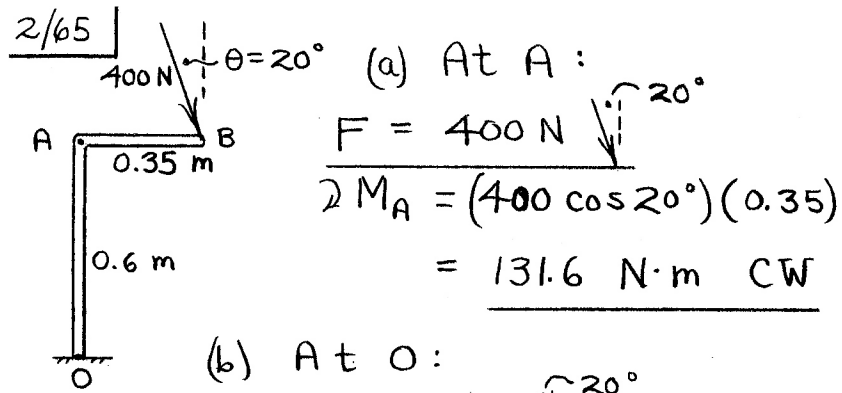
The weight  $W$  of the door is

$$W = (\rho g) V = 489 (20)(16)\left(\frac{1}{8}\right) \frac{1}{12^3} = 11.32 \text{ lb}$$

The equivalent force-couple system at  $O$  is

$$\begin{cases} R = 11.32 \text{ lb down} \\ M_o = 11.32(11) = 124.5 \text{ lb-in. CW} \end{cases}$$

Assumption: Neglect weight of small tab of steel near hinge.



$$F = 400 \text{ N}$$

$$\sum M_A = (400 \cos 20^\circ)(0.35)$$

$$= \underline{131.6 \text{ N}\cdot\text{m CW}}$$

(b) At O:

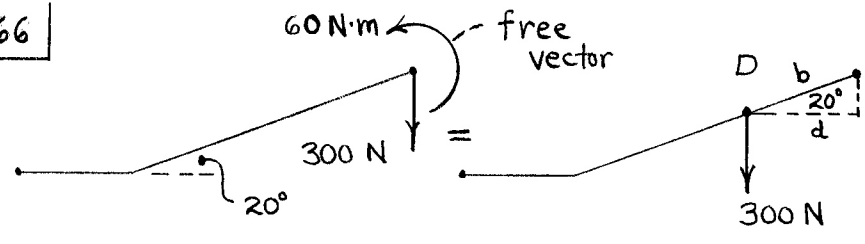
$$F = 400 \text{ N}$$

$$\sum M_o = 400 \cos 20^\circ (0.35) + 400 \sin 20^\circ (0.6)$$

$$= \underline{214 \text{ N}\cdot\text{m CW}}$$

Part (a) and (b) results are the same if  $\theta = 0$  or  $180^\circ$ .

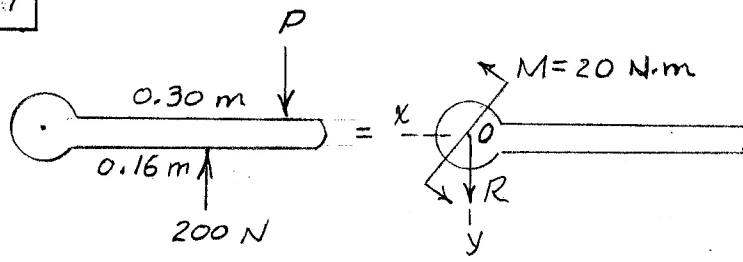
2/66



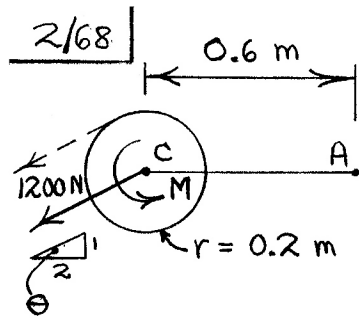
$$M = Fd: \quad 60 = 300 (b \cos 20^\circ), \quad b = 0.213 \text{ m}$$

or  $b = 213 \text{ mm}$

2/67



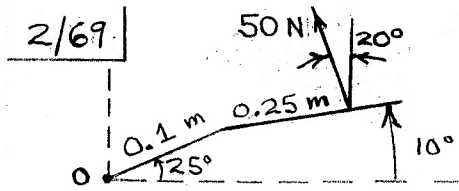
$$M = \sum Fd; \quad 20 = 200(0.16) - 0.30P, \quad P = 40\text{ N}$$
$$\underline{P = 40\text{ j N}}$$
$$\underline{R = -200\text{ j} + 40\text{ j} = -160\text{ j N}}$$



$$\text{f) } M = 1200(0.2) = 240 \text{ N}\cdot\text{m}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$$

$$\text{f) } M_A = 1200 \sin 26.6 (0.6) + 240 = \underline{562 \text{ N}\cdot\text{m}}$$



Use principle of moments.

$$\begin{aligned} \uparrow \Sigma M_o &= 50 \cos 20^\circ [0.1 \cos 25^\circ + 0.25 \cos 10^\circ] \\ &+ 50 \sin 20^\circ [0.1 \sin 25^\circ + 0.25 \sin 10^\circ] \\ &= 17.29 \text{ N}\cdot\text{m} \end{aligned}$$

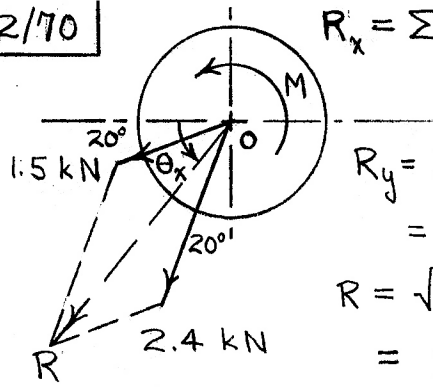
Force - Couple System at O:

$$\begin{cases} R = 50 \text{ N} \quad \nearrow 110^\circ \\ M_o = 17.29 \text{ N}\cdot\text{m} \quad \curvearrowright \end{cases}$$


---



2/70



$$R_x = \sum F_x = 1.5 \cos 20^\circ + 2.4 \sin 20^\circ$$
$$= 2.23 \text{ kN}$$

$$R_y = \sum F_y = 1.5 \sin 20^\circ + 2.4 \cos 20^\circ$$
$$= 2.77 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.23^2 + 2.77^2}$$
$$= \underline{3.56 \text{ kN}}$$

$$\theta_x = \tan^{-1} \left( \frac{2.77}{2.23} \right) = \underline{51.1^\circ}$$

$$M = 1.5 (200) \cos 20^\circ - 2.4 (120) \cos 20^\circ$$
$$= \underline{11.28 \text{ N}\cdot\text{m} \text{ CCW}}$$

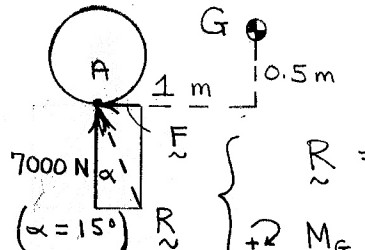
Initial rotation would be CCW.

2/71

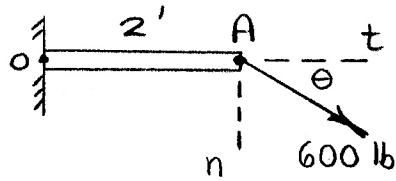
$$\tan 15^\circ = \frac{F}{7000}$$

$$F = 1876 \text{ N}$$

At G :


$$\left. \begin{array}{l} \vec{R} = \sum \vec{F} = \underline{7250 \text{ N}} \\ +\curvearrowright M_G = 7000(1) + 1876(0.5) \\ = \underline{7940 \text{ N}\cdot\text{m}} \end{array} \right\} \begin{array}{l} \nearrow 105^\circ \end{array}$$

2/72



The equivalent force-couple system at O is

$$R_t = 600 \cos \theta \quad (lb)$$

$$R_n = 600 \sin \theta \quad (lb)$$

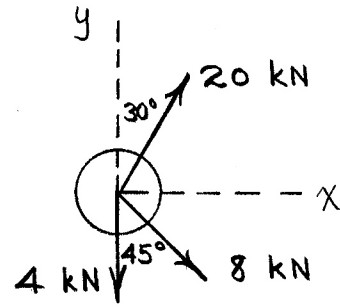
$$+ \curvearrowright M_o = 2(600 \sin \theta) = 1200 \sin \theta \quad (lb-ft)$$

Constraints:

$$\begin{cases} 600 \cos \theta \leq 550, & \theta \geq 23.6^\circ \\ 600 \sin \theta \leq 550, & \theta \leq 66.4^\circ \\ 1200 \sin \theta < 1000, & \theta \leq 56.4^\circ \end{cases}$$

All considered,  $23.6^\circ \leq \theta \leq 56.4^\circ$

2/73



$$R_x = \sum F_x = 20 \sin 30^\circ + 8 \sin 45^\circ = 15.66 \text{ kN}$$

$$R_y = \sum F_y = 20 \cos 30^\circ - 8 \cos 45^\circ - 4 = 7.66 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{17.43 \text{ kN}}$$

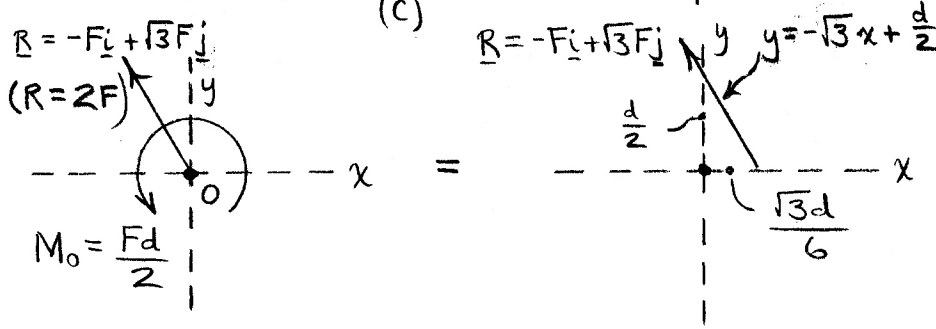
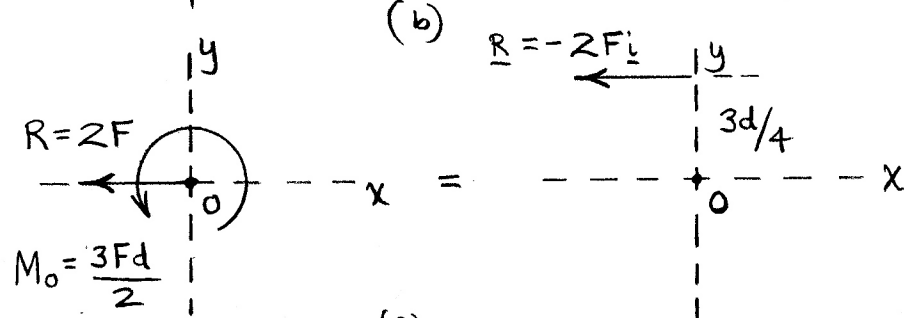
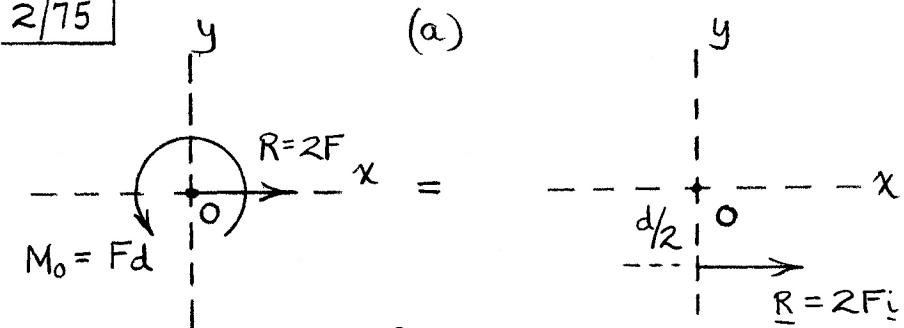
$$\theta_x = \tan^{-1} (R_y/R_x) = \underline{26.1^\circ}$$

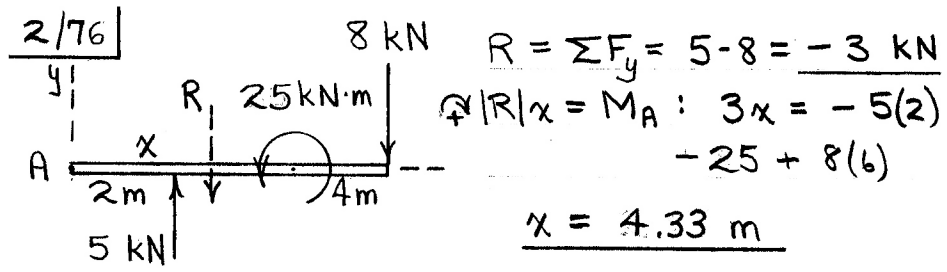
$$\underline{2/74} \quad (a) \quad \underline{R} = -2F\underline{j}, \quad \underline{M}_o = \underline{0}$$

$$(b) \quad \underline{R} = \underline{0}, \quad \underline{M}_o = Fd\underline{k} \quad (+\underline{k} \text{ is out})$$

$$(c) \quad \underline{R} = -F\underline{i} + F\underline{j}, \quad \underline{M}_o = \underline{0}$$

2/75





$$\frac{2}{77} \quad M_o = 0, \text{ so}$$

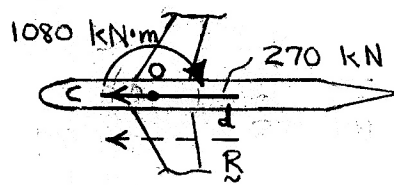
$$\curvearrowright M - 400(0.150 \cos 30^\circ) - 320(0.300) = 0$$

$$\underline{M = 148.0 \text{ N}\cdot\text{m}}$$



2/78 | Force - Couple system at point O:

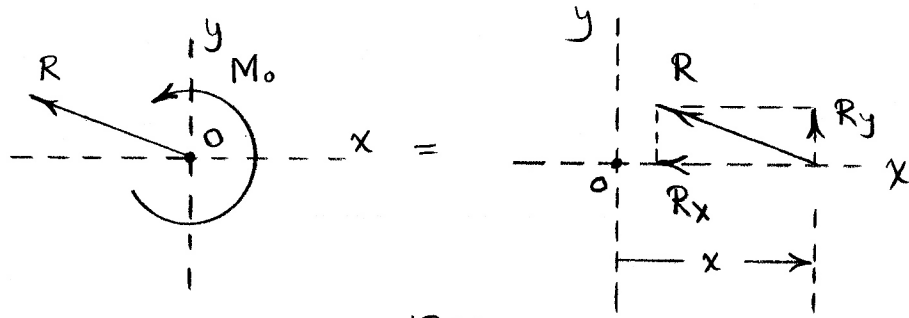
$$\begin{cases} R = 3(90) = 270 \text{ kN} (\leftarrow) \\ +2 M_o = 12(90) = 1080 \text{ kN}\cdot\text{m} \end{cases}$$



$$d = \frac{M_o}{R} = \frac{1080}{270} \\ = \underline{4 \text{ m}}$$

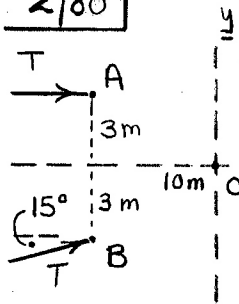
$$\underline{2/79} \quad \underline{\underline{R = -50\hat{i} + 20\hat{j} \text{ lb}}}$$

$$\curvearrowright M_o = -40(10) + 60(20) + 50(10) = 1300 \text{ lb-in.}$$



$$R_y x = M_o, \quad x = \frac{1300}{20} = \underline{\underline{65 \text{ in. (off pipe)}}}$$

2/80



$$\underline{R} = \Sigma \underline{F} = T \underline{i} + T(\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j})$$

$$= 1.966T \underline{i} + 0.259T \underline{j}$$

$$+2M_o = 3T - T \cos 15^\circ (3)$$

$$+ T \sin 15^\circ (10) = \underline{2.69T}$$

$$-R_y x = M_o: -0.259T(x) = 2.69T$$

$$\underline{x = -10.39 \text{ m}}$$

$$\frac{2}{81} \quad R_x = \sum F_x = 2 \cos 70^\circ + 1.2 \left(\frac{4}{5}\right) = 1.644 \text{ kN}$$

$$R_y = \sum F_y = 2 \sin 70^\circ - 1.2 \left(\frac{3}{5}\right) = 1.159 \text{ kN}$$

$$\curvearrowright M_A = -2 \cos 70^\circ (0.15) + 2 \sin 70^\circ (1.5 + 0.5)$$

$$+ 1.2 \left(\frac{4}{5}\right) (0.15) - 1.2 \left(\frac{3}{5}\right) (1.5) - 0.5$$

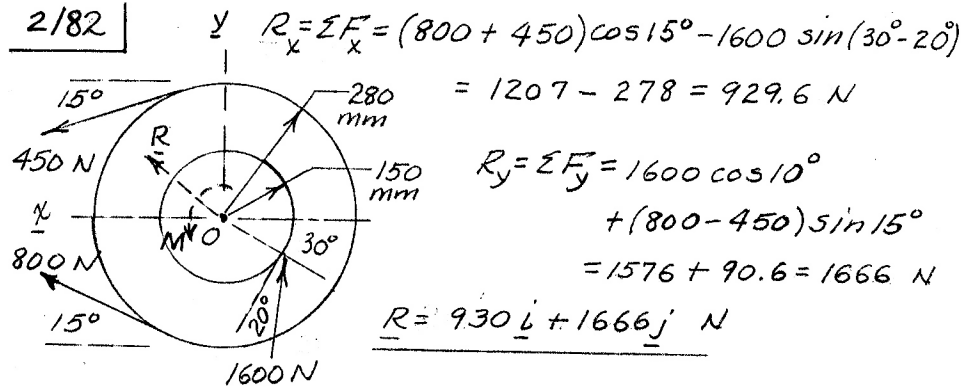
$$= 2.22 \text{ kN}\cdot\text{m} \quad \text{CCW}$$

So the force-couple system is

$$\left\{ \begin{array}{l} \underline{R} = 1.644 \underline{i} + 1.159 \underline{j} \text{ kN} \\ \underline{M}_A = 2.22 \text{ kN}\cdot\text{m} \quad \text{CCW} \end{array} \right.$$

$$\left\{ \begin{array}{l} \underline{R} = 1.644 \underline{i} + 1.159 \underline{j} \text{ kN} \\ \underline{M}_A = 2.22 \text{ kN}\cdot\text{m} \quad \text{CCW} \end{array} \right.$$

2/82



$$M = \sum M_O \uparrow); M = 1600 \cos 20^\circ (0.150) + (450 - 800) 0.280$$

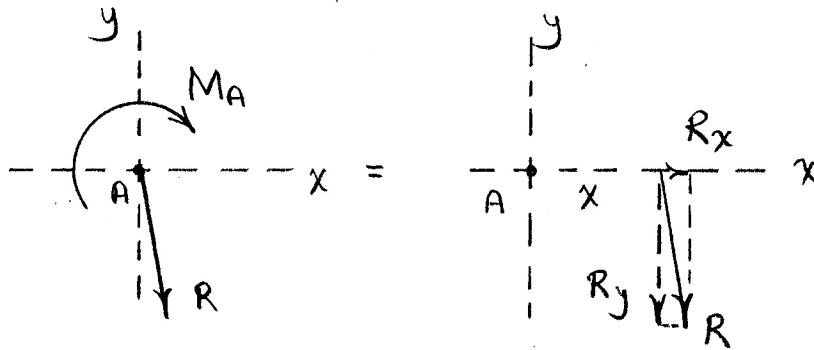
$$= 225.5 - 98.0 = \underline{127.5 \text{ N}\cdot\text{m} \text{ CCW}}$$

so unit is speeding up in CCW dir.

2/83 | Equivalent force-couple system at A:

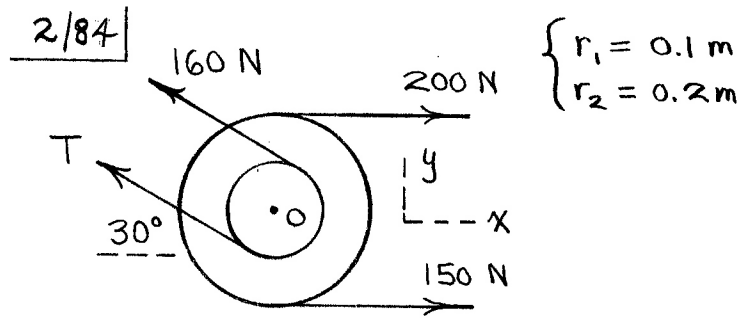
$$\begin{aligned}\underline{R} &= -2500\underline{j} - 1200\underline{j} + 800(\sin 30^\circ\underline{i} + \cos 30^\circ\underline{j}) \\ &= \underline{400\underline{i} - 3010\underline{j} \text{ lb}}\end{aligned}$$

$$\begin{aligned}\Rightarrow M_A &= 2500(4) + 1200(4 + 4\cos 30^\circ + 3) \\ &\quad - 800\sin 30^\circ(2\sin 30^\circ) - 800\cos 30^\circ(4 + 2\cos 30^\circ) \\ &= \underline{18,190 \text{ lb-ft CW}}\end{aligned}$$



Condition:  $x|R_y| = M_A$

$$x = \frac{18,190}{3010} = \underline{6.05 \text{ ft}}$$



$$+2 M_o = 0 : 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$$

$$\underline{T = 60 \text{ N}}$$

$$R_x = \sum F_x = 200 + 150 - (160 + 60) \cos 30^\circ = 159.5 \text{ N}$$

$$R_y = \sum F_y = (160 + 60) \sin 30^\circ = 110 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7 \text{ N}}$$

$$\theta = \tan^{-1}(R_y/R_x) = \underline{34.6^\circ}$$

$$\frac{2}{85} \mid \underline{R} = 45\underline{i} - 15\underline{j} \text{ lb}$$

$$\Rightarrow \underline{M}_A = 25(30) + 15(60) = 1650 \text{ lb-in.}$$

$$\text{or } \underline{M}_A = -1650\underline{k} \text{ lb-in.}$$

For final line of action,  $\underline{r} \times \underline{R} = \underline{M}_A$

$$(x\underline{i} + y\underline{j}) \times (45\underline{i} - 15\underline{j}) = -1650\underline{k}$$

$$\Rightarrow -15x - 45y = -1650 \text{ or } \underline{y = -\frac{1}{3}x + \frac{110}{3}}$$

(Axis intercepts:  $x = 110''$ ,  $y = 110/3''$ )

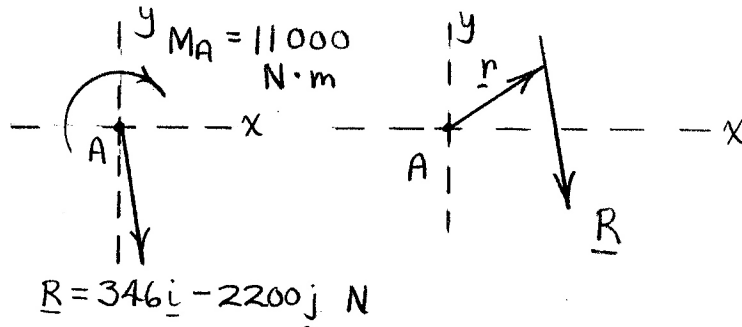




2/86 | Equivalent force-couple system at A:

$$\underline{R} = \sum \underline{F} = [-2(250) - 3(500)]\underline{j} + 400[\cos 30^\circ \underline{i} - \sin 30^\circ \underline{j}]$$
$$= 346 \underline{i} - 2200 \underline{j} \text{ N}$$

$$\curvearrowright M_A = 500[2.5 + 5 + 7.5] + 250[10] + 400(2.5)$$
$$= 11,000 \text{ N}\cdot\text{m} \text{ CW}$$

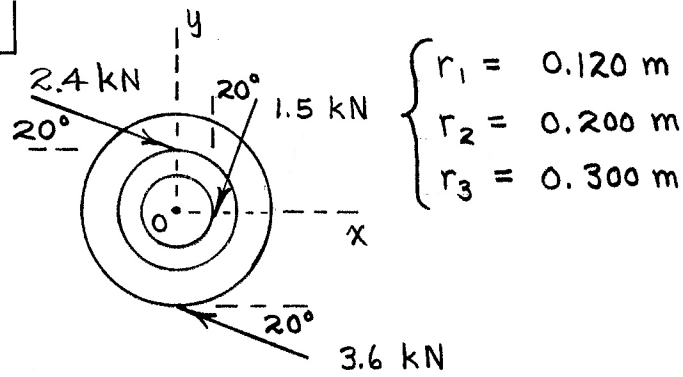


Condition:  $\underline{M}_A = \underline{r} \times \underline{R}$

$$-11000 \underline{k} = (x \underline{i} + y \underline{j}) \times (346 \underline{i} - 2200 \underline{j})$$
$$= (-2200x - 346y) \underline{k}$$

Set  $y = 0$  & obtain  $\underline{x} = 5 \text{ m}$  (!)

2/87



$$\underline{R} = \sum \underline{F} = 2.4 (\cos 20^\circ \underline{i} - \sin 20^\circ \underline{j}) + 1.5 (-\sin 20^\circ \underline{i} - \cos 20^\circ \underline{j}) + 3.6 (-\cos 20^\circ \underline{i} + \sin 20^\circ \underline{j}) = -1.641 \underline{i} - 0.999 \underline{j} \text{ kN}$$

$$\sum M_o = (2.4(0.2) + 1.5(0.12) + 3.6(0.3)) \cos 20^\circ = 1.635 \text{ kN}\cdot\text{m}$$

$$\underline{r} \times \underline{R} = M_o : (x \underline{i} + y \underline{j}) \times (-1.641 \underline{i} - 0.999 \underline{j}) = -1.635$$

$$\Rightarrow -0.999x + 1.641y = -1.635$$

$$\text{Axis intercepts : } \underline{x = 1.637 \text{ m}, y = -0.997 \text{ m}}$$

$$\underline{2/88} \quad \underline{R} = \sum \underline{F} = 400(\cos 45^\circ \underline{i} - \sin 45^\circ \underline{j}) + 500(\sin 15^\circ \underline{i} - \cos 15^\circ \underline{j})$$

$$= 412 \underline{i} - 766 \underline{j} \text{ N}$$

$$\Rightarrow M_o = (500 - 400)(0.060) = 6 \text{ N}\cdot\text{m}$$

For the line of action of the standalone force:

$$\underline{r} \times \underline{R} = M_o$$

$$(x \underline{i} + y \underline{j}) \times (412 \underline{i} - 766 \underline{j}) = -6 \underline{k}$$

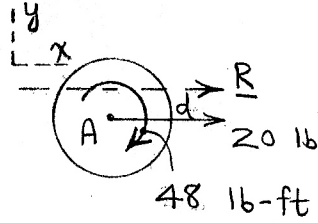
$$-766x - 412y = -6$$

$$\left\{ \begin{array}{l} \text{For } x=0: \quad y = 0.01455 \text{ m or } \underline{y = 14.55 \text{ mm}} \\ \text{For } y=0: \quad x = 0.00783 \text{ m or } \underline{x = 7.83 \text{ mm}} \end{array} \right.$$

2/89 | Force - Couple System at point A:

$$\underline{R} = \sum \underline{F} = -500\underline{j} + 60\underline{i} - 100\underline{j} - 40\underline{i} + 600\underline{j}$$
$$= \underline{20\underline{i} \quad 16\underline{j}}$$

$$\curvearrowright M_A = 2 - 40\left(\frac{15}{12}\right) = -48 \text{ lb-ft}$$



$$Rd = M_A$$

$$d = \frac{M_A}{R} = \frac{48}{20}$$

$$= \underline{2.40 \text{ ft}}$$

2/90 | Use  $\begin{matrix} y \\ \uparrow \\ \rightarrow x \end{matrix}$  system at G:

$$\underline{R} = \Sigma \underline{F} = (80 + 40 + 40 + 50 \sin 30^\circ) \underline{i} \\ + (50 \cos 30^\circ + 70) \underline{j}$$

$$= \underline{185 \underline{i} + 113.3 \underline{j} \text{ lb}}$$

$$M_G = 70(66) + 50 \sin 30^\circ (36) = 5520 \text{ lb-in.} \\ = \underline{460 \text{ lb-ft}} \quad (\curvearrowright)$$


For line of action of resultant:

$$\underline{r} \times \underline{R} = \underline{M}_G$$

$$(x \underline{i} + y \underline{j}) \times (185 \underline{i} + 113.3 \underline{j}) = 460 \underline{k}$$

$$113.3x - 185y = 460$$

$$\underline{x = 4.06 \text{ ft}} \quad \text{when } y = 0.$$

2/91 | For a zero force-couple system  
at point O: 

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30^\circ + F_D \sin 30^\circ) \underline{i} \\ + (50 - 10 - 100 - 50 + F_B \\ + F_C \cos 30^\circ + F_D \cos 30^\circ) \underline{j} = \underline{0}$$

$$\Rightarrow F_C = F_D = F$$

$$\Sigma M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) \\ - 50(2.5) + 2F \cos 30^\circ (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}}, \quad \underline{F_B = 98.9 \text{ N}}$$

2/92 | Use the x-y coordinates of the figure:

$$\underline{R} = \Sigma \underline{F} = 20(\sin 5^\circ \underline{i} + \cos 5^\circ \underline{j}) + 40(\sin 15^\circ \underline{i} - \cos 15^\circ \underline{j}) = \underline{12.10 \underline{i} - 18.71 \underline{j}} \text{ lb}$$

$$\begin{aligned} \underline{M}_o &= 6.5(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j}) \times 40(\sin 15^\circ \underline{i} - \cos 15^\circ \underline{j}) \\ &+ 6.5(-\cos 30^\circ \underline{i} - \sin 30^\circ \underline{j}) \times 20(\sin 5^\circ \underline{i} + \cos 5^\circ \underline{j}) \\ &= \underline{-358 \underline{k}} \text{ lb-in.} \end{aligned}$$

Condition for line of action of resultant:

$$\underline{r} \times \underline{R} = \underline{M}_o$$

$$(x \underline{i} + y \underline{j}) \times (12.10 \underline{i} - 18.71 \underline{j}) = -358 \underline{k}$$

$$(-18.71x - 12.10y) \underline{k} = -358 \underline{k}$$

$$\text{or } -18.71x - 12.10y = -358$$

$$\text{(or } \underline{y = -1.547x + 29.6 \text{ in.}})$$

$$\begin{aligned} \underline{2/93} \quad \underline{T} &= T \underline{n}_{AB} \\ \underline{T} &= 12 \left[ \frac{35\underline{i} - 25\underline{j} - 60\underline{k}}{\sqrt{35^2 + 25^2 + 60^2}} \right] \\ &= \underline{5.69\underline{i} - 4.06\underline{j} - 9.75\underline{k} \text{ kN}} \end{aligned}$$



2/94

$$\underline{F} = F \underline{n}_{AB} = 400 \left[ \frac{-0.2\underline{i} + 0.5\underline{j} - 0.1\underline{k}}{\sqrt{0.2^2 + 0.5^2 + 0.1^2}} \right]$$

$$= \underline{-146.1\underline{i} + 365\underline{j} - 73.0\underline{k} \text{ N}}$$

Projection onto  $x$ -axis  $\underline{F}_x = \underline{-146.1 \text{ N}}$

$$\begin{aligned} \underline{2/95} \quad \underline{\underline{T}} &= T \frac{\underline{\underline{CD}}}{\underline{\underline{CD}}} = 1.2 \frac{1.5\mathbf{i} + 3\mathbf{j} - 4.5\mathbf{k}}{\sqrt{1.5^2 + 3^2 + 4.5^2}} \\ &= \underline{\underline{0.321\mathbf{i} + 0.641\mathbf{j} - 0.962\mathbf{k} \text{ kN}}} \end{aligned}$$

The two indicated coordinate systems are equivalent for the question at hand.

$$\begin{aligned} \underline{2/96} \quad T_{GF} &= \underline{T} \cdot \underline{n}_{GF} \\ &= (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) \cdot \frac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}} \\ &= \underline{0.978 \text{ kN}} \end{aligned}$$

$$\frac{2/97}{|} \underline{F}_{hor} = 5 \cos 50^\circ = 3.21 \text{ kN}$$

$$\left\{ \begin{array}{l} F_x = 3.21 \cos 65^\circ = 1.358 \text{ kN} \\ F_y = 3.21 \sin 65^\circ = 2.91 \text{ kN} \\ F_z = 5 \sin 50^\circ = 3.83 \text{ kN} \end{array} \right.$$

$$\left\{ \begin{array}{l} F_y = 3.21 \sin 65^\circ = 2.91 \text{ kN} \\ F_z = 5 \sin 50^\circ = 3.83 \text{ kN} \end{array} \right.$$

$$\text{So } \underline{F} = 1.358\underline{i} + 2.91\underline{j} + 3.83\underline{k} \text{ kN}$$

$$\text{Projection onto y-axis: } \underline{F}_y = 2.91 \text{ kN}$$

Projection onto OB:

$$F_{OB} = \underline{F} \cdot \underline{n}_{OB}$$

$$= (1.358\underline{i} + 2.91\underline{j} + 3.83\underline{k}) \cdot (\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$

$$= \underline{2.63 \text{ kN}}$$

$$2/98 \quad \overline{AB} = \sqrt{1.2^2 + 0.5^2 + 0.7^2} = 1.304 \text{ m}$$

$$\left. \begin{array}{l} l = -1.2/1.304 = -0.920 \\ m = 0.5/1.304 = 0.383 \\ n = 0.1/1.304 = 0.0767 \end{array} \right\} \underline{T} = 2 \cdot (-0.920\underline{i} + 0.383\underline{j} + 0.0767\underline{k})$$

kN

2/99 | The coordinates of point B are  
 $(x_B, y_B, z_B) = (1.6, -0.8 \sin 30^\circ, 0.8 \cos 30^\circ)$   
 $= (1.6, -0.4, 0.693) \text{ m}$

The position vector  $\underline{BC}$  is

$$\underline{BC} = (0-1.6)\underline{i} + (0.7-(-0.4))\underline{j} + (1.2-0.693)\underline{k}$$
$$= -1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k} \text{ m}$$

The unit vector which characterizes  $\underline{BC}$  is

$$\underline{n}_{BC} = \frac{-1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k}}{\sqrt{1.6^2 + 1.1^2 + 0.507^2}}$$
$$= -0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k}$$

Then  $\underline{T} = T \underline{n}_{BC}$

$$= 750 (-0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k})$$
$$= \underline{-598\underline{i} + 411\underline{j} + 189.5\underline{k} \text{ N}}$$

$$\frac{2}{100} \quad \underline{F} = F \underline{n}_{AM} = F \frac{AM}{AM}$$

$$F = 500 \left[ \frac{16\underline{i} - 10\underline{j} + 8\underline{k}}{\sqrt{16^2 + 10^2 + 8^2}} \right]$$

$$= 500 [0.781\underline{i} - 0.488\underline{j} + 0.390\underline{k}] \text{ lb}$$

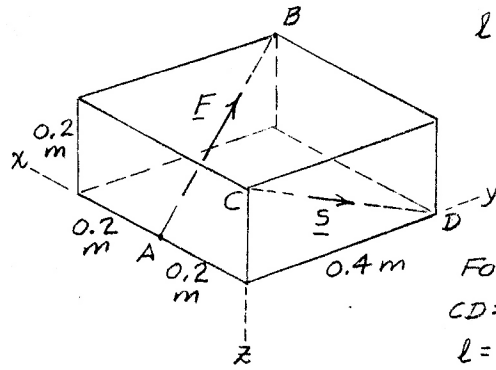
$$\underline{F}_x = 390 \text{ lb} , \quad \underline{F}_y = -244 \text{ lb} , \quad \underline{F}_z = 195.2 \text{ lb}$$

2/101 |  $F = 2 \text{ kN}$ ; For  $\underline{F}$ ,  $AB = \sqrt{0.2^2 + 0.4^2 + 0.2^2} = \sqrt{0.24} \text{ m}$

$$l = \frac{0.2}{\sqrt{0.24}} = \frac{1}{\sqrt{6}}, \quad m = \frac{0.4}{\sqrt{0.24}} = \frac{2}{\sqrt{6}}$$

$$n = -\frac{1}{\sqrt{6}}$$

$$\underline{F} = \frac{2}{\sqrt{6}} (\underline{i} + 2\underline{j} - \underline{k}) \text{ kN}$$



For unit vector  $\underline{s}$   
 $CD = \sqrt{0.2^2 + 0.4^2} = \sqrt{0.20} \text{ m}$

$$l = 0, \quad m = \frac{0.4}{\sqrt{0.2}} = \frac{2}{\sqrt{5}}, \quad n = \frac{1}{\sqrt{5}}$$

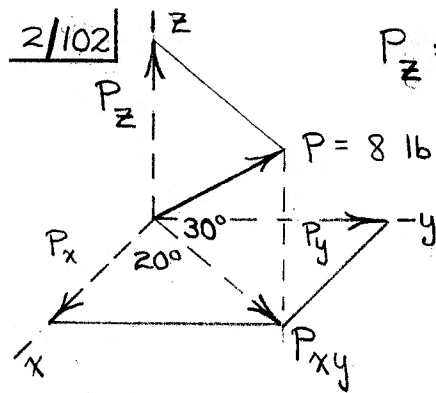
$$\underline{s} = \frac{1}{\sqrt{5}} (2\underline{j} + \underline{k})$$

$$F_{CD} = \underline{F} \cdot \underline{s} = \frac{2}{\sqrt{6}} (\underline{i} + 2\underline{j} - \underline{k}) \cdot \frac{1}{\sqrt{5}} (2\underline{j} + \underline{k})$$

$$= \frac{2}{\sqrt{30}} (4 - 1) = \frac{6}{\sqrt{30}} = \sqrt{6/5} \text{ kN}$$

$$\cos \theta = \frac{\underline{F} \cdot \underline{s}}{F} = \frac{\sqrt{6/5}}{2}, \quad \theta = 56.8^\circ$$





$$P_z = P \sin 30^\circ = 8 \sin 30^\circ = 4 \text{ lb}$$

$$P_{xy} = P \cos 30^\circ = 6.93 \text{ lb}$$

$$P_y = P_{xy} \sin 20^\circ = 2.37 \text{ lb}$$

$$P_x = P_{xy} \cos 20^\circ = 6.51 \text{ lb}$$

$$\underline{P} = 6.51 \underline{i} + 2.37 \underline{j} + 4.00 \underline{k} \text{ lb}$$

$$\cos \theta_x = P_x/P = 6.51/8, \quad \theta_x = 35.5^\circ$$

$$\cos \theta_y = P_y/P = 2.37/8, \quad \theta_y = 72.8^\circ$$

$$\cos \theta_z = P_z/P = 4/8, \quad \theta_z = 60.0^\circ$$

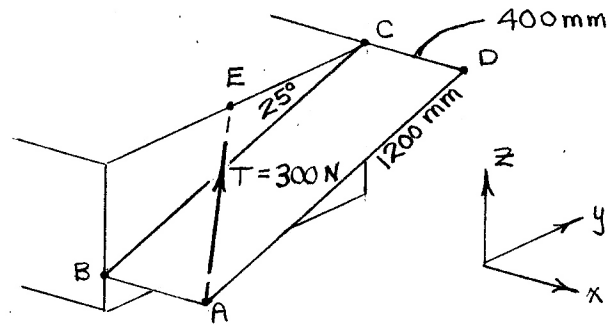
$$\begin{aligned} \underline{2/103} \quad \underline{T} &= T \underline{n}_{BC} \\ \underline{T} &= 800 \left[ \frac{+1\underline{i} - 7\underline{j} + 1.5\underline{k}}{\sqrt{1^2 + 7^2 + 1.5^2}} \right] \\ &= \underline{+110.7\underline{i} - 775\underline{j} + 166.0\underline{k} \text{ lb}} \end{aligned}$$

$$\cos \theta_x = \frac{+1}{7.23} \quad , \quad \theta_x = 82.0^\circ$$

$$\cos \theta_y = \frac{-7}{7.23} \quad , \quad \theta_y = 165.6^\circ$$

$$\cos \theta_z = \frac{1.5}{7.23} \quad , \quad \theta_z = 78.0^\circ$$

2/104



$$\begin{aligned}\underline{T} &= T \underline{n}_{AE} = 300 \left[ \frac{-400\underline{i} + 544\underline{j} + 507\underline{k}}{\sqrt{400^2 + 544^2 + 507^2}} \right] \\ &= 300 [-0.474\underline{i} + 0.644\underline{j} + 0.601\underline{k}] \text{ N}\end{aligned}$$

$$\underline{n}_{BC} = \cos 25^\circ \underline{j} + \sin 25^\circ \underline{k}$$

Carry out  $T_{BC} = \underline{T} \cdot \underline{n}_{BC}$  to obtain

$$\underline{T}_{BC} = \underline{251 \text{ N}}$$

$$\frac{2}{105} \quad x_B - x_A = 120 - 90 \sin 40^\circ = 62.1 \text{ mm}$$

$$y_B - y_A = 0 - (-90 \cos 40^\circ) = 68.9 \text{ mm}$$

$$z_B - z_A = 0 - 140 = -140 \text{ mm}$$

$$\overline{AB} = \sqrt{62.1^2 + 68.9^2 + 140^2}$$

$$= 168.0 \text{ mm}$$

$$l = \frac{62.1}{168.0} = 0.370$$

$$m = \frac{68.9}{168.0} = 0.410$$

$$n = \frac{-140}{168.0} = -0.833$$

$$\therefore \underline{T} = 500 (0.370 \underline{i} + 0.410 \underline{j} - 0.833 \underline{k}) \text{ N}$$

$$\overline{DC} = \sqrt{140^2 + 100^2} = 172.0 \text{ mm}$$

$$\underline{n}_{DC} = 0 \underline{i} + \frac{100}{172.0} \underline{j} - \frac{140}{172.0} \underline{k} = 0.581 \underline{j} - 0.814 \underline{k}$$

$$\underline{T}_{DC} = \underline{T} \cdot \underline{n}_{DC} = 500 (0.371 \underline{i} + 0.410 \underline{j} - 0.833 \underline{k}) \cdot (0.581 \underline{j} - 0.814 \underline{k})$$

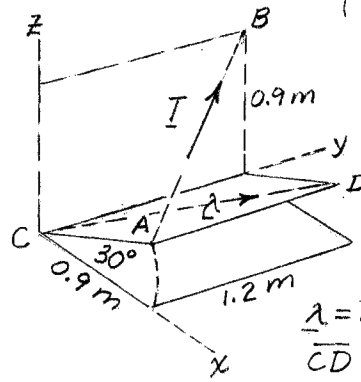
$$= \underline{458 \text{ N}}$$

2/106 |  $T = 150 \text{ N}$

Coordinates of A are

$$(0.9 \cos 30^\circ, 0, 0.9 \sin 30^\circ)$$

$$\text{or } (0.779, 0, 0.45) \text{ m}$$



$$\overline{AB} = \sqrt{0.779^2 + 1.2^2 + (0.9 - 0.45)^2}$$

$$= 1.50 \text{ m}$$

$$l = -0.779/1.5, m = 1.2/1.5, n = \frac{0.45}{1.5}$$

$$\underline{T} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k}) \text{ N}$$

$\lambda =$  unit vector along CD

$$\overline{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5 \text{ m}$$

$$l = 0.779/1.5, m = 1.2/1.5, n = 0.45/1.5$$

$$\lambda = \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$$

$$\underline{T}_{CD} = \underline{T} \cdot \lambda = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k}) \cdot \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$$

$$= \frac{100}{2.25}(-0.779^2 + 1.2^2 + 0.45^2) = \frac{100}{2.25}(-0.6075 + 1.44 + 0.2025)$$

$$= \frac{400}{9}(1.035) = \underline{46.0 \text{ N}}$$

2/107

$$\theta = \tan^{-1} \frac{4.5}{30} = 8.53^\circ$$

$$T_{xy} = 200 \cos 15^\circ = 193.2 \text{ lb}$$

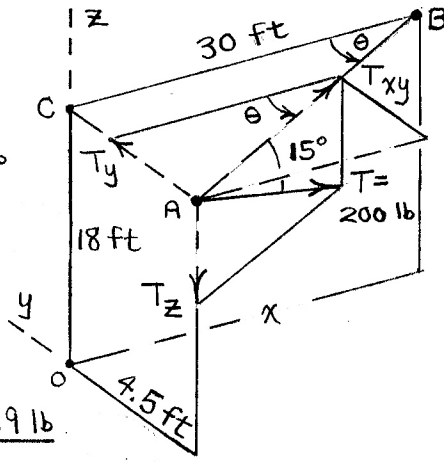
$$T_x = T_{xy} \cos \theta = 191.0 \text{ lb}$$

$$T_y = T_{xy} \sin \theta = 28.7 \text{ lb}$$

$$T_z = -T \sin 15^\circ = -51.8 \text{ lb}$$

$$T_{xz} = \sqrt{T_x^2 + T_z^2} = 197.9 \text{ lb}$$

$$\underline{T} = 191.0 \underline{i} + 28.7 \underline{j} - 51.8 \underline{k} \text{ lb}$$



► 2/108 | The position of point A is

$$\begin{aligned}\underline{r}_A &= 10 \cos 15^\circ \underline{i} + L \underline{j} + 10 \sin 15^\circ \underline{k} \\ &= 9.66 \underline{i} + L \underline{j} + 2.59 \underline{k} \text{ in.} \quad \left[ \begin{array}{l} L = \text{distance from } O \\ \text{to disk center} \end{array} \right.\end{aligned}$$

$$\begin{aligned}\underline{r}_B &= 8 \cos 30^\circ \underline{i} + (L+36) \underline{j} - 8 \sin 30^\circ \underline{k} \\ &= 6.93 \underline{i} + (L+36) \underline{j} - 4 \underline{k} \text{ in.}\end{aligned}$$

$$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A = -2.73 \underline{i} + 36 \underline{j} - 6.59 \underline{k} \text{ in.}$$

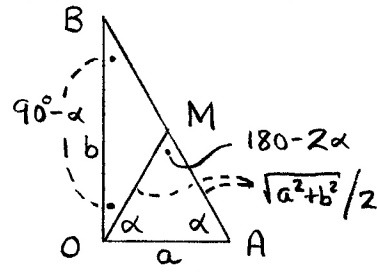
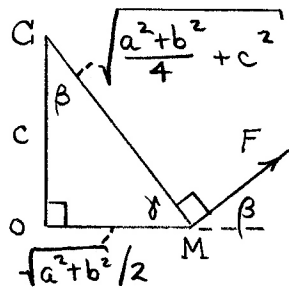
$$r_{AB} = \sqrt{2.73^2 + 36^2 + 6.59^2} = 36.7 \text{ in.} \quad \left[ \begin{array}{l} \text{unstretched length} \\ \text{is } \sqrt{(8-10)^2 + 36^2} \\ = 36.1 \text{ in.} \end{array} \right.$$

The spring force is  $F = k\delta = 15(36.7 - 36.1) = 9.66 \text{ lb}$

As a vector:  $\underline{F} = F \underline{n}_{AB} = F \frac{\underline{r}_{AB}}{r_{AB}}$

$$\begin{aligned}\underline{F} &= 9.66 \left[ \frac{-2.73 \underline{i} + 36 \underline{j} - 6.59 \underline{k}}{36.7} \right] \\ &= \underline{-0.719 \underline{i} + 9.48 \underline{j} - 1.734 \underline{k} \text{ lb}}\end{aligned}$$

2/109

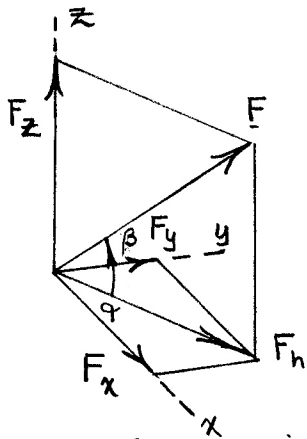


$$\tan \gamma = \frac{c}{\sqrt{a^2 + b^2}/2} = \frac{2c}{\sqrt{a^2 + b^2}}$$

$$\gamma + 90^\circ + \beta = 180^\circ$$

$$\beta = 90^\circ - \gamma = 90^\circ - \tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}}$$

$$\begin{cases} \tan \alpha = \frac{b}{a} \\ \cos \alpha = \frac{a}{\sqrt{a^2 + b^2}} \\ \sin \alpha = \frac{b}{\sqrt{a^2 + b^2}} \end{cases}$$



$$\begin{cases} F_z = F \sin \beta \\ F_h = F \cos \beta \\ F_x = F_h \cos \alpha = F \cos \beta \cos \alpha \\ F_y = F_h \sin \alpha = F \cos \beta \sin \alpha \end{cases}$$

Now simplify  $\sin \beta$  &  $\cos \beta$  expressions:

$$\begin{aligned} \sin \beta &= \sin \left[ 90^\circ - \tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}} \right] \\ &= \sin 90^\circ \cos \left[ \tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}} \right] - \cos 90^\circ \sin \left[ \tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}} \right] \\ &= \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + 4c^2}} \quad (\text{Also see that } \sin \beta = \cos \gamma!) \end{aligned}$$

$$\cos \beta = \sin \gamma = \frac{c}{\sqrt{\frac{a^2 + b^2}{4} + c^2}} = \frac{2c}{\sqrt{a^2 + b^2 + 4c^2}}$$

Finally,

$$F_x = F \frac{2c}{\sqrt{a^2 + b^2 + 4c^2}} \frac{a}{\sqrt{a^2 + b^2}} = \frac{2acF}{\sqrt{a^2 + b^2} \sqrt{a^2 + b^2 + 4c^2}}$$

$$F_y = F \frac{2c}{\sqrt{a^2 + b^2 + 4c^2}} \frac{b}{\sqrt{a^2 + b^2}} = \frac{2bcF}{\sqrt{a^2 + b^2} \sqrt{a^2 + b^2 + 4c^2}}$$

$$F_z = F \sqrt{\frac{a^2 + b^2}{a^2 + b^2 + 4c^2}}$$

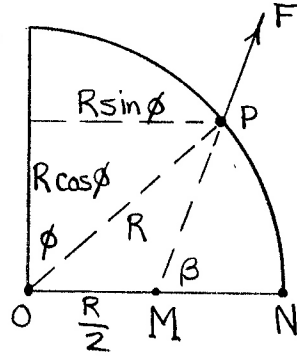




►2/110 |  $F_x = F_{xy} \cos \theta$ ,  $F_y = F_{xy} \sin \theta$   
 $F_z = F \sin \beta$ ,  $F_{xy} = F \cos \beta$   
 $\tan \beta = \frac{R \cos \phi}{R \sin \phi - \frac{R}{2}} = \frac{2 \cos \phi}{2 \sin \phi - 1}$

So  $\sin \beta = \frac{2 \cos \phi}{\sqrt{(2 \sin \phi - 1)^2 + (2 \cos \phi)^2}}$

$\cos \beta = \frac{2 \sin \phi - 1}{\sqrt{(2 \sin \phi - 1)^2 + (2 \cos \phi)^2}}$

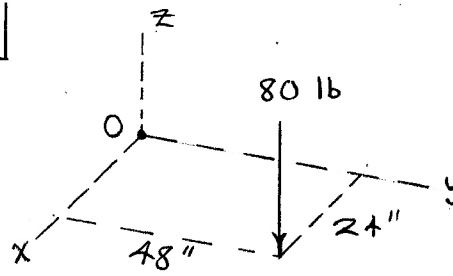


Note that  $\sqrt{(2 \sin \phi - 1)^2 + (2 \cos \phi)^2} = \sqrt{5 - 4 \sin \phi}$

So  $\underline{F} = F [\cos \theta \cos \beta \underline{i} + \sin \theta \cos \beta \underline{j} + \sin \beta \underline{k}]$

$= \frac{F}{\sqrt{5 - 4 \sin \phi}} [(2 \sin \phi - 1)(\cos \theta \underline{i} + \sin \theta \underline{j}) + 2 \cos \phi \underline{k}]$

2/III



$$\begin{cases} M_{Ox} = -80 \left( \frac{48}{12} \right) = -320 \text{ lb-ft} \\ M_{Oy} = 80 \left( \frac{24}{12} \right) = 160 \text{ lb-ft} \\ M_{Oz} = 0 \end{cases}$$

$$\therefore \underline{M}_O = -320\mathbf{i} + 160\mathbf{j} \text{ lb-ft}$$

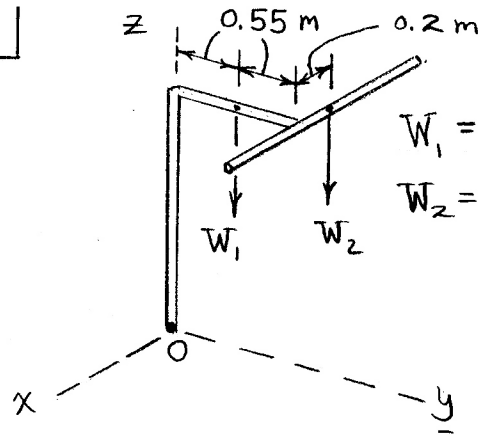
$$M_O = \sqrt{320^2 + 160^2} = \underline{358 \text{ lb-ft}}$$

$$\frac{2}{112} \quad \underline{M_o} = F(-c_i + a_k)$$
$$\underline{M_A} = F a_k$$

$$\begin{aligned} \underline{2/113} \quad \underline{M} &= -150(0.250 + 0.250)\underline{i} + 150(0.150)\underline{j} \\ &= -75\underline{i} + 22.5\underline{j} \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} 2/114 \quad \underline{M}_o &= \underline{r} \times \underline{F} \\ &= (-6\underline{i} + 0.8\underline{j} + 1.2\underline{k}) \times (-400\underline{j}) \\ &= \underline{480\underline{i} + 2400\underline{k} \text{ N}\cdot\text{m}} \end{aligned}$$

2/115



$$W_1 = 1.1(7)(9.81) = 75.5 \text{ N}$$

$$W_2 = 2(7)(9.81) = 137.3 \text{ N}$$

$$\begin{cases} M_{Ox} = -75.5(0.55) - 137.3(1.1) = -192.6 \text{ N}\cdot\text{m} \\ M_{Oy} = -137.3(0.2) = -27.5 \text{ N}\cdot\text{m} \\ M_{Oz} = 0 \end{cases}$$

$$\therefore \underline{M_O} = -192.6 \underline{i} - 27.5 \underline{j} \text{ N}\cdot\text{m}$$

$$\underline{M_O} = 194.6 \text{ N}\cdot\text{m}$$

$$\frac{2/116}{|} \bar{AB} = \sqrt{0.8^2 + 1.5^2 + 2^2} = 2.62 \text{ m}$$

$$\underline{T} = \frac{1.2}{2.62} (0.8\underline{i} + 1.5\underline{j} - 2\underline{k}) \text{ kN}$$

$$\text{Take } \underline{r} = \underline{OA} = 1.6\underline{i} + 2\underline{k} \text{ m}$$

$$\underline{M}_O = \underline{r} \times \underline{T} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.6 & 0 & 2 \\ 0.8 & 1.5 & -2 \end{vmatrix} \frac{1.2}{2.62}$$

$$\underline{M}_O = 0.457(-3\underline{i} + 4.8\underline{j} + 2.40\underline{k}) \text{ kN}\cdot\text{m}$$

$$M_O = |\underline{M}_O| = 0.457\sqrt{3^2 + 4.8^2 + 2.40^2} = \underline{2.81 \text{ kN}\cdot\text{m}}$$

2/117 From the solution to Prob. 2/99, the force is  $\underline{R} = \underline{T} = -598\underline{i} + 411\underline{j} + 189.5\underline{k}$  N

The moment associated with the couple is  $\underline{M}_o = \underline{r}_{oc} \times \underline{T}$ , where  $\underline{r}_{oc} = 0.7\underline{j} + 1.2\underline{k}$  m

Carry out the cross product to obtain

$$\underline{M}_o = -361\underline{i} - 718\underline{j} + 419\underline{k} \text{ N}\cdot\text{m}$$



$$\underline{2/118} \quad T = 400 \left[ \frac{6\mathbf{i} + 2\mathbf{j} - 9\mathbf{k}}{\sqrt{6^2 + 2^2 + 9^2}} \right]$$

$$= 218\mathbf{i} + 72.7\mathbf{j} - 327\mathbf{k} \quad \text{N}$$

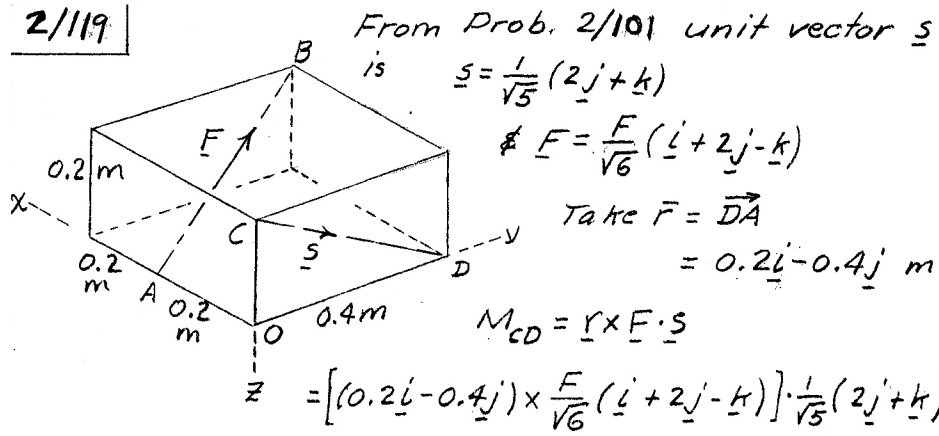
$$\underline{r}_{CA} = 6\mathbf{j} + 3\mathbf{k} \quad \text{m}$$

Carry out  $\underline{M}_C = \underline{r}_{CA} \times \underline{T}$  to obtain

$$\underline{M}_C = -2180\mathbf{i} + 655\mathbf{j} - 1309\mathbf{k} \quad \text{N}\cdot\text{m}$$

$$M_C = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{2630 \text{ N}\cdot\text{m}}$$

2/119



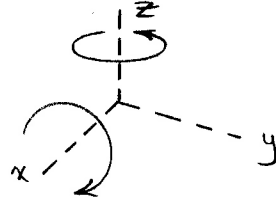
So

$$50 = \frac{F}{\sqrt{30}} \begin{vmatrix} 0.2 & -0.4 & 0 \\ 1 & 2 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \frac{F}{\sqrt{30}} (0.8 + 0.4) = \frac{1.2F}{\sqrt{30}}, F = \frac{50\sqrt{30}}{1.2} = \underline{228 \text{ N}}$$

$$\underline{2/120} \quad \underline{M} = 1.2(40)\underline{k} - 1.2(50)\underline{i}$$
$$= -60\underline{i} + 48\underline{k} \quad \text{lb-in.}$$

The spacecraft will begin to rotate about its x- and z axes.



$$\underline{2/121} \quad \underline{F} = 50(\cos 15^\circ \underline{i} - \sin 15^\circ \underline{k}) \text{ lb}$$

$$\underline{r}_{oc} = -8 \sin 15^\circ \underline{i} + 7 \underline{j} - (6 + 8 \cos 15^\circ) \underline{k}$$

Carry out  $\underline{M}_o = \underline{r}_{oc} \times \underline{F}$  to obtain

$$\underline{M}_o = -90.6 \underline{i} - 690 \underline{j} - 338 \underline{k} \text{ lb-in.}$$

---

$$\underline{M}_{oA} = \underline{M}_{oY} = -690 \text{ lb-in.}$$

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$$\frac{2/122}{\left\{ \begin{array}{l} x_B - x_A = 0 - (-14 \cos 60^\circ) = 7 \text{ in.} \\ y_B - y_A = 14 - 14 \sin 60^\circ = 1.876 \text{ in.} \\ z_B - z_A = 0 - 16 = -16 \text{ in.} \end{array} \right.}$$

$$\overline{AB} = \sqrt{7^2 + 1.876^2 + 16^2} = 17.56 \text{ in.}$$

$$\underline{T} = 120 \left( \frac{7\underline{i} + 1.876\underline{j} - 16\underline{k}}{17.56} \right) = 47.8\underline{i} + 12.8\underline{j} - \frac{109.3\underline{k}}{16}$$

$$\underline{r}_{OB} = 14\underline{j} \text{ in.}$$

Carry out  $\underline{M}_O = \underline{r}_{OB} \times \underline{T}$  to obtain

$$\underline{M}_O = -1530\underline{i} - 670\underline{k} \text{ lb-in.}$$

$$\underline{2/123} \quad \underline{F} = 300 (\sin 60^\circ \underline{j} - \cos 60^\circ \underline{k}) \text{ N}$$

$$\underline{r}_{oA} = 0.115 \underline{i} + 0.350 \cos 40^\circ \underline{j} + 0.350 \sin 40^\circ \underline{k} \text{ m}$$

Carry out  $\underline{M}_o = \underline{r}_{oA} \times \underline{F}$  to obtain

$$\underline{M}_o = -98.7 \underline{i} + 17.25 \underline{j} + 29.9 \underline{k} \text{ N}\cdot\text{m}$$

---

$$\underline{M}_x = -98.7 \text{ N}\cdot\text{m}$$

2/124

$$M_o = (250 \sin 60^\circ)12 + (250 \cos 60^\circ) \sin 40^\circ (8 - 4.2)$$
$$= \underline{2900 \text{ lb-in.}}$$

$$\frac{2}{125} \quad \underline{R} = 50(-\cos 40^\circ \underline{i} - \sin 40^\circ \underline{j})$$
$$= -38.3 \underline{i} - 32.1 \underline{j} \text{ N}$$

$$\underline{M}_o = (50 \sin 40^\circ)(20) \underline{i} + (50 \cos 40^\circ)(20) \underline{j}$$
$$+ 50(125) \underline{k}$$
$$= 643 \underline{i} - 766 \underline{j} + 6250 \underline{k} \text{ N}\cdot\text{mm}$$

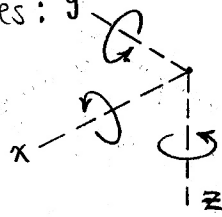


$$\underline{2/126} \quad \underline{M} = (1700)(2)\underline{i} - (1700)(30)\underline{j} - (1700)(30)\underline{k}$$

$$= 3400\underline{i} + 51,000\underline{j} - 51,000\underline{k} \text{ N}\cdot\text{m}$$


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The orbiter would acquire rotational motion about all three axes:



2/127

$$\begin{aligned}\underline{M}_o &= 0\underline{i} - (200)(0.2 + 0.125 \sin 20^\circ)\underline{j} \\ &\quad - 200(0.125 \cos 20^\circ - 0.070)\underline{k} \\ &= \underline{-48.6\underline{j} - 9.49\underline{k} \text{ N}\cdot\text{m}}\end{aligned}$$

There would be no z-component of  $\underline{M}_o$  if  
 $d \cos 20^\circ - 70 = 0$ ,  $d = 74.5 \text{ mm}$

$$2/128 \quad \underline{M}_o = \underline{r}_{oB} \times \underline{T}, \quad \underline{r}_{oB} = 6\underline{i} + 13\underline{j} \text{ m}$$

$$\underline{T} = T \underline{n}_{AB} = 24 \left[ \frac{6\underline{i} - 5\underline{j} - 30\underline{k}}{\sqrt{6^2 + 5^2 + 30^2}} \right]$$

$$= 4.65\underline{i} - 3.87\underline{j} - 23.2\underline{k} \text{ kN}$$

Carry out  $\underline{r}_{oB} \times \underline{T}$  to obtain

$$\underline{M}_o = -302\underline{i} + 139.4\underline{j} - 83.6\underline{k} \text{ N}\cdot\text{m}$$

---

2/129 |

$$\underline{T} = T \left[ \frac{-0.35\mathbf{i} - 0.45 \cos 20^\circ \mathbf{j} + (0.4 + 0.45 \sin 20^\circ) \mathbf{k}}{\sqrt{(0.35)^2 + (0.45 \cos 20^\circ)^2 + (0.4 + 0.45 \sin 20^\circ)^2}} \right]$$

$$= 143.4 [-0.449\mathbf{i} - 0.542\mathbf{j} + 0.710\mathbf{k}] \text{ N}$$

Moment of this force about the  $x$ -axis is

$$M_{O_x} = (0.710)(143.4)(0.45 \cos 20^\circ) - 0.542(143.4)(0.45 \sin 20^\circ) = \underline{31.1 \text{ N}\cdot\text{m}}$$

The moment of the weight  $W$  of the 15-kg plate about the  $x$ -axis is

$$(M_{O_x})_W = -15(9.81) \frac{0.45 \cos 20^\circ}{2} = \underline{-31.1 \text{ N}\cdot\text{m}}$$

The moment of  $\underline{T}$  about the line  $OB$  is zero, because  $\underline{T}$  intersects  $OB$ .

2/130 | Moment of couple is  $240(\underline{j}\cos 30^\circ - \underline{k}\sin 30^\circ)$   
 $= 207.8\underline{j} - 120\underline{k} \text{ N}\cdot\text{m}$

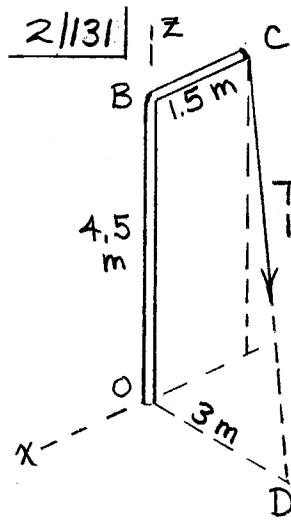
Moment of force is

$$1200 \cos 30^\circ (-0.250\underline{i} + 0.200\underline{k}) + 1200 \sin 30^\circ (0.200\underline{j})$$
$$= -259.8\underline{i} + 120\underline{j} + 207.8\underline{k} \text{ N}\cdot\text{m}$$

Thus total moment is

$$\underline{M}_0 = -259.8\underline{i} + 327.8\underline{j} + 87.8\underline{k} \text{ N}\cdot\text{m}$$

or  $\underline{M}_0 = -260\underline{i} + 328\underline{j} + 88\underline{k} \text{ N}\cdot\text{m}$



From the solution to Prob. 2/95,

$$\underline{T} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k} \text{ kN}$$

(a)  $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$ ,  $\underline{r}_{OD} = 3\underline{j} \text{ m}$

$$\underline{M}_O = -2.89\underline{i} - 0.962\underline{k} \text{ kN}\cdot\text{m}$$

Could also use

$$\underline{M}_O = \underline{r}_{OC} \times \underline{T}, \text{ where}$$

$$\underline{r}_{OC} = -1.5\underline{i} + 4.5\underline{k} \text{ m}$$

Result for  $\underline{M}_O$  is the same.

(b) About z-axis:

$$M_z = -1.5T_y = -1.5(0.641) = -0.962 \text{ kN}\cdot\text{m}$$

$$\text{Also, } M_z = \underline{M}_O \cdot \underline{k} = (-2.89\underline{i} - 0.962\underline{k}) \cdot \underline{k} = -0.962 \text{ kN}\cdot\text{m}$$

$$\underline{2/132} \quad \underline{F} = 180 (-\cos 60^\circ \underline{i} - \sin 60^\circ \underline{j}) \text{ lb}$$

$$\underline{r}_{BA} = +15 \sin 45^\circ \underline{i} - (15 \cos 45^\circ + 13) \underline{j} - 12 \underline{k} \text{ in.}$$

$$\underline{r}_{CA} = (+15 \sin 45^\circ - 6) \underline{i} - (15 \cos 45^\circ + 13) \underline{j} \text{ in.}$$

$$\underline{M}_B = \underline{r}_{BA} \times \underline{F}, \quad \underline{M}_C = \underline{r}_{CA} \times \underline{F}$$

Carry out to obtain

$$\underline{M}_B = -1871 \underline{i} + 1080 \underline{j} - 3780 \underline{k} \text{ lb-in.}$$

$$\underline{M}_C = -2840 \underline{k} \text{ lb-in.}$$

The unit vector which characterizes line BC

$$\text{is } \underline{n}_{BC} = \frac{6 \underline{i} - 12 \underline{k}}{\sqrt{6^2 + 12^2}} = 0.447 \underline{i} - 0.894 \underline{k}$$

$$M_{BC} = \underline{M}_B \cdot \underline{n}_{BC} = \underline{2540 \text{ lb-in.}}$$

$$(\underline{M}_C \cdot \underline{n}_{BC} = 2540 \text{ lb-in.}, \text{ also})$$

2/133 | Using the coordinates of the figure:

$$\underline{M}_A = \underline{r} \times \underline{F}, \quad \underline{F} = -1.8 \underline{k} \text{ lb}$$

$$\underline{r} = [(2+1) \cos 30^\circ] \underline{i} + 3 \underline{j} + [(2+1) \sin 30^\circ] \underline{k}$$

$$\therefore \underline{M}_A = -5.40 \underline{i} + 4.68 \underline{j} \text{ lb-in.}$$

$$\underline{M}_{AB} = (\underline{M}_A \cdot \underline{n}_{AB}) \underline{n}_{AB}, \quad \underline{n}_{AB} = \cos 30^\circ \underline{i} + \sin 30^\circ \underline{k}$$

$$\therefore \underline{M}_{AB} = -4.05 \underline{i} - 2.34 \underline{k} \text{ lb-in.}$$



$$2/134 \quad \underline{M}_o = \underline{r}_{oA} \times \underline{F}$$

$$\begin{aligned}\underline{r}_{oA} &= (0.050 + 0.130 \sin 60^\circ) \underline{i} \\ &\quad + (-0.140 - 0.130 \cos 60^\circ) \underline{j} + 0.150 \underline{k} \\ &= 0.1626 \underline{i} - 0.205 \underline{j} + 0.150 \underline{k} \text{ m}\end{aligned}$$

$$\begin{aligned}\underline{F} &= 600 (\cos 45^\circ \sin 60^\circ \underline{i} - \cos 45^\circ \cos 60^\circ \underline{j} + \sin 45^\circ \underline{k}) \\ &= 600 (0.612 \underline{i} - 0.354 \underline{j} + 0.707 \underline{k}) \\ &= 367 \underline{i} - 212 \underline{j} + 424 \underline{k} \text{ N}\end{aligned}$$

Carry out  $\underline{M}_o = \underline{r}_{oA} \times \underline{F}$  to obtain

$$\underline{M}_o = -55.2 \underline{i} - 13.8 \underline{j} + 40.8 \underline{k} \text{ N}\cdot\text{m}$$

$$\underline{2/135} \quad \underline{R} = \underline{W} + \underline{L} + \underline{D}$$

$$= -5\underline{j} + L(-\sin \theta \underline{j} - \cos \theta \underline{k}) - 1.7\underline{i}$$

$$= -1.7\underline{i} - (5 + L \sin \theta) \underline{j} - L \cos \theta \underline{k}$$

$$\begin{cases} -L \cos \theta = -0.866 \\ -(5 + L \sin \theta) = -5.500 \end{cases} \Rightarrow \begin{array}{l} \theta = 30^\circ \\ \underline{L = 1.02} \end{array}$$

$$R = \left[ (1.7)^2 + (5 + 1 \sin 30^\circ)^2 + (1 \cos 30^\circ)^2 \right]^{1/2}$$

$$= \underline{5.82 \text{ oz}}$$

$$\frac{2/136}{\left\{ \begin{array}{l} \underline{R} = -3F\underline{k} \\ \underline{M}_o = -\frac{\sqrt{3}}{2} bF\underline{i} \end{array} \right.}$$
$$\underline{R} \cdot \underline{M}_o = 0 \quad \text{so} \quad \underline{R} \perp \underline{M}_o$$

$$\frac{2}{137} \quad \underline{R} = -2F\underline{k} + F\underline{k} + F(\cos 30^\circ \underline{k} + \sin 30^\circ \underline{j})$$

$$= \frac{F}{2}\underline{j} + F\left(\frac{\sqrt{3}}{2} - 1\right)\underline{k} = F\left(\frac{1}{2}\underline{j} + \left(\frac{\sqrt{3}}{2} - 1\right)\underline{k}\right)$$

$$\underline{M}_o = 2Fb\underline{j} + Fb\underline{i} + \frac{F}{2}(2b)\underline{k} + \frac{\sqrt{3}}{2}Fb\underline{i} - \frac{\sqrt{3}}{2}F(2b)\underline{j}$$

$$= Fb \left[ \left(1 + \frac{\sqrt{3}}{2}\right)\underline{i} + (2 - \sqrt{3})\underline{j} + \underline{k} \right]$$

$$\underline{R} \cdot \underline{M}_o = \left[ \frac{1}{2}(2 - \sqrt{3}) + \left(\frac{\sqrt{3}}{2} - 1\right)(1) \right] F^2 b = 0, \text{ so}$$

$$\underline{R} \perp \underline{M}_o.$$

$$\frac{2}{138} \quad \underline{R} = (1.2 - 1.2 - 1.2)\underline{j} = -1.2\underline{j} \text{ lb}$$

$$\underline{M}_G = 1.2(3)(20)\underline{k} + (1.2 - 1.2 - 1.2)(25)\underline{i}$$

$$\underline{M}_G = -30\underline{i} + 72\underline{k} \text{ lb-in.}$$

$$\underline{2/139} \quad \underline{R} = (200 + 800) \underline{i} + 1200 (\cos 10^\circ \underline{j} - \sin 10^\circ \underline{i})$$

$$= \underline{792 \underline{i} + 1182 \underline{j} \text{ N}}$$

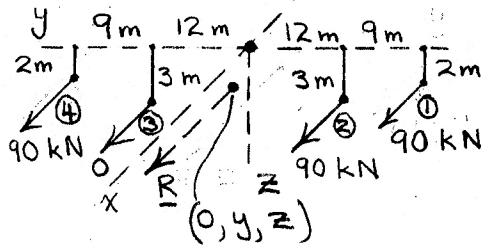
$$\underline{M}_o = [(200 - 800)(0.1) + (1200 \cos 10^\circ)(0.075)] \underline{k}$$

$$+ [-(200 + 800)(0.220 + 0.330) + 1200 \sin 10^\circ (0.220)] \underline{j}$$

$$+ [1200 \cos 10^\circ (0.220)] \underline{i}$$

$$= \underline{260 \underline{i} - 504 \underline{j} + 28.6 \underline{k} \text{ N}\cdot\text{m}}$$

2/140



$$R = \sum F = 3(90) = 270 \text{ kN}$$

$$\sum M_z = -R_y = 90(21)$$

$$+ 90(12) - 90(21),$$

$$y = -4 \text{ m}$$

$$\sum M_y = R_z = 2(90)(2) + 1(90)(3), \quad z = 2.33 \text{ m}$$

$$\underline{2/141} \quad \text{At } O: \quad \underline{R} = \Sigma \underline{F} = (200 + 400)\underline{j} = \underline{600j \text{ lb}}$$

$$\underline{M}_O = 600(8)\underline{k} + 400(3)\underline{i} = \underline{1200i + 4800k \text{ lb-ft}}$$

$$\underline{R} \cdot \underline{M}_O = 0 \Rightarrow \underline{R} \perp \underline{M}_O \quad (\text{loading system can be represented by single force})$$

Let P have coordinates  $(x, 0, z)$  and let  $\underline{R}$  act at P.

$$\underline{r}_{OP} \times \underline{R} = \underline{M}_O: \quad (x\underline{i} + z\underline{k}) \times 600\underline{j} = 1200\underline{i} + 4800\underline{k}$$

$$600x\underline{k} - 600z\underline{i} = 1200\underline{i} + 4800\underline{k}$$

$$\Rightarrow \underline{x = 8 \text{ ft}, \quad z = -2 \text{ ft}}$$





2/142 | The two 160-N forces constitute a couple  $160(0.250)\underline{j} = 40\underline{j}$  N·m

$$\underline{R} = \sum \underline{F} = 120\underline{i} - 180\underline{j} - 100\underline{k} \text{ N}$$

$$\underline{M} = \sum \underline{M}_A = [120(0.25) + 100(0.3) + 40]\underline{j} + 50\underline{k} \text{ N·m}$$
$$= 100\underline{j} + 50\underline{k} \text{ N·m}$$

$$\frac{2}{143} \quad R_x = \sum F_x = -20 \text{ kN}$$

$$R_y = \sum F_y = -40 \cos(\tan^{-1} \frac{1}{3}) = -37.9 \text{ kN}$$

$$R_z = \sum F_z = 40 \sin(\tan^{-1} \frac{1}{3}) = 12.65 \text{ kN}$$

$$\Rightarrow \underline{R} = -20\mathbf{i} - 37.9\mathbf{j} + 12.65\mathbf{k} \text{ kN}$$

$$M_{Ax} = 0$$

$$M_{Ay} = 20(1) + 40 \frac{1}{\sqrt{3^2+1^2}} (2) = 45.3 \text{ kN}\cdot\text{m}$$

$$M_{Az} = 40 \frac{3}{\sqrt{3^2+1^2}} (2) - 35 = 40.9 \text{ kN}\cdot\text{m}$$

$$\Rightarrow \underline{M} = 45.3\mathbf{j} + 40.9\mathbf{k} \text{ kN}\cdot\text{m}$$

$$\underline{2/144} \quad R = \sum F_z = 70 + 30 - 80 - 60 - 50 = -90 \text{ lb}$$

$$-R|y = \sum M_x: -90y = 30(12) + 70(12) - 60(6) - 50(12)$$

$$y = \underline{-2.67 \text{ in.}}$$

$$|R|x = \sum M_y: 90x = 80(10) - 30(10) - 50(8)$$

$$x = \underline{1.111 \text{ in.}}$$

$$2/145 \quad R_x = -120 \text{ N}, \quad R_y = 0, \quad R_z = -160 \text{ N}$$

$$R = \sqrt{120^2 + 160^2} = 200 \text{ N}, \quad \underline{R} = -120\underline{i} - 160\underline{k} \text{ N}$$

$$M_x = 25 - 160(0.2) = -7 \text{ N}\cdot\text{m}$$

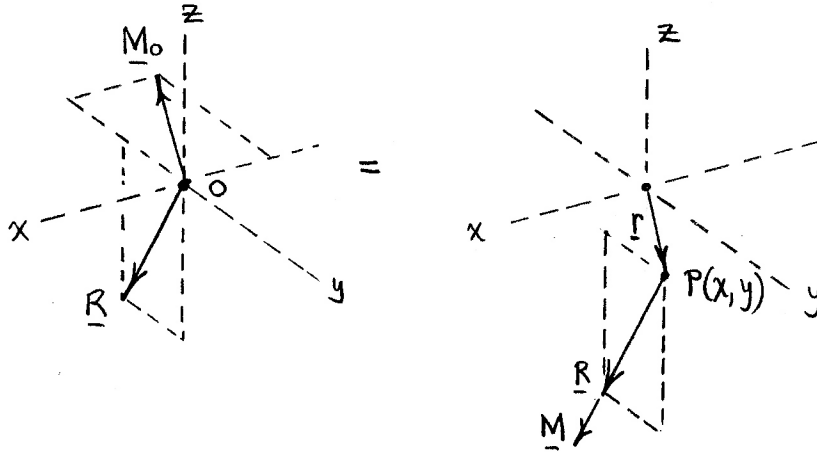
$$M_y = 160(0.075) - 120(0.100 - 0.075) = 9 \text{ N}\cdot\text{m}$$

$$M_z = 120(0.2) = 24 \text{ N}\cdot\text{m}$$

$$M = \sqrt{7^2 + 9^2 + 24^2} = 25.5 \text{ N}\cdot\text{m}$$

$$\underline{M} = -7\underline{i} + 9\underline{j} + 24\underline{k} \text{ N}\cdot\text{m}$$

$$\frac{2/146}{\left\{ \begin{array}{l} \underline{R} = -20\underline{j} - 40\underline{k} \text{ lb} \quad (= 44.7(-0.447\underline{j} - 0.894\underline{k})) \\ \underline{M}_o = -40(1.4)\underline{i} - 40(8)\underline{j} \\ = -56\underline{i} - 320\underline{j} \end{array} \right.}$$



$$\underline{M}_o: -56\underline{i} - 320\underline{j} = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + y\underline{j}) \times (-20\underline{j} - 40\underline{k}) + M(-0.447\underline{j} - 0.894\underline{k})$$

Equate coefficients:

$$\begin{cases} \underline{i}: -56 = -40y \\ \underline{j}: -320 = 40x - 0.447M \\ \underline{k}: 0 = -20x - 0.894M \end{cases}$$

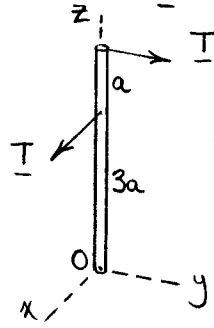
Solution:

$$x = -6.4 \text{ in.}$$

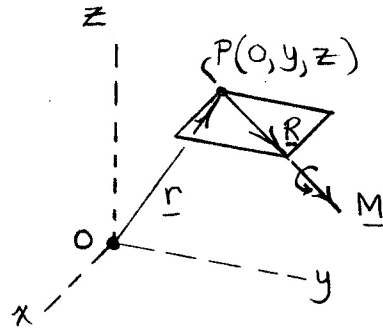
$$y = 1.4 \text{ in.}$$

$$2/147 \quad \underline{R} = \sum \underline{F} = T \underline{i} + T \underline{j} = \sqrt{2}T \left[ \frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right]$$

$$\sum \underline{M}_o = 3aT \underline{j} - 4aT \underline{i}$$



=



$$\sum \underline{M}_o = \underline{r} \times \underline{R} + \underline{M}$$

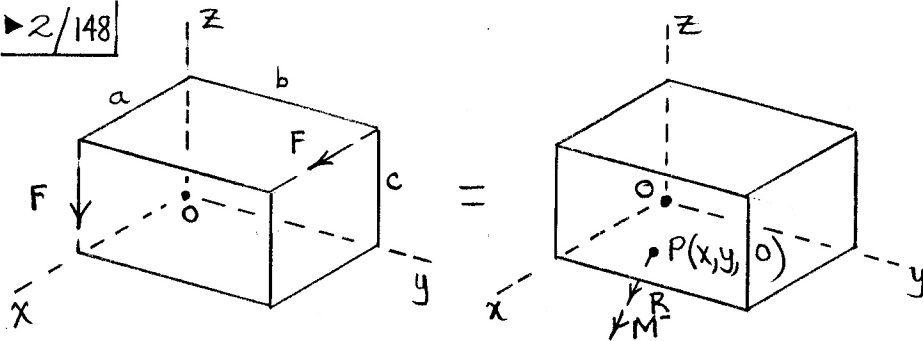
$$3aT \underline{j} - 4aT \underline{i} = (y \underline{j} + z \underline{k}) \times (T \underline{i} + T \underline{j}) + M \left( \frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right)$$

$$\Rightarrow \begin{cases} -4aT = -zT + \frac{M}{\sqrt{2}} \\ 3aT = zT + \frac{M}{\sqrt{2}} \\ 0 = -yT \end{cases}$$

$$\text{So } \begin{cases} y = 0 \\ z = \frac{7}{2}a \\ M = -\frac{\sqrt{2}}{2}aT \end{cases}$$

$$\text{So } \underline{M} = -\frac{\sqrt{2}}{2}aT \left( \frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right) \\ = \underline{\underline{-\frac{aT}{2} (\underline{i} + \underline{j})}} \quad (\text{a negative wrench})$$

2/148



$$\underline{R} = \sum \underline{F} = F \underline{i} - F \underline{k} = \sqrt{2} F \left( \frac{1}{\sqrt{2}} \underline{i} - \frac{1}{\sqrt{2}} \underline{k} \right)$$

$$\sum \underline{M}_O = aF \underline{j} + cF \underline{j} - bF \underline{k} = F((a+c) \underline{j} - b \underline{k})$$

$$= (x \underline{i} + y \underline{j}) \times \underline{R} + \underline{M}$$

$$= (x \underline{i} + y \underline{j}) \times (F \underline{i} - F \underline{k}) + M \left( \frac{1}{\sqrt{2}} \underline{i} - \frac{1}{\sqrt{2}} \underline{k} \right)$$

$$= \left( -Fy + \frac{M}{\sqrt{2}} \right) \underline{i} + (Fx) \underline{j} + \left( -Fy - \frac{M}{\sqrt{2}} \right) \underline{k}$$

Equate coefficients:

$$\left. \begin{aligned} 0 &= -Fy + \frac{M}{\sqrt{2}} \\ F(a+c) &= Fx \\ -Fb &= -Fy - \frac{M}{\sqrt{2}} \end{aligned} \right\} \text{Solution: } \begin{cases} x = a+c \\ y = \frac{b}{2} \\ M = \frac{Fb\sqrt{2}}{2} \end{cases}$$

$$\underline{M} = \frac{Fb\sqrt{2}}{2} \left( \frac{1}{\sqrt{2}} \underline{i} - \frac{1}{\sqrt{2}} \underline{k} \right) = \underline{\underline{\frac{Fb}{2} (\underline{i} - \underline{k})}}$$

$$\triangleright 2/149 \quad \underline{R} = \Sigma \underline{F} = 100 \underline{i} + 100 \underline{j} \text{ N}$$

Direction cosines.  $l = \frac{1}{\sqrt{2}}$ ,  $m = \frac{1}{\sqrt{2}}$ ,  $n = 0$

Let  $P = P(x, 0, z)$

$$\underline{M}_P = 100z \underline{i} + 100(0.4-x) \underline{k} + 100(0.4-z) \underline{j} \\ - 100(0.3) \underline{k} - 20z \underline{j}$$

$$= 100z \underline{i} + 100(0.2-z) \underline{j} + 100(0.1-x) \underline{k} \text{ N}\cdot\text{m}$$

Let  $M = |\underline{M}_P|$ . Equate direction cosines of  $\underline{R}$  &  $\underline{M}_P$  to obtain

$$\frac{100z}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.2-z)}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.1-x)}{M} = 0$$

$$\text{Solution: } \begin{cases} x = 0.1 \text{ m} \\ z = 0.1 \text{ m} \\ M = 10\sqrt{2} \text{ N}\cdot\text{m} \end{cases}$$

$$\underline{M} = 10\sqrt{2} \left( \frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right) = \underline{10i + 10j} \text{ N}\cdot\text{m}$$



$$\triangleright 2/150 \quad \underline{R} = \sum \underline{F} = -20\underline{i} - 40\sqrt{\frac{3}{3^2+1^2}}\underline{j} + 40\sqrt{\frac{1}{3^2+1^2}}\underline{k}$$

$$= -20\underline{i} - 37.9\underline{j} + 12.65\underline{k} \quad \text{kN}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 44.7 \text{ kN}$$

$$\text{Direction cosines of } \underline{R}: \begin{cases} l = -\frac{20}{44.7} = -0.447 \\ m = \frac{-37.9}{44.7} = -0.849 \\ n = \frac{12.65}{44.7} = 0.283 \end{cases}$$

Let  $P = P(x, y, 0)$ :

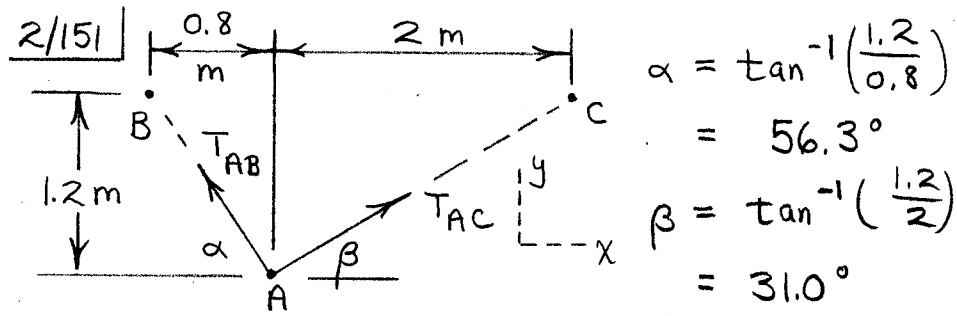
$$\underline{M} = \sum \underline{M}_p = 20(-y\underline{k} + \underline{j}) + 37.9(x\underline{k} - \underline{i}) + 12.65[(3-y)\underline{i} + x\underline{j}]$$

$$\dots - 35\underline{k} = -12.65y\underline{i} + (20 + 12.65x)\underline{j} + (37.9x - 20y - 35)\underline{k} \quad \text{kN}\cdot\text{m}$$

Equate direction cosines of  $\underline{R}$  and  $\underline{M}$ :

$$\left. \begin{array}{l} -0.447 = -12.65y/M \\ -0.849 = (20 + 12.65x)/M \\ 0.283 = (37.9x - 20y - 35)/M \end{array} \right\} \begin{array}{l} \text{Solution:} \\ \underline{x = 0.221 \text{ m}} \\ \underline{y = -0.950 \text{ m}} \\ \underline{M = -26.9 \text{ kN}\cdot\text{m}} \end{array}$$

$\underline{M}$  and  $\underline{R}$  have opposite directions.



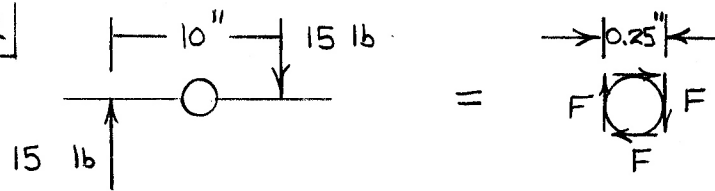
$$\underline{T}_{AB} = T_{AB} \underline{n}_{AB} = 0.858(60)(9.81) [-\cos 56.3^\circ \underline{i} + \sin 56.3^\circ \underline{j}]$$

$$= \underline{-280 \underline{i} + 420 \underline{j} \text{ N}}$$

$$\underline{T}_{AC} = T_{AC} \underline{n}_{AC} = 0.555(60)(9.81) [\cos 31.0^\circ \underline{i} + \sin 31.0^\circ \underline{j}]$$

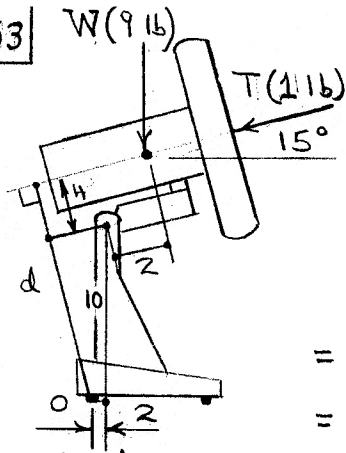
$$= \underline{280 \underline{i} + 168.1 \underline{j} \text{ N}}$$

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$$M = Fd = 15(10) = 2F(0.25), \underline{F = 300 \text{ lb}}$$

2/153

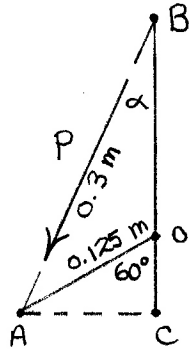


Dim. in inches

$$\begin{aligned} M_o &= Fd \\ &= 1(4 + 10 \cos 15^\circ - 2 \sin 15^\circ) \\ &= 13.14 \text{ lb-in. } (\checkmark) \end{aligned}$$

$$\begin{aligned} M_{o_w} &= W d_w \\ &= 9(2 + 2 \cos 15^\circ - 4 \sin 15^\circ) \\ &= 26.1 \text{ lb-in. } (2) \end{aligned}$$

2/154



$$AC = 0.125 \sin 60^\circ = 0.1083 \text{ m}$$

$$\alpha = \sin^{-1} \frac{0.1083}{0.300} = 21.2^\circ$$

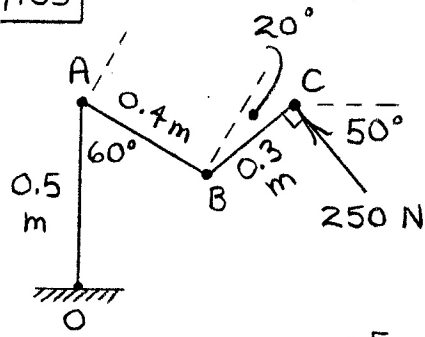
$$BC = 0.300 \cos \alpha = 0.280 \text{ m}$$

$$BO = 0.280 - 0.125 \cos 60^\circ = 0.217 \text{ m}$$

$$\curvearrowright M_o = 720 = P \sin \alpha (BO) = P \sin 21.2^\circ (0.217)$$

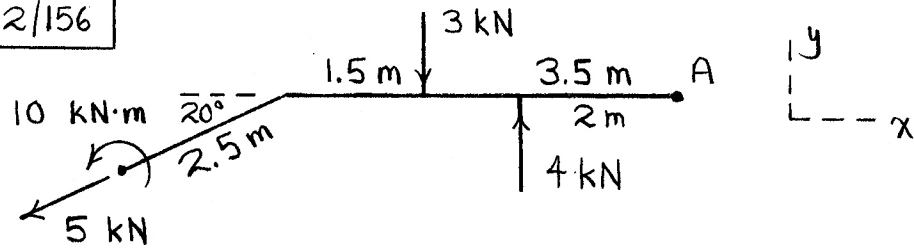
$$\underline{P = 9.18 \text{ kN}}$$

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$$\begin{aligned} \sum M_O &= 250 \cos 50^\circ [0.5 - 0.4 \cos 60^\circ + 0.3 \sin 40^\circ] \\ &\quad + 250 \sin 50^\circ [0.4 \sin 60^\circ + 0.3 \cos 40^\circ] \\ &= \underline{189.6 \text{ N}\cdot\text{m} \text{ CCW}} \end{aligned}$$

2/156



$$R_x = \sum F_x = -5 \cos 20^\circ = -4.70 \text{ kN}$$

$$R_y = \sum F_y = -5 \sin 20^\circ + 4 - 3 = -0.710 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = 4.75 \text{ kN}$$

$$\begin{aligned} \sum M_A &= 10 + 5 \sin 20^\circ (5) + 3(3.5) - 4(2) \\ &= \underline{21.1 \text{ kN}\cdot\text{m}} \end{aligned}$$

$$\underline{2|157} \text{ At } A: R = \sum F = 200 + 180 - 300 = \underline{80 \text{ lb } (\downarrow)}$$

$$\sum M_A = 200(8) + 180(28) - 300(18) = \underline{1240 \text{ lb-in.}}$$

$$80 \text{ lb} \downarrow A \text{ --- } x \quad = \quad A \text{ --- } x \downarrow 80 \text{ lb}$$
$$1240 \text{ lb-in.} \quad \quad \quad 80x = 1240, \underline{x = 15.5 \text{ in.}}$$



2/158 | Coordinates of A:  $(x_A, y_A, z_A) = (0, r, 0)$

Coordinates of B:  $(x_B, y_B, z_B) = (h, r \cos \theta, r \sin \theta)$

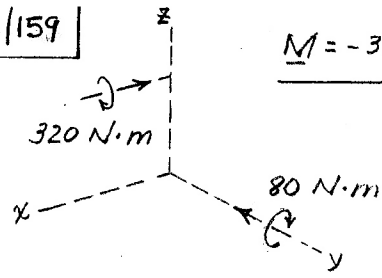
So  $\underline{r}_{AB} = h \underline{i} + (r \cos \theta - r) \underline{j} + r \sin \theta \underline{k}$

$$\text{and } \underline{F} = F \underline{r}_{AB} = F \left[ \frac{h \underline{i} + r (\cos \theta - 1) \underline{j} + r \sin \theta \underline{k}}{\sqrt{h^2 + [r (\cos \theta - 1)]^2 + [r \sin \theta]^2}} \right]$$

$$= F \left[ \frac{h \underline{i} + r (\cos \theta - 1) \underline{j} + r \sin \theta \underline{k}}{\sqrt{h^2 + 2r^2 (1 - \cos \theta)}} \right]$$

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2/159



$$\underline{M} = -320\underline{i} - 80\underline{j} \text{ N}\cdot\text{m}$$

$$\cos \theta_x = \frac{M_x}{|M|} = \frac{-320}{\sqrt{320^2 + 80^2}} = -0.970$$

$$\underline{2/160} \quad \underline{\underline{R = P(0.6\mathbf{j} + 0.8\mathbf{k})}}$$

$$\begin{cases} M_{Bx} = Pb(-0.6 + 0.8) = 0.2Pb \\ M_{By} = 0.8Pb \\ M_{Bz} = -0.6Pb \end{cases}$$

$$\text{So } \underline{\underline{M_B = Pb(0.2\mathbf{i} + 0.8\mathbf{j} - 0.6\mathbf{k})}}$$

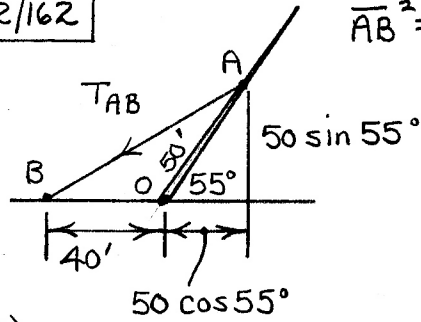
$$\frac{2}{161} \quad \underline{M}_{100} = -100 (0.200) \underline{i} = -20 \underline{i} \text{ N}\cdot\text{m}$$

$$\begin{aligned} \underline{M}_{80} &= 80 (0.180 \cos 20^\circ) (-\underline{j} \sin 30^\circ - \underline{k} \cos 30^\circ) \\ &= -6.77 \underline{j} - 11.72 \underline{k} \text{ N}\cdot\text{m} \end{aligned}$$

$$\underline{M}_{120} = -120 (0.300 \cos 45^\circ) \underline{k} = -25.5 \underline{k} \text{ N}\cdot\text{m}$$

$$\text{So } \underline{M} = \underline{-20i - 6.77j - 37.2k} \text{ N}\cdot\text{m}$$

2/162



$$\overline{AB}^2 = 40^2 + 50^2 - 2(40)(50) \cos 125^\circ$$

$$\overline{AB} = 80.0'$$

(a)

$$\begin{aligned} \underline{T}_{AB} &= 600 \left[ -\frac{40 + 50 \cos 55^\circ}{80.0} \cos 40^\circ \underline{i} - \frac{40 + 50 \cos 55^\circ}{80.0} \sin 40^\circ \underline{j} \right. \\ &\quad \left. - \frac{50 \sin 55^\circ}{80.0} \underline{k} \right] \\ &= \underline{-395 \underline{i} - 331 \underline{j} - 307 \underline{k} \text{ lb}} \end{aligned}$$

(b) Carry out  $\underline{M}_O = \underline{r}_{OB} \times \underline{T}_{AB}$ , where  $\underline{r}_{OB} = 40'(-\cos 40^\circ \underline{i} - \sin 40^\circ \underline{j})$ :  $\underline{M}_O = \underline{7900 \underline{i} - 9420 \underline{j} \text{ lb-ft}}$   
 $\underline{M}_{Ox} = 7900 \text{ lb-ft}, \underline{M}_{Oy} = -9420 \text{ lb-ft}, \underline{M}_{Oz} = 0$

(c)  $\underline{T}_{Ao} = \underline{T}_{AB} \cdot \underline{n}_{Ao}$ , where  $\underline{n}_{Ao} = -\cos 55^\circ \cos 40^\circ \underline{i} - \cos 55^\circ \sin 40^\circ \underline{j} - \sin 55^\circ \underline{k}$ . Carry out to obtain  $\underline{T}_{Ao} = \underline{547 \text{ lb}}$

$$\underline{2/163} \quad \underline{R} = \underline{\Sigma F} = 500 \cos 45^\circ (\underline{i}) + 400 \underline{j} - (600 + 500 \sin 45^\circ) \underline{k}$$

$$= 354 \underline{i} + 400 \underline{j} - 954 \underline{k} \quad \text{lb}$$

$$R = \sqrt{354^2 + 400^2 + 954^2} = \underline{1093 \text{ lb}}$$

$$\underline{M} = [500 \cos 45^\circ (3) - 600(3) - 400(10)] \underline{i}$$

$$+ [500 \cos 45^\circ (6) + 500 \sin 45^\circ (7) + 600(6)] \underline{j}$$

$$+ [500 \sin 45^\circ (3) + 400(3)] \underline{k}$$

$$= -4739 \underline{i} + 8196 \underline{j} + 2261 \underline{k} \quad \text{lb-ft}$$

$$M = \sqrt{4739^2 + 8196^2 + 2261^2} = \underline{9730 \text{ lb-ft}}$$

$$\underline{2/164} \quad \underline{R} = 800 \left[ -\sin 30^\circ \cos 20^\circ \underline{i} + \sin 30^\circ \sin 20^\circ \underline{j} + \cos 30^\circ \underline{k} \right]$$

$$= \underline{-376 \underline{i} + 137 \underline{j} + 693 \underline{k} \quad \text{N}}$$

$$\underline{M}_O = \underline{r}_{OB} \times \underline{F}$$

$$\underline{r}_{OB} = [300 \sin 20^\circ \underline{i} + 300 \cos 20^\circ \underline{j} + 250 \underline{k}] \text{mm}$$

$$\underline{M}_O = \underline{161 \underline{i} - 165 \underline{j} + 120 \underline{k} \quad \text{N}\cdot\text{m}}$$

\*2/165

$$\Sigma F_x = 0: -360 - 240 \sin \theta + T \sin 30^\circ + 400 \cos 30^\circ = 0 \quad (1)$$

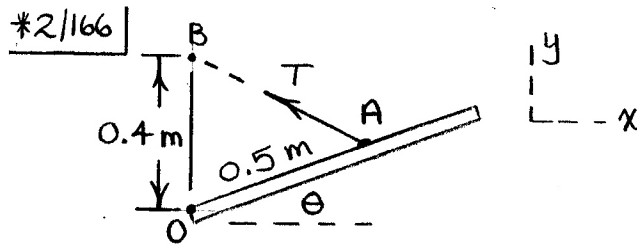
$$\Sigma F_y = 600: 240 \cos \theta + T \cos 30^\circ + 400 \sin 30^\circ = 600 \quad (2)$$

Numerical solution of Eqs. (1) & (2):

$$\underline{\theta = 21.7^\circ, \quad T = 204 \text{ lb}}$$

(We could eliminate  $T$  between Eqs. (1) & (2),  
but the resulting equation is still transcendental.)





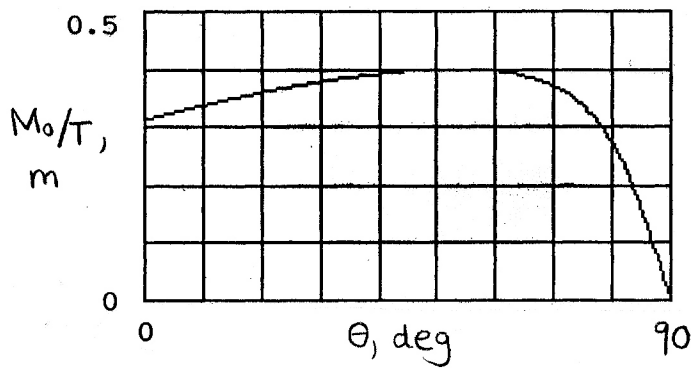
$$\underline{n}_{AB} = \frac{\underline{r}_{AB}}{r_{AB}} = \frac{\underline{r}_B - \underline{r}_A}{r_{AB}}$$

$$\begin{aligned} \underline{r}_B - \underline{r}_A &= 0.4\mathbf{j} - 0.5(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) \\ &= -0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j} \\ \therefore \underline{n}_{AB} &= \frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{(0.5\cos\theta)^2 + (0.4 - 0.5\sin\theta)^2}} \end{aligned}$$

Then  $\underline{T} = T\underline{n}_{AB}$  and  $\underline{M}_O = \underline{r}_{OB} \times \underline{T} = 0.4\mathbf{j} \times \underline{T}$

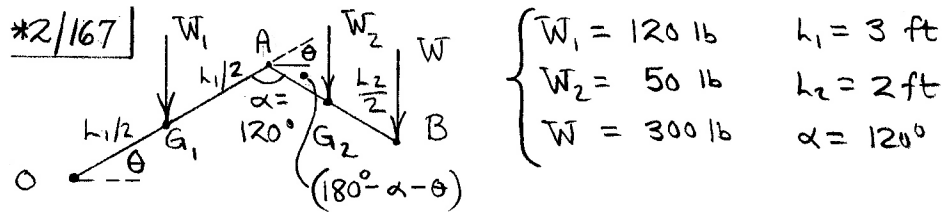
Carry out  $\hat{\imath}$  obtain

$$\frac{M_O}{T} = \frac{0.2 \cos\theta}{\sqrt{0.41 - 0.4 \sin\theta}} \quad (\text{in m})$$



$$\left(\frac{M_O}{T}\right)_{\min} = 0 \quad @ \quad \theta = 90^\circ$$

$$\left(\frac{M_O}{T}\right)_{\max} = 0.4 \text{ m} \quad @ \quad \theta = 53.1^\circ$$



$$+2 M_o = W_1 \frac{l_1}{2} \cos \theta + W_2 \left( l_1 \cos \theta + \frac{l_2}{2} \cos (180^\circ - \alpha - \theta) \right) + W \left( l_1 \cos \theta + l_2 \cos (180^\circ - \alpha - \theta) \right)$$

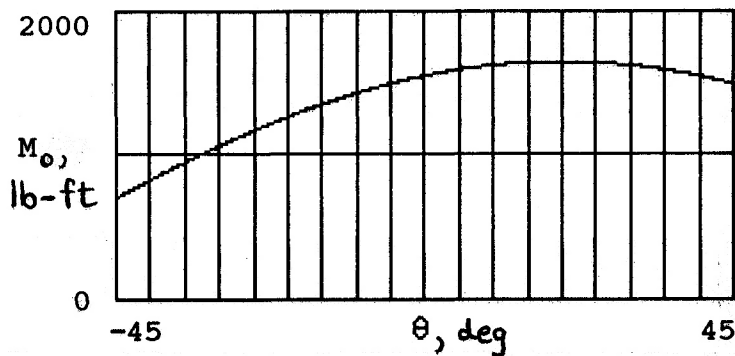
With the above numbers :

$$M_o = 1230 \cos \theta + 650 \cos (60^\circ - \theta) \quad (\text{in lb-ft})$$

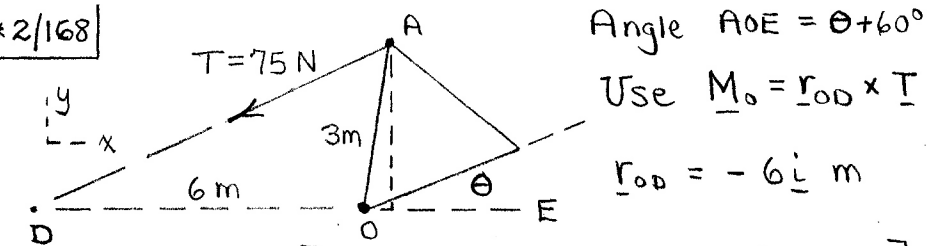
(see plot below)

$$\text{For } (M_o)_{\max} : \frac{dM_o}{d\theta} = -1230 \sin \theta + 650 \sin (60^\circ - \theta) = 0$$

$$\text{Numerical solution : } \theta = 19.90^\circ, \quad (M_o)_{\max} = 1654 \text{ lb-ft}$$



\*2/168



Angle  $\text{AOE} = \theta + 60^\circ$

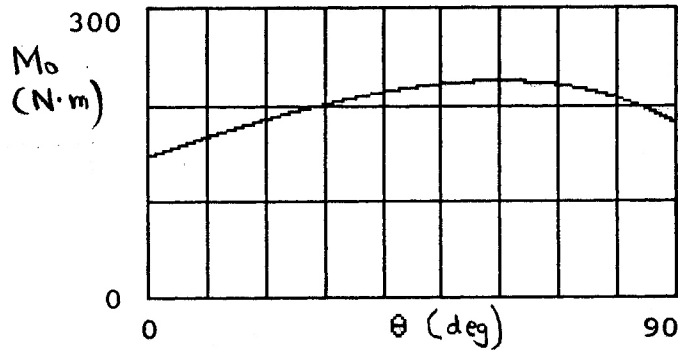
Use  $\underline{M}_o = \underline{r}_{oD} \times \underline{T}$

$$\underline{r}_{oD} = -6\hat{i} \text{ m}$$

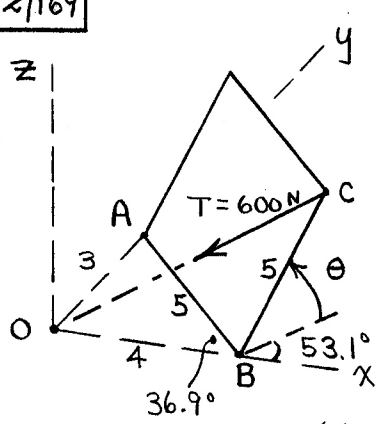
$$\underline{T} = T \underline{n}_{AD} = 75 \left[ \frac{-(6 + 3 \cos(\theta + 60^\circ))\hat{i} - 3 \sin(\theta + 60^\circ)\hat{j}}{\sqrt{[6 + 3 \cos(\theta + 60^\circ)]^2 + [3 \sin(\theta + 60^\circ)]^2}} \right] \text{ N}$$

$$\underline{M}_o = \underline{r}_{oD} \times \underline{T} = \frac{1350 \sin(\theta + 60^\circ) \hat{k}}{\sqrt{45 + 36 \cos(\theta + 60^\circ)}} \text{ N}\cdot\text{m}$$

$M_o$  is a max @  $\theta = 60^\circ : M_o = 225 \text{ N}\cdot\text{m}$



\* 2/169



$$C = C(4 + 5 \cos 53.1^\circ \cos \theta, 5 \sin 53.1^\circ \cos \theta, 5 \sin \theta) \text{ m}$$

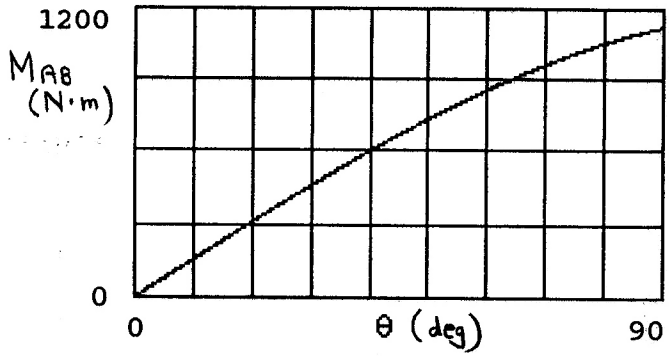
$$\underline{n}_{AB} = 0.8 \underline{i} - 0.6 \underline{j}$$

$$\underline{T} = T \underline{n}_{co} = 600 \left[ \frac{-(4 + 3 \cos \theta) \underline{i} - (4 \cos \theta) \underline{j} - (5 \sin \theta) \underline{k}}{\sqrt{(4 + 3 \cos \theta)^2 + (4 \cos \theta)^2 + (5 \sin \theta)^2}} \right]$$

$$= \frac{600 [-(4 + 3 \cos \theta) \underline{i} - (4 \cos \theta) \underline{j} - (5 \sin \theta) \underline{k}]}{\sqrt{41 + 24 \cos \theta}} \text{ N}$$

$$\underline{M}_B = \underline{r}_{OB} \times \underline{T} = \frac{600}{\sqrt{41 + 24 \cos \theta}} (-20 \sin \theta \underline{j} + 16 \cos \theta \underline{k}) \text{ N}\cdot\text{m}$$

$$\text{Now, } M_{AB} = \underline{M}_B \cdot \underline{n}_{AB} = \frac{7200 \sin \theta}{\sqrt{41 + 24 \cos \theta}} \text{ N}\cdot\text{m}$$



\*2/170 | Length of spring  $L = \sqrt{x^2 + 0.1^2 + 0.15^2}$   
 $= \sqrt{x^2 + 0.0325}$  m

Deflection  $\delta = L - 0.15 = \sqrt{x^2 + 0.0325} - 0.15$  m

Spring force  $F = k\delta = 200[\sqrt{x^2 + 0.0325} - 0.15]$

As a vector,  $\underline{F} = F\underline{n}_{A_0}$ , where

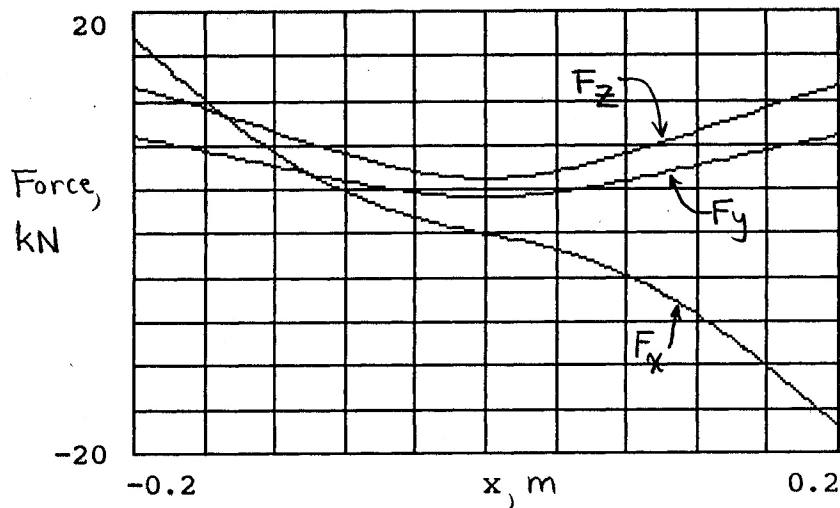
$$\underline{n}_{A_0} = \left[ \frac{-x_A \underline{i} + 0.1 \underline{j} + 0.15 \underline{k}}{\sqrt{x^2 + 0.0325}} \right]$$

So the required force components are

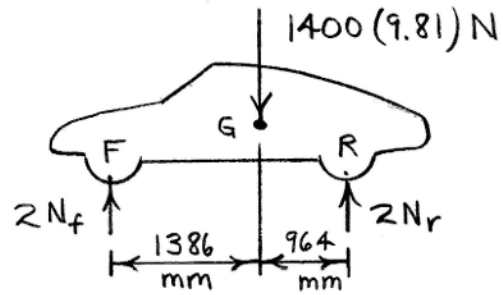
$$F_x = 200[\sqrt{x^2 + 0.0325} - 0.15] \left[ \frac{-x_A}{\sqrt{x^2 + 0.0325}} \right]$$

$$F_y = 200[\sqrt{x^2 + 0.0325} - 0.15] \left[ \frac{0.1}{\sqrt{x^2 + 0.0325}} \right]$$

$$F_z = 200[\sqrt{x^2 + 0.0325} - 0.15] \left[ \frac{0.15}{\sqrt{x^2 + 0.0325}} \right]$$



3/1



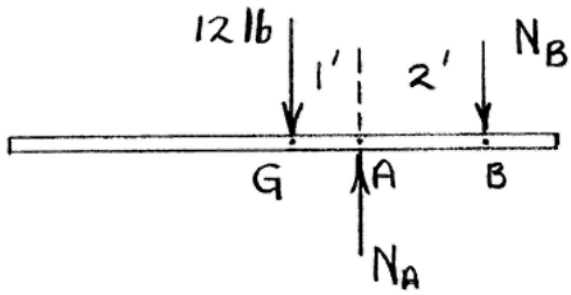
$$\uparrow \Sigma F = 0 : 2N_f + 2N_r - 1400(9.81) = 0$$

$$\curvearrowright \Sigma M_F = 0 : -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$$

$$\text{Solution : } \begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$$

Assumes  $G$  midway between left and right wheels.

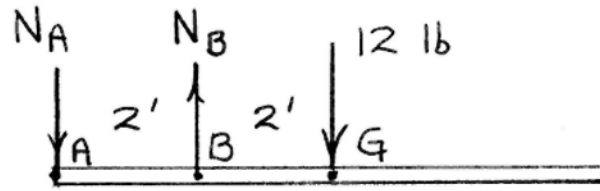
3/2



$$\uparrow + \sum M_B = 0 : 12(3) - N_A(2) = 0$$

$$\underline{N_A = 18 \text{ lb}}$$

3/3

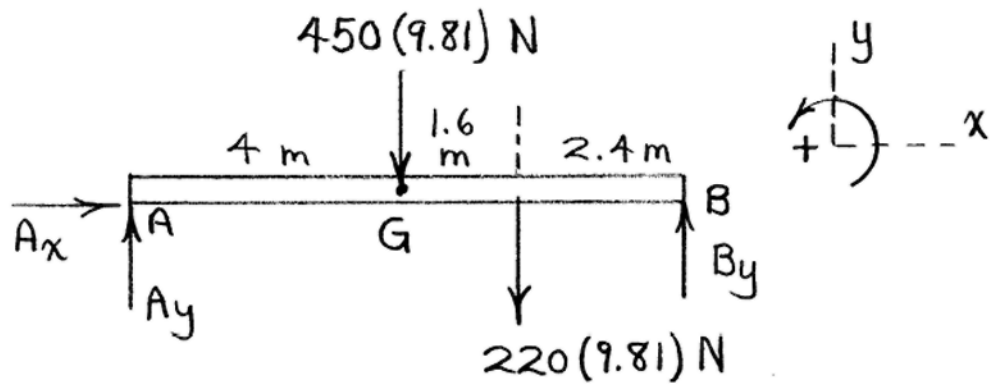


$$\uparrow \sum M_A = 0 : N_B(2) - 12(4) = 0, \quad \underline{N_B = 24 \text{ lb}}$$

$$\uparrow \sum F = 0 : -N_A + 24 - 12 = 0, \quad \underline{N_A = 12 \text{ lb}}$$



3/4

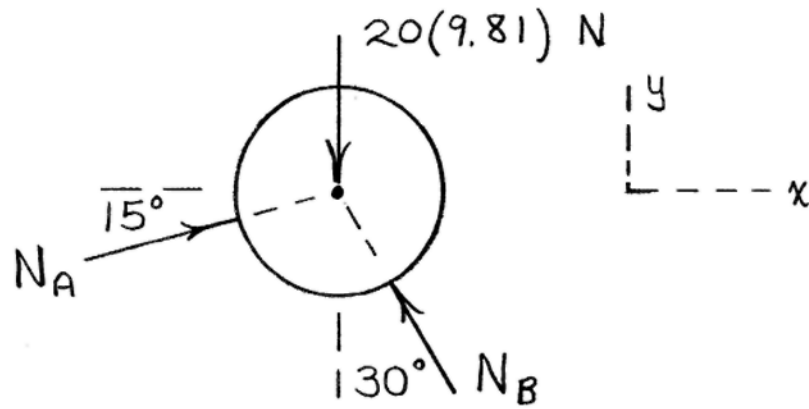


From  $\Sigma F_x = 0$ ,  $A_x = 0$

$$\Sigma M_A = 0 : -450(9.81)4 - 220(9.81)(5.6) + B_y(8) = 0 \quad , \quad \underline{B_y = 3720 \text{ N}}$$

$$\Sigma F_y = 0 : A_y - 450(9.81) - 220(9.81) + 3720 = 0$$
$$\underline{A_y = 2850 \text{ N}}$$

3/5

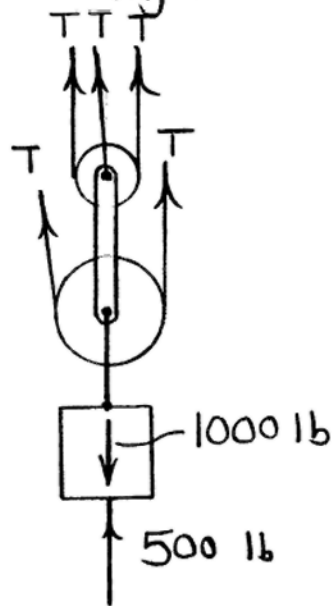


$$\left\{ \begin{array}{l} \sum F_x = 0: N_A \cos 15^\circ - N_B \sin 30^\circ = 0 \quad (1) \\ \sum F_y = 0: N_A \sin 15^\circ + N_B \cos 30^\circ - 20(9.81) = 0 \quad (2) \end{array} \right.$$

Solution :

$$\begin{cases} N_A = 101.6 \text{ N} \\ N_B = 196.2 \text{ N} \end{cases}$$

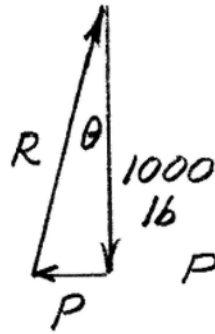
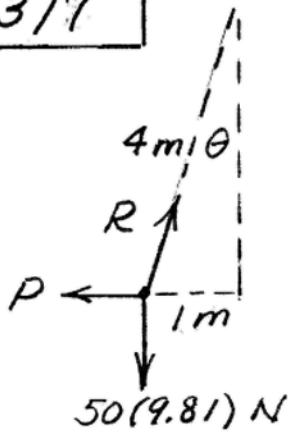
3/6 | FBD of 1000-lb weight and lower pair of pulleys:



$$\uparrow \Sigma F = 0: 5T + 500 - 1000 = 0, \quad \underline{T = 100 \text{ lb}}$$

(We assume that the nonverticality of some of the cables is negligible.)

3/7



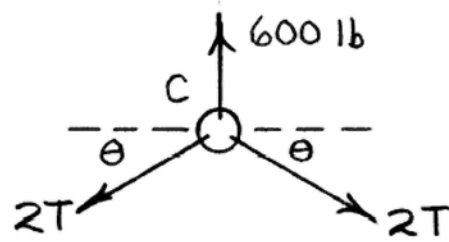
$$P = 50(9.81) \tan \theta$$

$$\sin \theta = 1/4$$

$$\tan \theta = 1/\sqrt{4^2 - 1^2} = 0.258$$

$$P = 50(9.81)(0.258) = \underline{126.6 \text{ N}}$$

3/8 | FBD of junction ring C:

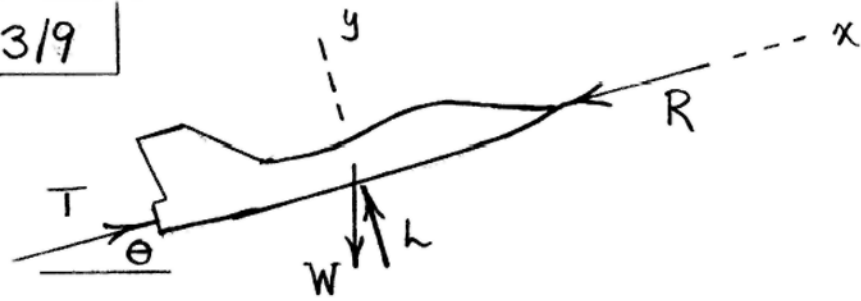


$$\theta = \tan^{-1} \frac{10}{36/2}$$
$$= 29.1^\circ$$

$$\uparrow + \sum F = 0 : 600 - 4T \sin 29.1^\circ = 0$$

$$\underline{T = 309 \text{ lb}}$$

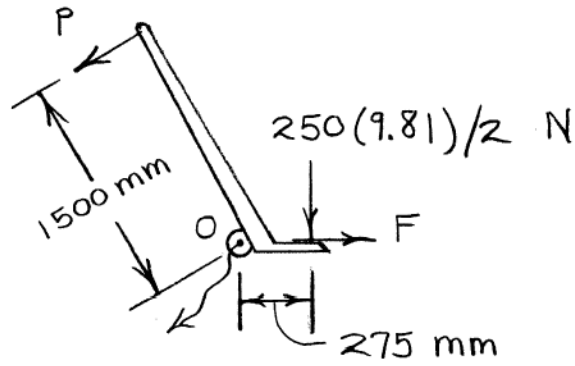
3/9



$$\Sigma F_x = 0 : T - R - W \sin \theta = 0$$

$$n = \frac{T - R}{W} = \underline{\sin \theta}$$

3/10

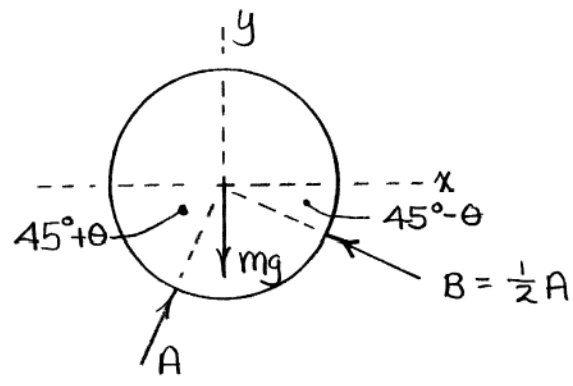


$$\sum M_O = 0 : P(1500) - \frac{1}{2}(250)(9.81)(275) = 0$$

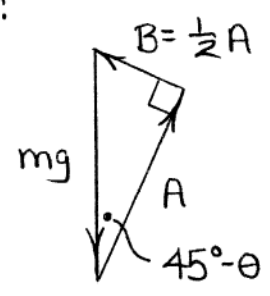
$$\underline{P = 225 \text{ N}}$$

(assumes that the moment of the friction force  $F$  is small compared to the other moments)

3/11



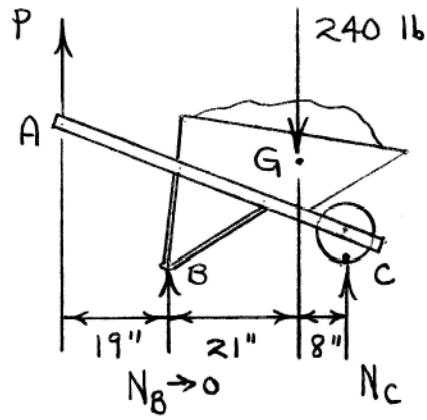
$\Sigma F = 0 :$



$$\tan(45^\circ - \theta) = \frac{A/2}{A} = \frac{1}{2}$$
$$45^\circ - \theta = 26.6^\circ$$
$$\theta = \underline{18.43^\circ}$$



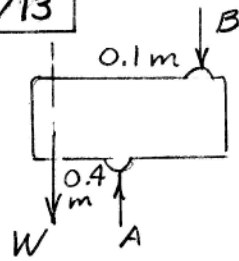
3/12



$$\sum M_c = 0: P(48) - 240(8) = 0$$

$$\underline{P = 40 \text{ lb}}$$

3/13



$$W = 300(9.81) = 2943 \text{ N}$$

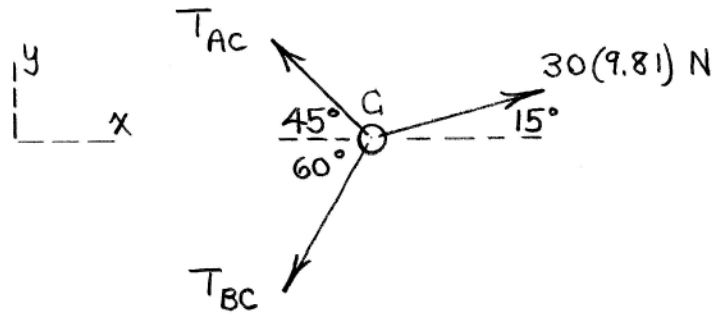
$$\sum M_A = 0; 2943(0.4) - B(0.6) = 0$$

$$B = 1962 \text{ N or } 1.962 \text{ kN}$$

$$\sum F = 0; A = 2943 + 1962$$

$$= 4910 \text{ N or } 4.91 \text{ kN}$$

3/14 | FBD of junction ring C:

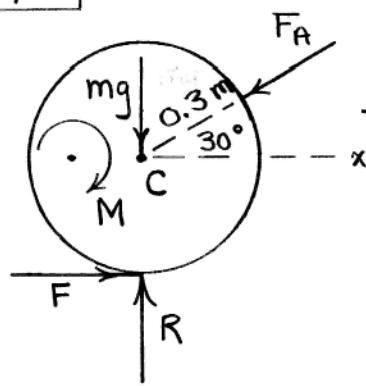


$$\begin{cases} \sum F_x = 0 : -T_{AC} \cos 45^\circ - T_{BC} \cos 60^\circ + 30(9.81) \cos 15^\circ = 0 \\ \sum F_y = 0 : T_{AC} \sin 45^\circ - T_{BC} \sin 60^\circ + 30(9.81) \sin 15^\circ = 0 \end{cases}$$

Solve simultaneously to obtain

$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

3/15



$$mg = 100(9.81) = 981 \text{ N}$$

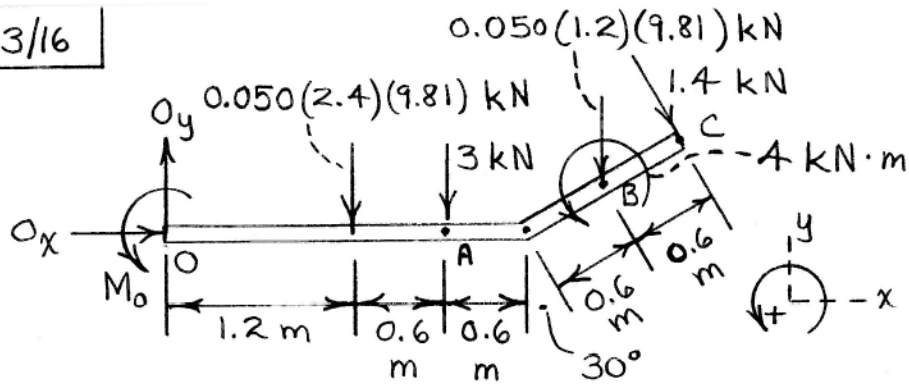
$$M = 60 \text{ N}\cdot\text{m}$$

$$\sum M_C = 0: 60 - 0.3F = 0$$

$$F = 200 \text{ N}$$

$$\sum F_x = 0: -F_A \cos 30^\circ + 200 = 0$$

$$\underline{F_A = 231 \text{ N}}$$



$$\sum F_x = 0 : O_x + 1.4 \sin 30^\circ = 0$$

$$O_x = -0.7 \text{ kN}$$

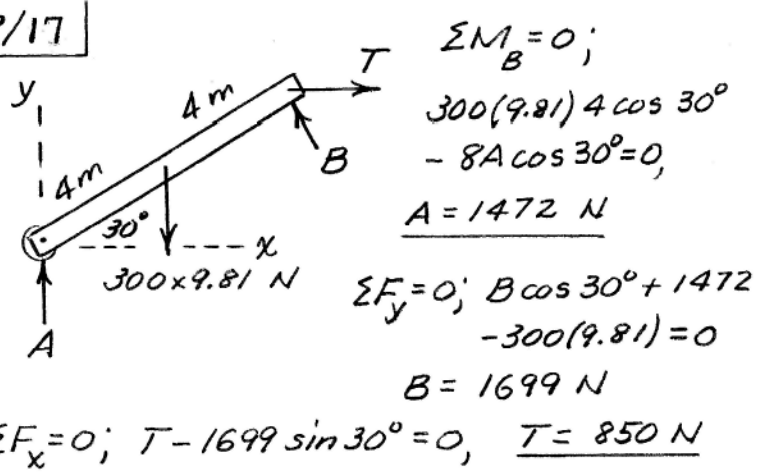
$$\sum F_y = 0 : O_y - 0.050(2.4)(9.81) - 3 - 1.4 \cos 30^\circ - 0.050(1.2)(9.81) = 0, \quad O_y = 5.98 \text{ kN}$$

$$\sum M_o = 0 : M_o - 0.050(2.4)(9.81)(1.2) - 3(1.8)$$

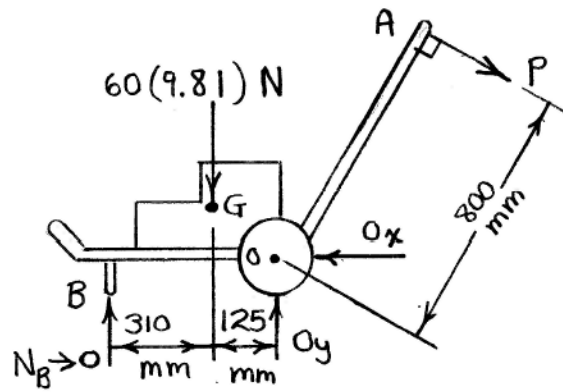
$$- 0.050(1.2)(9.81)(2.4 + 0.6 \cos 30^\circ) + 4$$

$$- 1.4(2.4 \cos 30^\circ + 1.2) = 0, \quad M_o = 9.12 \text{ kN}\cdot\text{m}$$

3/17



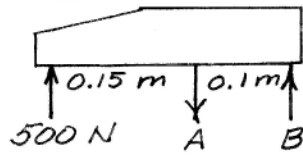
3/18



$$\sum M_O = 0: 60(9.81)(125) - P(800) = 0$$

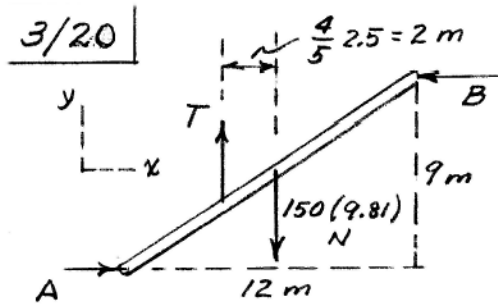
$$\underline{P = 92.0 \text{ N}}$$

3/19



$$\sum M_B = 0; 500(0.25) - 0.1A = 0$$
$$\underline{A = 1250 \text{ N}}$$





$$\sum F_y = 0; T - 150(9.81) = 0$$

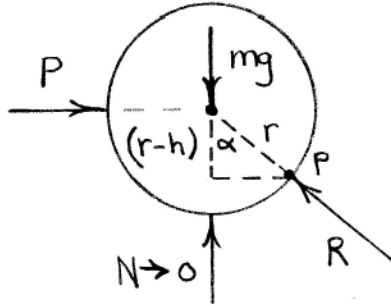
$$T = 1472 \text{ N}$$

$$\sum F_x = 0; A = B$$

$$\sum M = 0; 1472(2) - A(9) = 0$$

$$\underline{A = B = 327 \text{ N}}$$

3/21



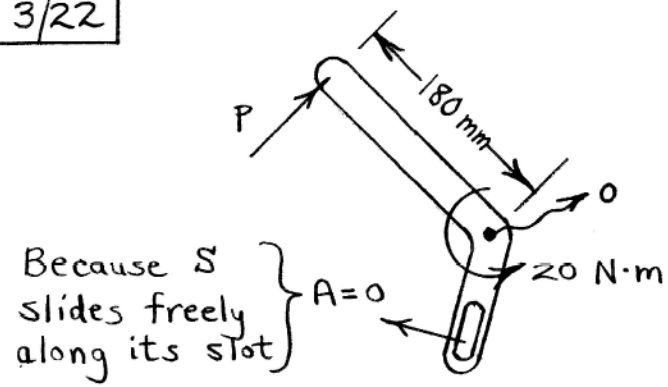
$$\cos \alpha = \frac{r-h}{r}$$

$$\frac{r}{(r-h)} = \frac{\sqrt{r^2 - (r-h)^2}}{\sqrt{2rh - h^2}}$$

$$\sin \alpha = \frac{\sqrt{2rh - h^2}}{r}$$

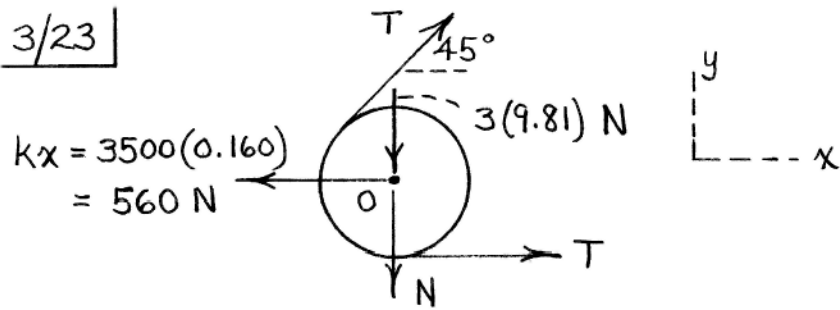
$$\begin{aligned} +\curvearrowright \sum M_P = 0: & P(r-h) - mgr \sin \alpha = 0 \\ \Rightarrow P = & \frac{mg \sqrt{2rh - h^2}}{r-h} \end{aligned}$$

3/22



$$\sqrt{+} \sum M_o = 0: 20 - P(0.180) = 0, \quad \underline{P = 111.1 \text{ N}}$$

3/23



$$\sum F_x = 0 : T(1 + \cos 45^\circ) - 560 = 0$$

$$\underline{T = 328 \text{ N}}$$

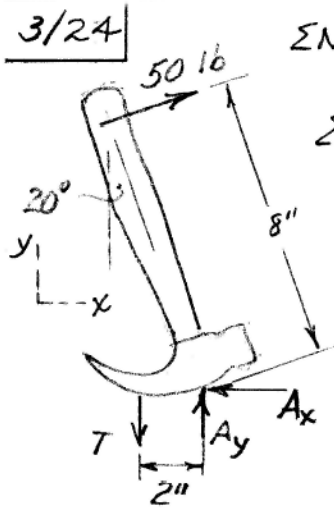
$$\sum F_y = 0 : 328(\sin 45^\circ) - 3(9.81) - N = 0$$

$$N = 203 \text{ N (down)}$$

The force on the guide is then

$$\underline{N = 203 \text{ N up}}$$

3/24



$$\sum M_A = 0; 50(8) - 2T = 0, \underline{T = 200 \text{ lb}}$$

$$\sum F_x = 0; 50 \cos 20^\circ - A_x = 0$$

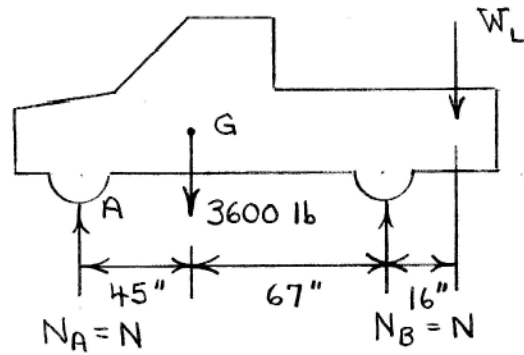
$$A_x = 46.98 \text{ lb}$$

$$\sum F_y = 0; A_y + 50 \sin 20^\circ - 200 = 0$$

$$A_y = 182.9 \text{ lb}$$

$$A = \sqrt{(46.98)^2 + (182.9)^2} = \underline{188.8 \text{ lb}}$$

3/25



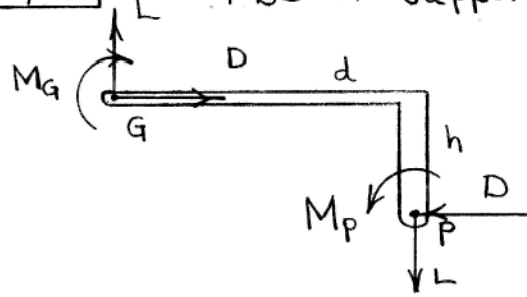
$$\curvearrowright \sum M_A = 0: 3600(45) - N(112) + W_L(128) = 0$$

$$+\uparrow \sum F = 0: 2N - 3600 - W_L = 0$$

Solve to obtain  $N = 2075 \text{ lb}$

$$\underline{W_L = 550 \text{ lb}}$$

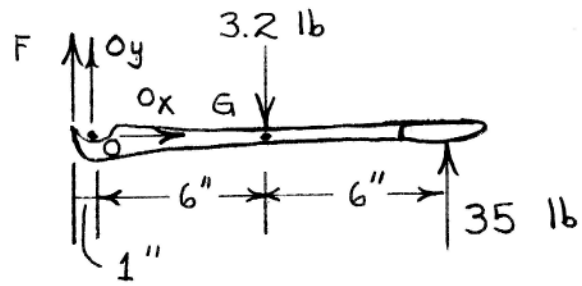
3/26 | FBD of support & model :



$$\curvearrow + \sum M_P = 0: M_P - M_G - Ld - Dh = 0$$

$$\underline{M_G = M_P - Ld - Dh}$$

3/27

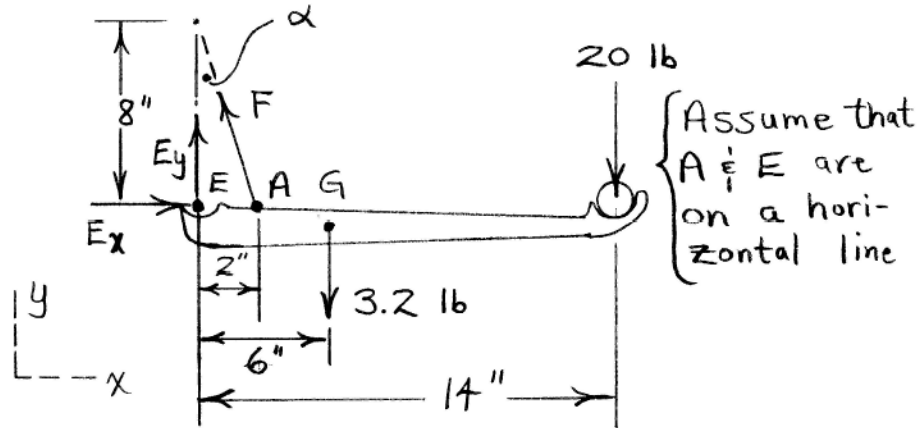


$$\begin{aligned} \sum M_o = 0: & -F(1) - 3.2(6) + 35(12) = 0 \\ & \underline{F = 401 \text{ lb}} \end{aligned}$$



3/28

$$\alpha = \tan^{-1} \frac{2}{8} = 14.04^\circ$$



$$\sum M_E = 0: F \cos 14.04^\circ (2) - 3.2(6) - 20(14) = 0$$

$$F = 154.2 \text{ lb}$$

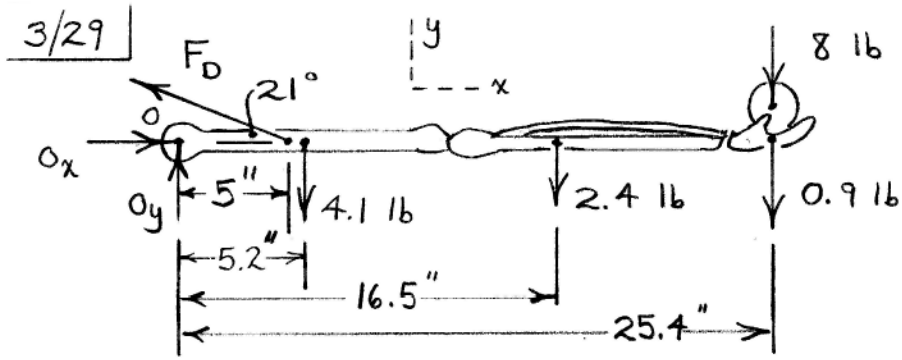
$$\sum F_x = 0: -154.2(\sin 14.04^\circ) + E_x = 0$$

$$E_x = 37.4 \text{ lb}$$

$$\sum F_y = 0: 154.2 \cos 14.04^\circ - 3.2 - 20 + E_y = 0$$

$$E_y = -126.4 \text{ lb}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{37.4^2 + 126.4^2} = 131.8 \text{ lb}$$

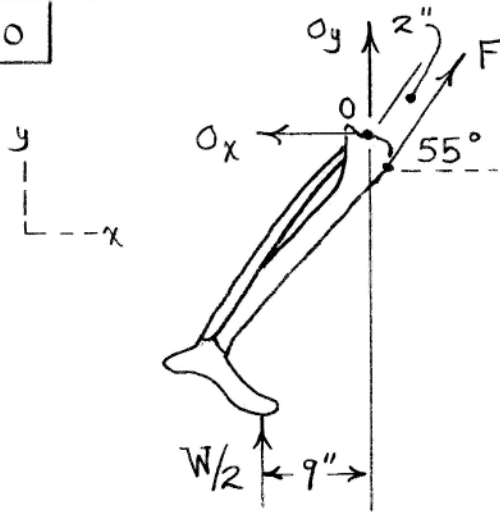


$$\begin{aligned} \curvearrowright \sum M_o = 0 : & (F_D \sin 21^\circ)(5) - 4.1(5.2) - 2.4(16.5) \\ & - (8 + 0.9)(25.4) = 0, \quad \underline{F_D = 160.2 \text{ lb}} \end{aligned}$$

$$\begin{aligned} \rightarrow \sum F_x = 0 : & O_x - 160.2 \cos 21^\circ = 0 \\ & \underline{O_x = 149.5 \text{ lb}} \end{aligned}$$

$$\begin{aligned} \uparrow \sum F_y = 0 : & O_y + 160.2 \sin 21^\circ - 4.1 - 2.4 \\ & - 8.9 = 0, \quad \underline{O_y = -42.2 \text{ lb}} \end{aligned}$$

3/30



$$\sum M_o = 0 : F(2) - \frac{W}{2}(9) = 0, \quad \underline{F = 2.25W}$$

$$\sum F_x = 0 : -O_x + 2.25W \cos 55^\circ = 0, \quad O_x = 1.291W$$

$$\sum F_y = 0 : \frac{W}{2} + O_y + 2.25W \sin 55^\circ = 0$$

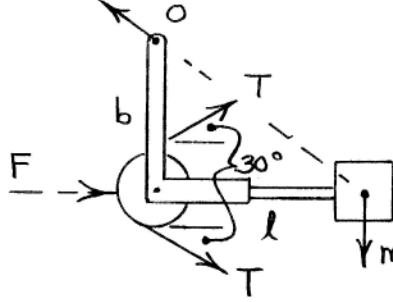
$$O_y = -2.34W$$

$$O = \sqrt{O_x^2 + O_y^2} = \sqrt{(1.291W)^2 + (2.34W)^2}$$

$$= \underline{2.67W}$$

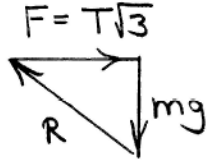
3/31 | R

Replace 2 T's by  $F = 2T \cos 30^\circ$   
 $= T\sqrt{3}$



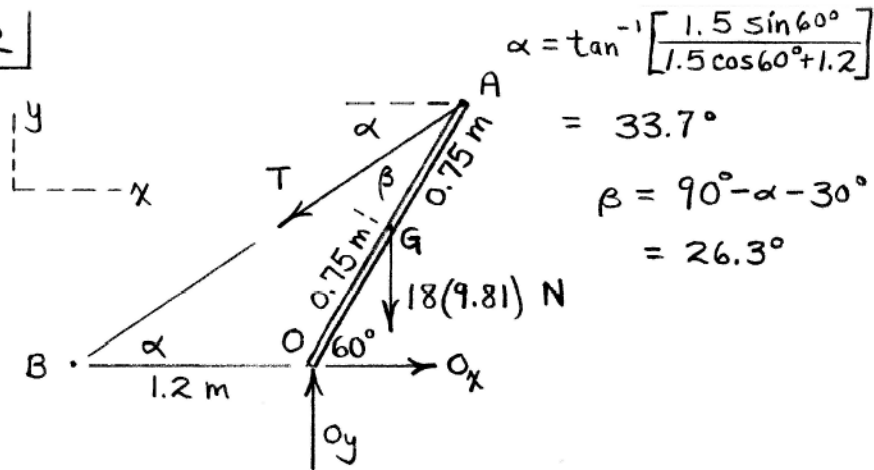
$$\sum M_o = 0: mg l - T\sqrt{3} b = 0$$

$$l = \frac{T\sqrt{3}}{mg} b$$



$$R = \sqrt{(T\sqrt{3})^2 + (mg)^2} = \sqrt{3T^2 + m^2 g^2}$$

3/32



$$\sum M_O = 0: T \sin 33.7^\circ (1.2) - 18(9.81)(0.75) \cos 60^\circ = 0$$

$$T = 99.5 \text{ N}$$

$$\sum F_x = 0: -99.5 \cos 33.7^\circ + O_x = 0$$

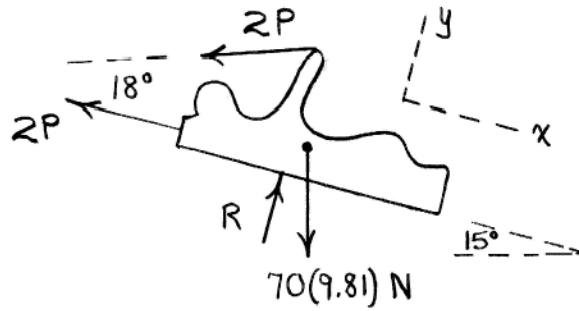
$$O_x = 82.8 \text{ N}$$

$$\sum F_y = 0: -99.5 \sin 33.7^\circ - 18(9.81) + O_y = 0$$

$$O_y = 232 \text{ N}$$

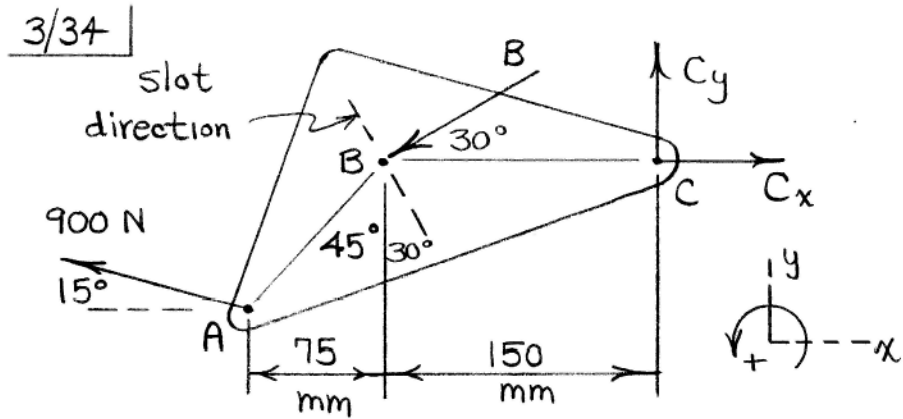
So  $O = 246 \text{ N}$  @  $70.3^\circ$  CCW from  $+x$ -axis

3/33



$$\Sigma F_x = 0: 70(9.81) \sin 15^\circ - 2P - 2P \cos 18^\circ = 0$$
$$P = \underline{45.5 \text{ N}}$$

$$\Sigma F_y = 0: R - 70(9.81) \cos 15^\circ - 2(45.5) \sin 18^\circ = 0$$
$$R = \underline{691 \text{ N}}$$

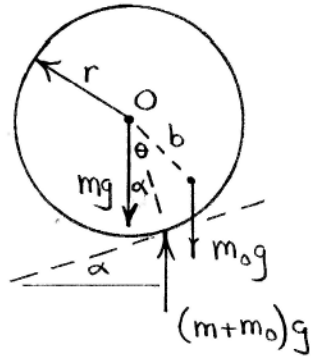


$$\begin{cases} \sum F_x = 0: C_x - B \cos 30^\circ - 900 \cos 15^\circ = 0 \\ \sum F_y = 0: C_y - B \sin 30^\circ + 900 \sin 15^\circ = 0 \\ \sum M_C = 0: B \sin 30^\circ (150) - 900 \cos 15^\circ (75) - 900 \sin 15^\circ (225) = 0 \end{cases}$$

Solution :  $B = 1568 \text{ N}$

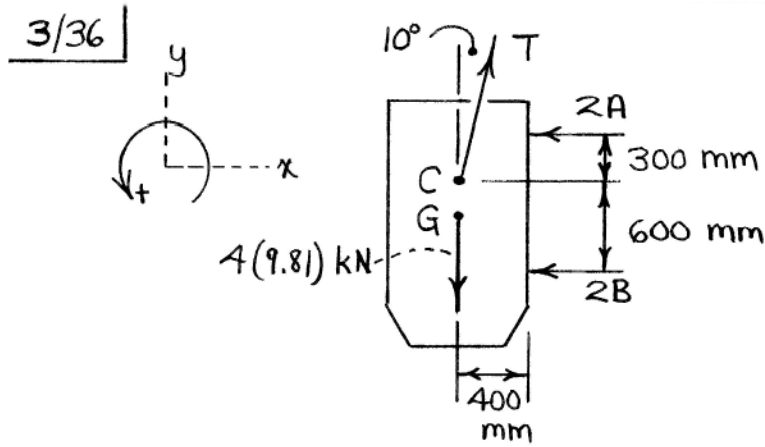
$C_x = 2230 \text{ N}$ ,  $C_y = 551 \text{ N} \Rightarrow C = \underline{2290 \text{ N}}$

(where  $C = \sqrt{C_x^2 + C_y^2}$  )



$$\begin{aligned} \sum M_o = 0 : (m+m_0)g r \sin \alpha - m_0 g b \sin \theta &= 0 \\ \Rightarrow \theta &= \sin^{-1} \left\{ \frac{r}{b} \left( 1 + \frac{m}{m_0} \right) \sin \alpha \right\} \end{aligned}$$





$$\sum F_y = 0: T \cos 10^\circ - 4(9.81) = 0, \quad T = 39.8 \text{ kN}$$

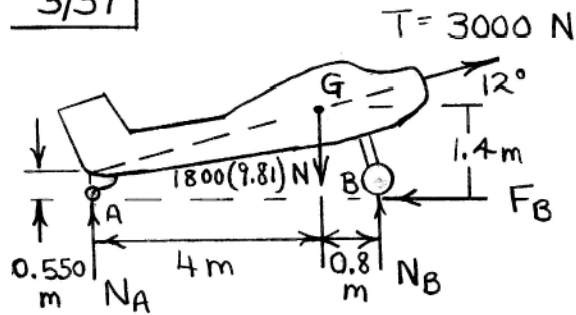
$$\sum M_c = 0: 2A(300) - 2B(600) = 0, \quad A = 2B$$

$$\sum F_x = 0: 39.8 \sin 10^\circ - 2(2B) - 2B = 0$$

$$\underline{B = 1.153 \text{ kN}}$$

$$A = 2B = 2(1.153) = \underline{2.31 \text{ kN}}$$

3/37



Engine off :  $T = 0$  ,  $F_B = 0$

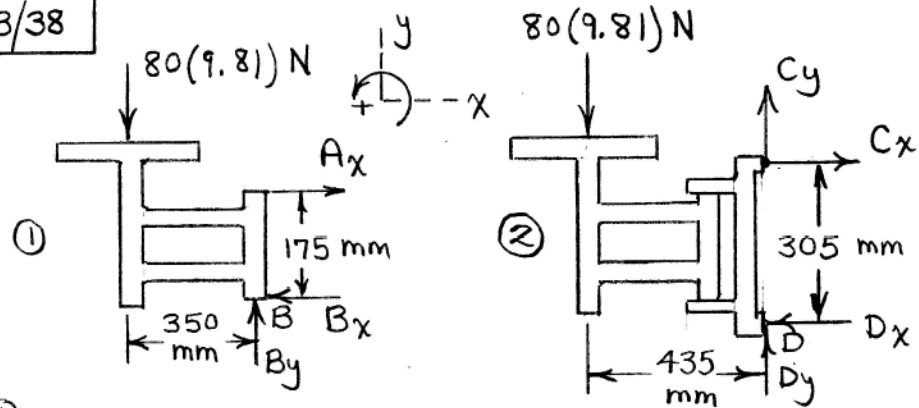
$$\left\{ \begin{array}{l} \sum M_A = 0 : 1800(9.81)4 - N_B(4.8) = 0 \quad N_B = 14720 \text{ N} \\ \sum F_y = 0 : N_A + 14720 - 1800(9.81) = 0, \quad N_A = 2940 \text{ N} \end{array} \right.$$

$$\sum M_A = 0 : 1800(9.81)4 - N'_B(4.8) + 3000 \cos 12^\circ (0.550) = 0$$

$$N'_B = 15,050 \text{ N}$$

$$\sum F_y = 0 : N'_A + 15,050 - 1800(9.81) + 3000 \sin 12^\circ = 0, \quad N'_A = 1983 \text{ N}$$

$$n_A = \frac{N'_A - N_A}{N_A} (100) = \underline{\underline{-32.6\%}}, \quad n_B = \frac{N'_B - N_B}{N_B} = \underline{\underline{2.28\%}}$$



①

$$\sum F_y = 0 : B_y = 80(9.81) = \underline{785 \text{ N}}$$

$$\sum M_B = 0 : -A_x(175) + 80(9.81)(350) = 0$$

$$\underline{A_x = B_x = 1570 \text{ N}}$$

②

$$\sum M_D = 0 : -C_x(305) + 80(9.81)(435) = 0$$

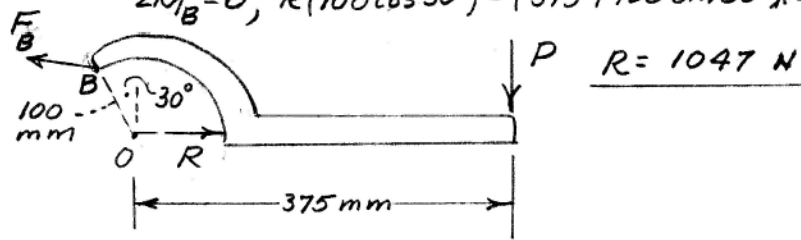
$$\underline{C_x = D_x = 1119 \text{ N}}$$

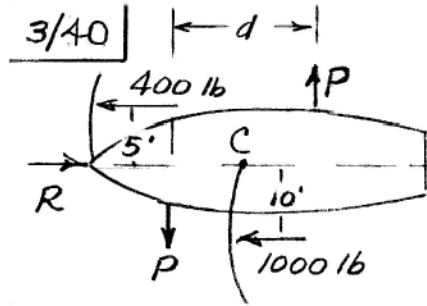
(All results due to L only)

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$$M = Fd; 80 = P(0.375), P = 213 \text{ N}$$

$$\sum M_B = 0; R(100 \cos 30^\circ) - (375 + 100 \sin 30^\circ)(213) = 0$$

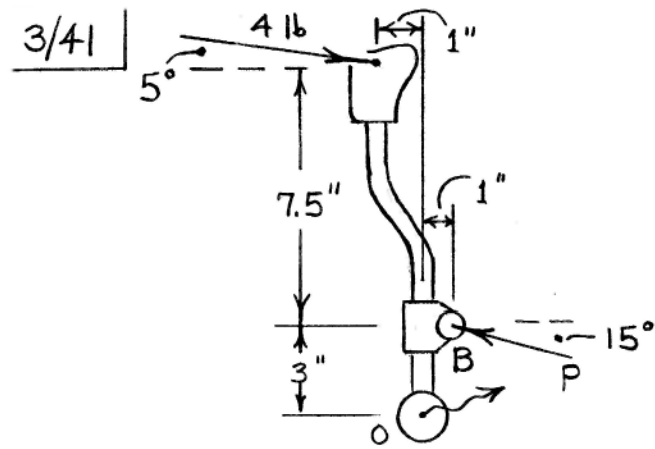




$$\sum M_C = 0; Pd + 400(5) - 1000(10) = 0$$

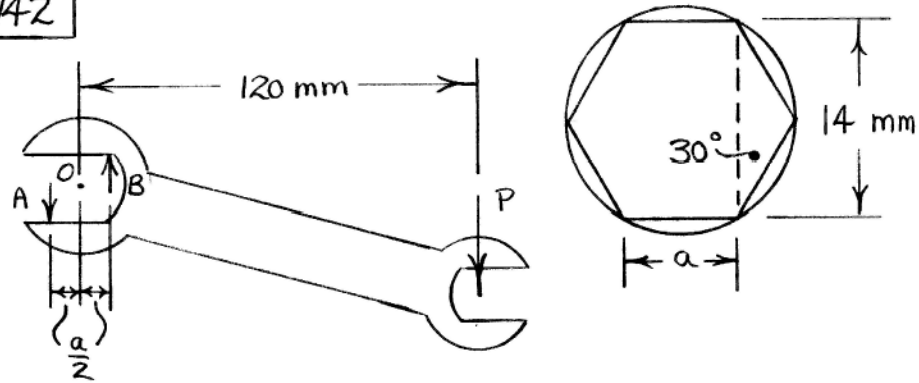
Resultant of lateral forces is couple

$$M = Pd = \underline{8000 \text{ lb-ft}}$$



$$\begin{aligned} \sum M_o = 0: & -4 \cos 5^\circ (10.5) + 4 \sin 5^\circ (1) \\ & + P \cos 15^\circ (3) + P \sin 15^\circ (1) = 0 \\ & \underline{P = 13.14 \text{ lb}} \end{aligned}$$

3/42



$$2a \cos 30^\circ = 14, \quad \frac{a}{2} = 4.04 \text{ mm}$$

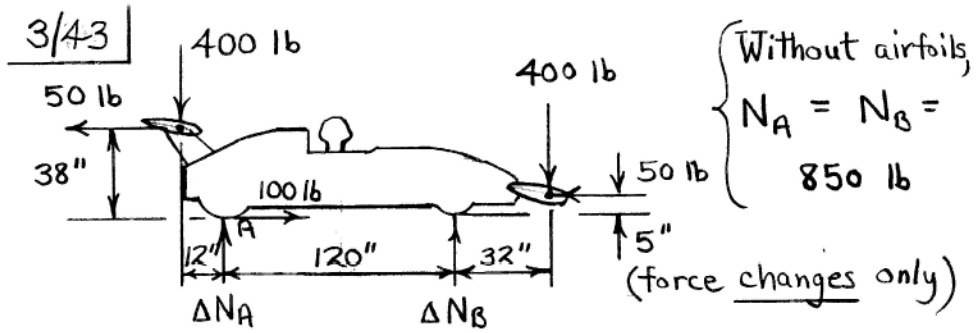
$$\sqrt{+} \sum M_O = 0: 0.120P - 24 = 0, \quad \underline{P = 200 \text{ N}}$$

(for wrench and bolt)

For wrench alone,

$$\sqrt{+} \sum M_A = 0: 200(0.120 + 0.00404) - B(2 \cdot 0.00404) = 0, \quad \underline{B = 3070 \text{ N}}$$

$$+\uparrow \sum F = 0: -A + 3070 - 200 = 0, \quad \underline{A = 2870 \text{ N}}$$



With airfoils,

$$+\uparrow \sum F_y = 0 : \Delta N_A + \Delta N_B - 2(400) = 0$$

$$+\curvearrowleft \sum M_A = 0 : 50(38) + 400(12) + \Delta N_B(120)$$

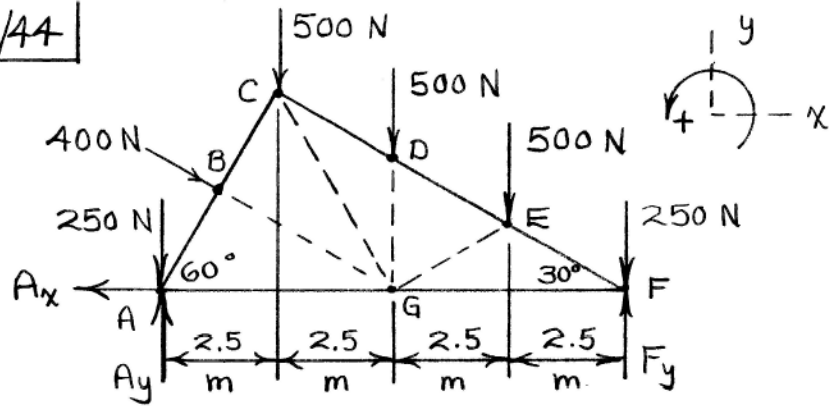
$$+ 50(5) - 400(152) = 0$$

$$\left. \begin{array}{l} \Delta N_A = 351 \text{ lb} \\ \Delta N_B = 449 \text{ lb} \end{array} \right\} \Rightarrow \begin{array}{l} N_A = 850 + 351 = 1201 \text{ lb} \quad (48.0\%) \\ N_B = 850 + 449 = 1299 \text{ lb} \quad (52.0\%) \end{array}$$

Note that a 100-lb propulsive force has been added (at A) to maintain equilibrium.



3/44



$$\sum F_x = 0: -A_x + 400 \cos 30^\circ = 0, \quad \underline{A_x = 346 \text{ N}}$$

$$\sum M_A = 0: 400 \left(\frac{10}{4}\right) + 500(2.5) + 500(5) + 500(7.5) + 250(10) - 10F_y = 0$$

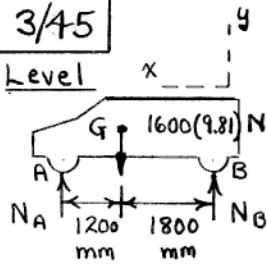
$$\underline{F_y = 1100 \text{ N}}$$

$$\sum F_y = 0: -250 - 400 \sin 30^\circ - 500(3) - 250$$

$$+ 1100 + A_y = 0, \quad \underline{A_y = 1100 \text{ N}}$$

3/45

Level



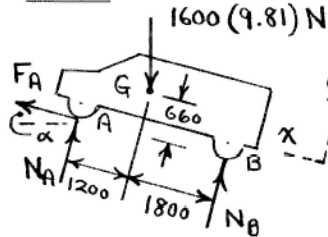
$$\sum M_A = 0: 1600(9.81)(1.2) - N_B(3) = 0$$

$$N_B = 6280 \text{ N}$$

$$\sum F_y = 0: N_A + N_B - 1600(9.81) = 0$$

$$N_A = 9420 \text{ N}$$

Climb



$$\sum M_A = 0: 1600(9.81)(1.2 \cos \alpha)$$

$$+ 1600(9.81)(0.66) \sin \alpha - 3N_B = 0$$

$$N_B = 6590 \text{ N}$$

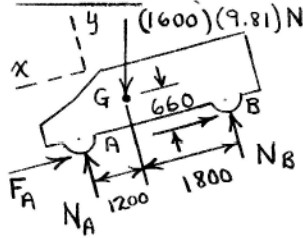
$$\sum F_y = 0: N_A + N_B - 1600(9.81) \cos \alpha = 0$$

$$N_A = 9030 \text{ N}$$

$$\alpha = \tan^{-1}(0.1) = 5.71^\circ$$

$$n_A = \frac{9030 - 9420}{9420} = -4.14\%, \quad n_B = \frac{6590 - 6278}{6278} = 4.98\%$$

Descend



$$\sum M_A = 0: 1600(9.81)(1.2 \cos \alpha)$$

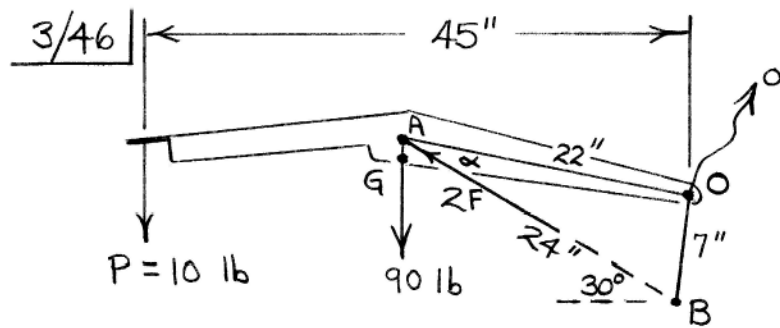
$$- 1600(9.81)(0.66) \sin \alpha - 3N_B = 0$$

$$N_B = 5900 \text{ N}$$

$$\sum F_y = 0: N_A + N_B - 1600(9.81) \cos \alpha = 0$$

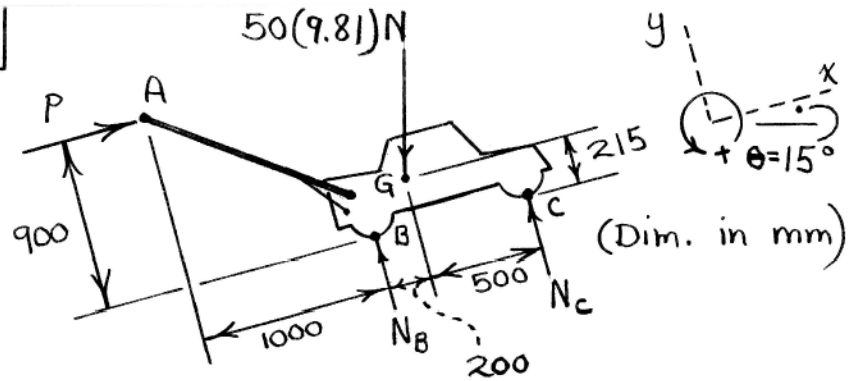
$$N_A = 9710 \text{ N}$$

$$n_A = \frac{9710 - 9420}{9420} = 3.15\%, \quad n_B = \frac{5900 - 6280}{6280} = -5.97\%$$



Law of cosines:  $7^2 = 22^2 + 24^2 - 2(22)(24) \cos \alpha$   
 $\alpha = 16.79^\circ$

$\sum M_O = 0: 10(45) - 2F(22 \sin \alpha) + 90(22 \cos(30^\circ - \alpha)) = 0,$   
 $F = \underline{\underline{187.1 \text{ lb}}}$



$$\Sigma F_x = 0: P - 50(9.81) \sin 15^\circ = 0 \quad (1)$$

$$\Sigma F_y = 0: N_B + N_C - 50(9.81) \cos 15^\circ = 0 \quad (2)$$

$$\Sigma M_C = 0: -P(900) - N_B(700) + 50(9.81)[500 \cos 15^\circ + 215 \sin 15^\circ] = 0 \quad (3)$$

Solution to Eqs. (1)-(3):

$$P = 127.0 \text{ N}$$

$$N_B = 214 \text{ N}$$

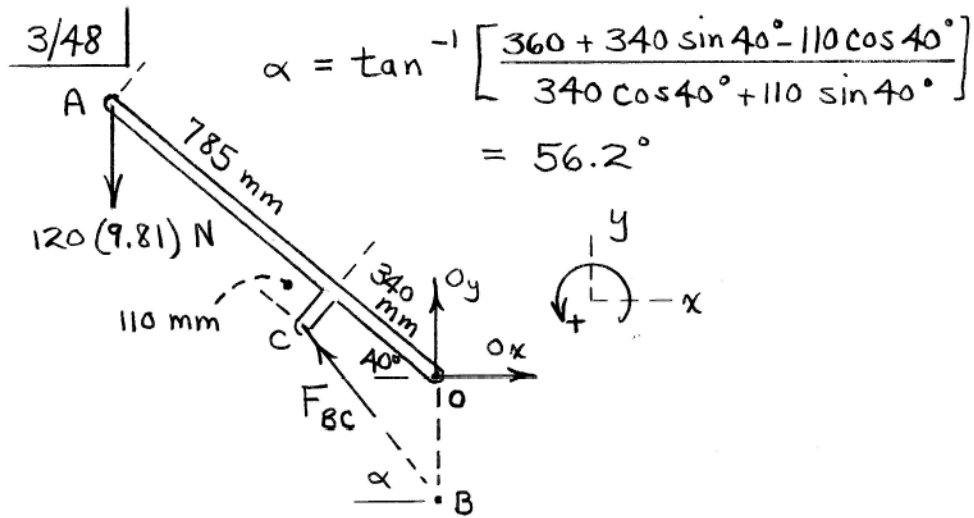
$$N_C = 260 \text{ N}$$

With  $\theta = P = 0$ :

$$P = 0$$

$$N_B = 350 \text{ N}$$

$$N_C = 140.1 \text{ N}$$



$$\sum M_O = 0 : 120(9.81)(785 + 340) \cos 40^\circ$$

$$- F_{BC} \cos \alpha (360) = 0, \quad F_{BC} = 5060 \text{ N}$$

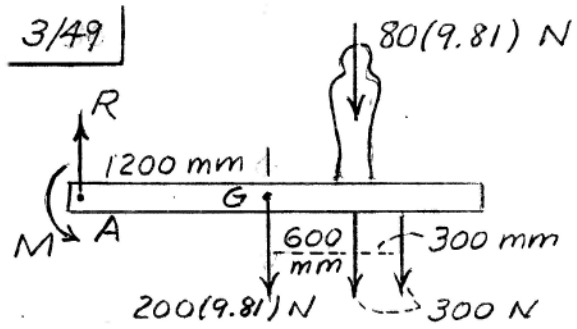
$$\sum F_x = 0 : O_x - 5060 \cos \alpha = 0, \quad O_x = 2820 \text{ N}$$

$$\sum F_y = 0 : O_y - 120(9.81) + 5060 \sin \alpha = 0$$

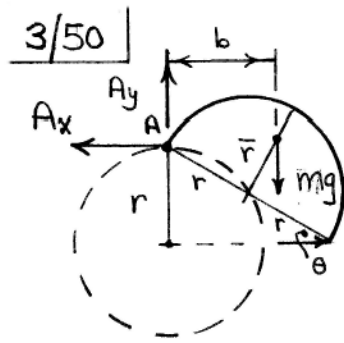
$$O_y = -3030 \text{ N}$$

$$O = \sqrt{O_x^2 + O_y^2} = 4140 \text{ N}$$

$$p = \frac{F_{BC}}{\pi d^2/4} = \frac{5060}{\pi 50^2/4} = 2.58 \frac{\text{N}}{\text{mm}^2} (2.58(10^6) \text{ Pa})$$



$$\begin{aligned} \sum M_A = 0; & \quad 80(9.81)(1800) + 200(9.81)(1200) \\ & \quad + 300(1800 + 2100) - M = 0 \\ M = & \quad 4.94(10^6) \text{ N}\cdot\text{mm} \text{ or } \underline{M = 4.94 \text{ kN}\cdot\text{m}} \end{aligned}$$



From Table D/3,  $\bar{r} = \frac{2r}{\pi}$

$$b = r \cos \theta + \bar{r} \sin \theta$$

Here,  $\sin \theta = \frac{1}{2}$ ,  $\cos \theta = \frac{\sqrt{3}}{2}$

$$\text{So } b = \frac{r}{2} \left( \sqrt{3} + \frac{2}{\pi} \right)$$

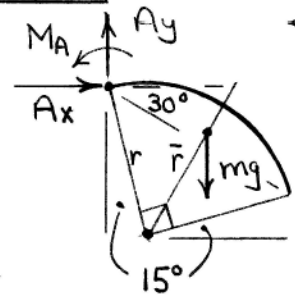
$$\sum M_A = 0: mg \frac{r}{2} \left( \sqrt{3} + \frac{2}{\pi} \right) - Cr = 0$$

$$C = \frac{mg}{2} \left( \sqrt{3} + \frac{2}{\pi} \right)$$

$$\left. \begin{array}{l} \sum F_x = 0: A_x = C \\ \sum F_y = 0: A_y = mg \end{array} \right\} F_A = \sqrt{C^2 + (mg)^2}$$

$$= \underline{1.550 mg}$$

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{ From Table D/3,  $\bar{r} = \frac{r \sin \alpha}{\alpha}$   
For  $\alpha = \pi/4$ ,  $\bar{r} = 2\sqrt{2} r/\pi$

$\sum M_A = 0 :$

$M_A - mg(r \sin 15^\circ + \bar{r} \cos 60^\circ) = 0$

$M_A = 0.709 mgr$



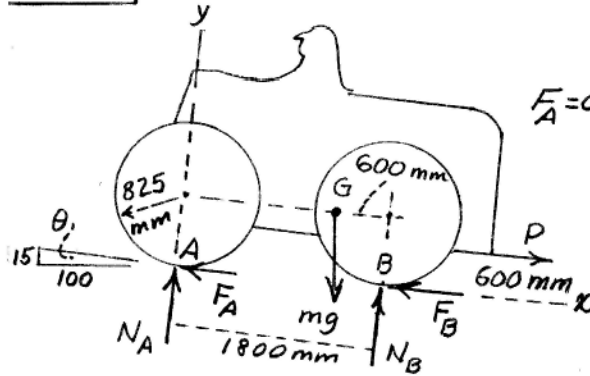
3/52

$$mg = 13.5(9.81) = 132.4 \text{ kN}; \tan \theta = 0.15$$

$$\sin \theta = 0.148$$

$$\cos \theta = 0.989$$

$$F_A = 0.8N_A, F_B = 0.8N_B$$



$$\Sigma M_A = 0; -P(0.6) - 132.4(0.148)(0.825) - 132.4(0.989)(1.8 - 0.6) + 1.8N_B = 0$$

$$\Sigma F_x = 0; P + 132.4(0.148) - 0.8(N_A + N_B) = 0$$

$$\Sigma F_y = 0; N_A + N_B - 132.4(0.989) = 0$$

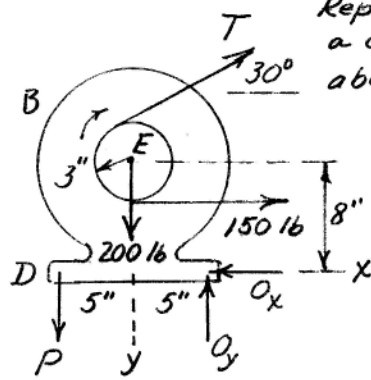
Solve & get  $N_A + N_B = 131.0 \text{ kN}$

$$P = 85.1 \text{ kN}, N_B = 124.7 \text{ kN}$$

3/53

$$\text{Torque } M = 900 = (150 - T)9, \quad T = 50 \text{ lb}$$

Replace  $T$  by force at  $E$  and a couple, so that moment of  $T$  about  $D$  is



$$(50 \cos 30^\circ)8 - (50 \sin 30^\circ)5 + 50(3) = 371.4 \text{ lb-in. CW}$$

$$\sum M_D = 0;$$

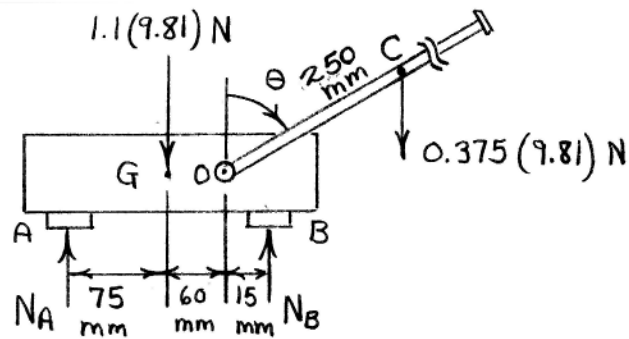
$$371.4 + 150(8-3) + 200(5)$$

$$-10O_y = 0, \quad O_y = 212.1 \text{ lb}$$

$$\sum F_x = 0; \quad 150 + 50 \cos 30^\circ - O_x = 0; \quad O_x = 193.3 \text{ lb}$$

$$R = \sqrt{O_x^2 + O_y^2} = \sqrt{(193.3)^2 + (212.1)^2} = \underline{287 \text{ lb}}$$

3/54

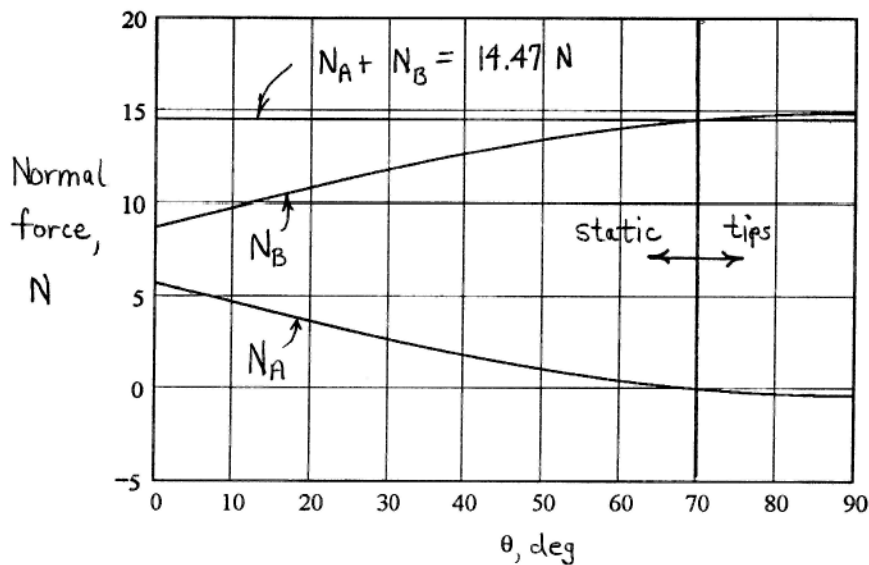


$$\uparrow \Sigma F = 0: N_A + N_B - (1.1 + 0.375)(9.81) = 0 \quad (1)$$

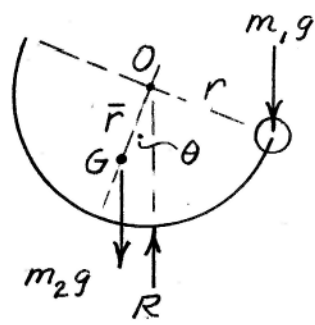
$$\begin{aligned} \curvearrowright \Sigma M_B = 0: & -N_A(150) + 1.1(9.81)(75) \\ & + (15 - 250 \sin \theta)(0.375)(9.81) = 0 \quad (2) \end{aligned}$$

$$\left. \begin{aligned} (2): N_A &= 5.76 - 6.13 \sin \theta \\ (1): N_B &= 8.71 + 6.13 \sin \theta \end{aligned} \right\} N_A, N_B \text{ in newtons}$$

Note that  $N_A$  goes to zero at  $\theta = 70.1^\circ$ , meaning that the receiver would tip if  $\theta$  exceeds this value.



3/55



$$\Sigma M_O = 0;$$

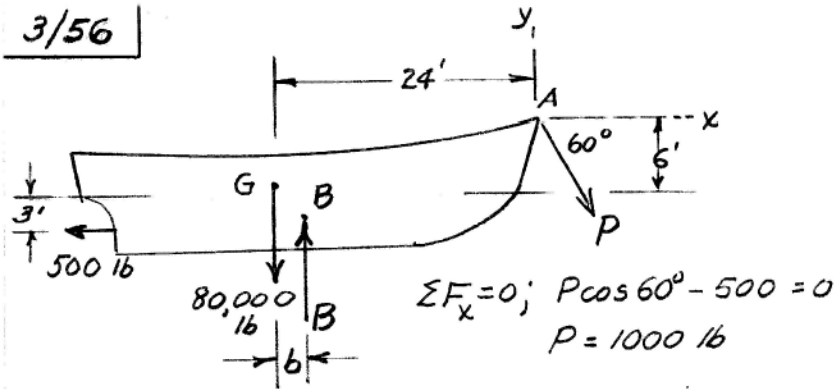
$$m_1 g r \cos \theta - m_2 g \bar{r} \sin \theta = 0$$

$$\text{where } \bar{r} = \frac{2r}{\pi}$$

$$\text{so } \tan \theta = \frac{m_1 r}{m_2 \bar{r}} = \frac{m_1 \pi}{2 m_2}$$

$$\theta = \tan^{-1} \frac{\pi m_1}{2 m_2}$$

3/56



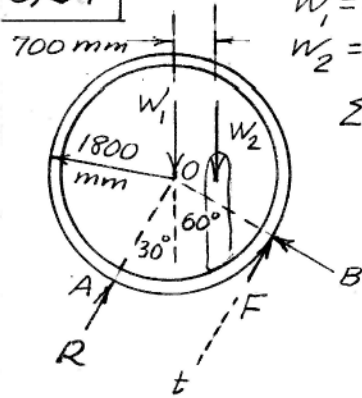
$$\Sigma F_y = 0; B - 80,000 - 1000 \sin 60^\circ = 0, B = 80,866 \text{ lb}$$

$$\Sigma M_A = 0; 80,000(24) - 80,866(24 - b) - 500(6 + 3) = 0$$

$$1,920,000 - 1,940,784 + 80,866b - 4500 = 0$$

$$b = \frac{25284}{80,866} = 0.3127 \text{ ft or } \underline{b = 3.75 \text{ in.}}$$

3/57



$$W_1 = m_1 g = 400(9.81) = 3924 \text{ N}$$

$$W_2 = m_2 g = 80(9.81) = 785 \text{ N}$$

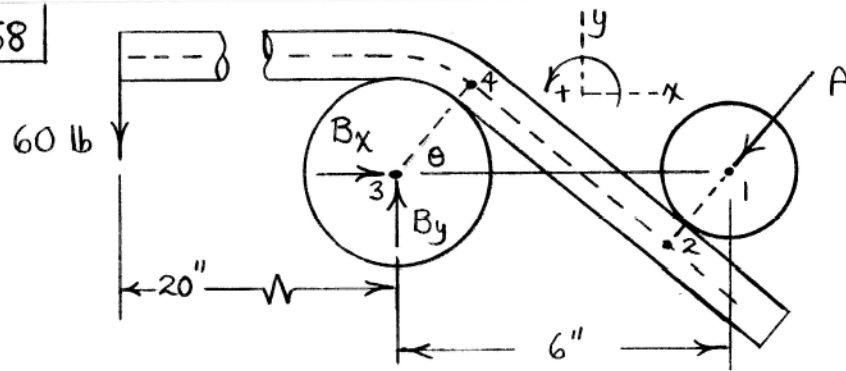
$$\sum M_O = 0; 785(0.7) - 1.8F = 0$$

$$F = 305 \text{ N}$$

$$\sum F_t = 0; R + 305 - (3924 + 785)\cos 30^\circ = 0$$

$$R = 3770 \text{ N}$$

3/58



$$\overline{1-3} = 6''; \quad \overline{1-2} = 1 + \frac{1.050}{2} = 1.525''; \quad \overline{3-4} = 1.75 + \frac{1.050}{2} = 2.275''$$

$$\theta = \cos^{-1}\left(\frac{1.525 + 2.275}{6}\right) = 50.7^\circ$$

$$\overline{2-4} = 6 \sin 50.7^\circ = 4.64''$$

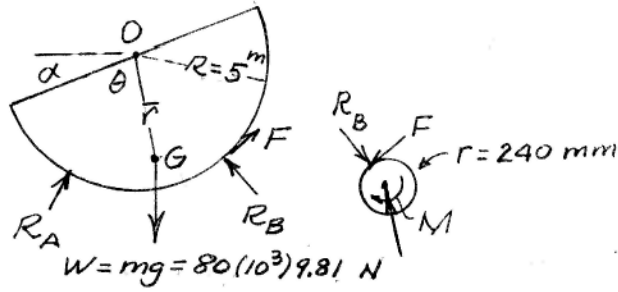
$$\sum M_3 = 0: 60(20) - 4.64A = 0, \quad \underline{A = 258 \text{ lb}}$$

$$\sum F_x = 0: B_x - 258 \cos 50.7^\circ = 0, \quad B_x = 163.7 \text{ lb}$$

$$\sum F_y = 0: B_y - 60 - 258 \sin 50.7^\circ = 0, \quad B_y = 260 \text{ lb}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{163.7^2 + 260^2} = \underline{307 \text{ lb}}$$

► 3/59 | For gear  $Fr = M$ ,  $F = \frac{M}{r} = \frac{M}{0.24}$



$$\alpha = 0, \theta < 90^\circ, M = 2460 \text{ N}\cdot\text{m}$$

$$\sum M_O = 0; \frac{2460}{r} R = mg \bar{r} \cos \theta \quad \text{--- (a)}$$

$$\alpha = 30^\circ, \theta + \alpha > 90^\circ, M = 4680 \text{ N}\cdot\text{m}$$

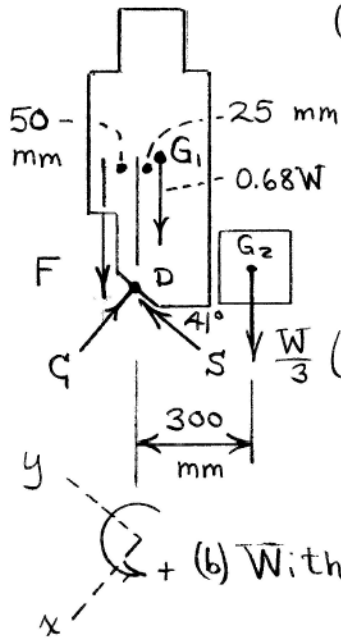
$$\sum M_O = 0; \frac{4680}{r} R = mg \bar{r} \sin (\theta + 30^\circ - 90^\circ) \quad \text{--- (b)}$$

Divide (a) by (b)  $\frac{2460}{4680} = \frac{\cos \theta}{\sin (\theta - 60^\circ)}$  solve & get  $\theta = \tan^{-1} 5.54 = 79.8^\circ$

From (a)  $\bar{r} = \frac{2460(5)}{(0.24) 80(10^3)(9.81) \cos 79.8^\circ} = 0.367 \text{ m}$  or 367 mm



►3/60 | Consider a FBD of the upper torso



$$(a) \sum M_D = 0: F(50) - 0.68W(25) = 0$$

$$F = 0.34W$$

$$\sum F_y = 0: S - 0.68W \sin 41^\circ$$

$$- F \sin 41^\circ = 0, \underline{S = 0.669W}$$

$$\sum F_x = 0: -C - 0.68W \cos 41^\circ$$

$$- F \cos 41^\circ = 0, \underline{C = 0.770W}$$

(b) With weight  $\frac{W}{3}$ :

$$\sum M_D = 0: F(50) - 0.68W(25) - \frac{W}{3}(300) = 0$$

$$F = 2.34W$$

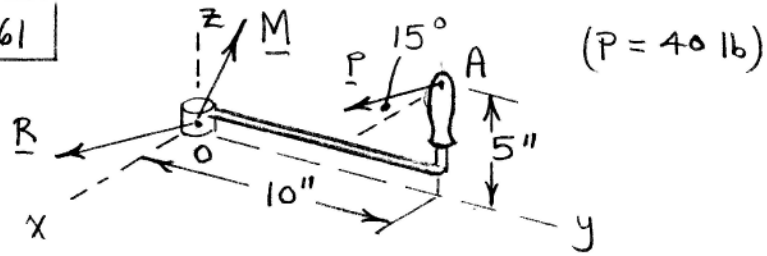
$$\sum F_y = 0: S - 0.68W \sin 41^\circ - F \sin 41^\circ - \frac{W}{3} \sin 41^\circ = 0$$

$$\underline{S = 2.20W}$$

$$\sum F_x = 0: -C + 0.68W \cos 41^\circ + F \cos 41^\circ + \frac{W}{3} \cos 41^\circ = 0$$

$$\underline{C = 2.53W}$$

3/61



$$\sum \underline{F} = \underline{0} : \underline{R} + \underline{P} = \underline{0}$$

$$\underline{R} = -\underline{P} = -40 [\cos 15^\circ \underline{i} + \sin 15^\circ \underline{k}]$$

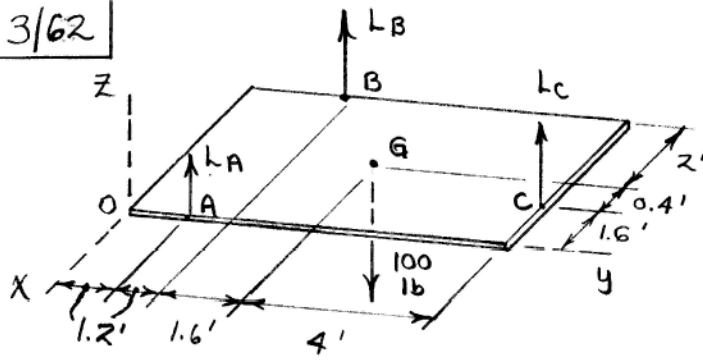
$$= -38.6 \underline{i} - 10.35 \underline{k} \text{ lb}$$

$$\sum \underline{M} = \underline{0} : \underline{M} + \underline{r} \times \underline{P} = \underline{0}$$

$$\underline{M} = -\underline{r} \times \underline{P} = -(10 \underline{j} + 5 \underline{k}) \times 40 [\cos 15^\circ \underline{i} + \sin 15^\circ \underline{k}]$$

$$= -103.5 \underline{i} - 193.2 \underline{j} + 386 \underline{k} \text{ lb-in.}$$

3/62



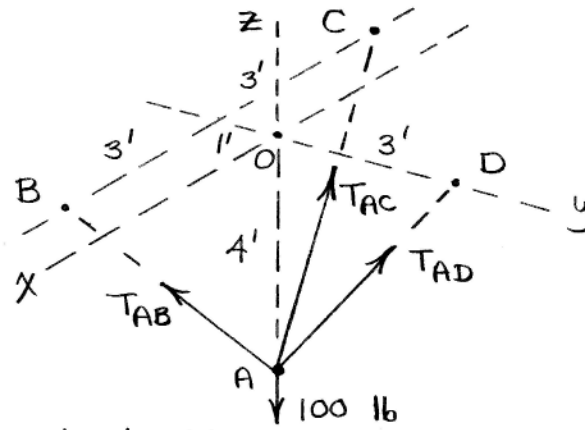
$$\Sigma F_z = 0 : L_A + L_B + L_C - 100 = 0$$

$$\Sigma M_{O_x} = 0 : L_A (1.2) + L_B (2.4) + L_C (8) - 100(4) = 0$$

$$\Sigma M_{O_y} = 0 : L_B (4) + L_C (1.6) - 100(2) = 0$$

$$\text{Solution : } \begin{cases} L_A = 29.1 \text{ lb} \\ L_B = 36.1 \text{ lb} \\ L_C = 34.8 \text{ lb} \end{cases}$$

3/63



$$\underline{T}_{AB} = T_{AB} \left[ \frac{3\mathbf{i} - \mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 1^2 + 4^2}} \right] = T_{AB} [0.588\mathbf{i} - 0.196\mathbf{j} + 0.784\mathbf{k}]$$

$$\underline{T}_{AC} = T_{AC} \left[ \frac{-3\mathbf{i} - \mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 1^2 + 4^2}} \right] = T_{AC} [-0.588\mathbf{i} - 0.196\mathbf{j} + 0.784\mathbf{k}]$$

$$\underline{T}_{AD} = T_{AD} \left[ \frac{3\mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 4^2}} \right] = T_{AD} [0.6\mathbf{j} + 0.8\mathbf{k}]$$

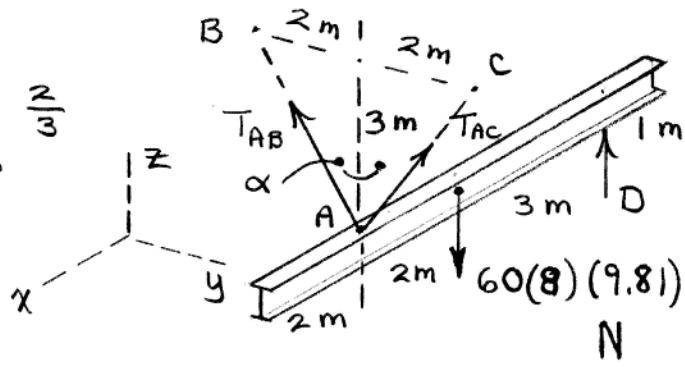
$$\left\{ \begin{array}{l} \sum F_x = 0 : 0.588T_{AB} - 0.588T_{AC} = 0 \quad (1) \\ \sum F_y = 0 : -0.196T_{AB} - 0.196T_{AC} + 0.6T_{AD} = 0 \quad (2) \\ \sum F_z = 0 : 0.784T_{AB} + 0.784T_{AC} + 0.8T_{AD} - 100 = 0 \quad (3) \end{array} \right.$$

$$\left\{ \begin{array}{l} \sum F_x = 0 : 0.588T_{AB} - 0.588T_{AC} = 0 \quad (1) \\ \sum F_y = 0 : -0.196T_{AB} - 0.196T_{AC} + 0.6T_{AD} = 0 \quad (2) \\ \sum F_z = 0 : 0.784T_{AB} + 0.784T_{AC} + 0.8T_{AD} - 100 = 0 \quad (3) \end{array} \right.$$

$$\text{Solution : } \begin{cases} T_{AB} = 47.8 \text{ lb} \\ T_{AC} = 47.8 \text{ lb} \\ T_{AD} = 31.2 \text{ lb} \end{cases}$$

3/64

$$\alpha = \tan^{-1} \frac{2}{3} \\ = 33.7^\circ$$



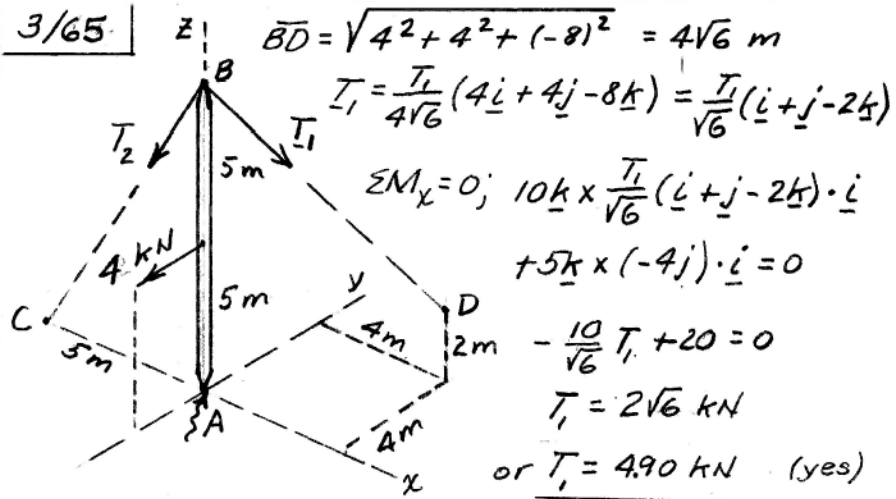
$$\text{From } \Sigma F_y = 0, \quad T_{AB} = T_{AC} = T$$

$$\Sigma M_{Ay} = 0 : -60(8)(9.81)(2) + D(5) = 0 \\ \underline{D = 1884 \text{ N}}$$

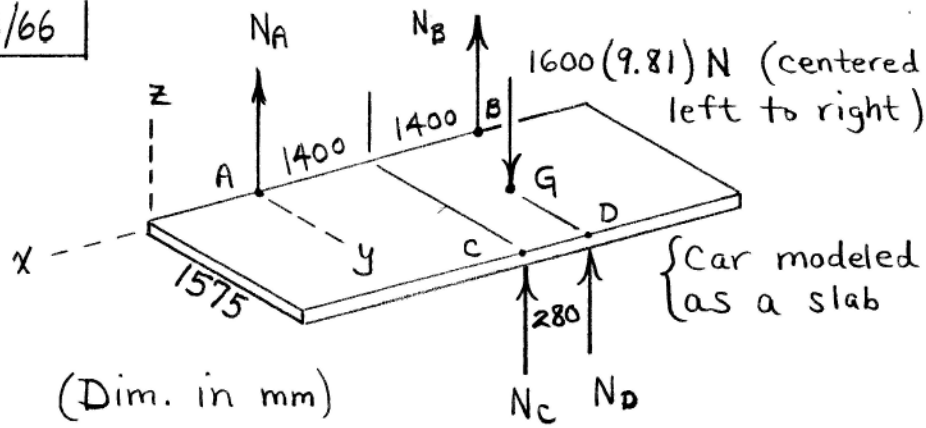
$$\Sigma F_z = 0 : 2T \cos \alpha + D - 60(8)(9.81) = 0$$

$$\underline{T = 1698 \text{ N} = T_{AB} = T_{AC}}$$

3/65



3/66



Jacking at C ( $N_D = 0$ ):

$$\sum M_x = 0 : -1600(9.81) \left( \frac{1575}{2} \right) + N_C (1575) = 0$$

$$\underline{N_C = 7850 \text{ N}}$$

$$\sum M_y = 0 : -1600(9.81)(1680) + N_B(2800) + N_C(1400) = 0$$

$$\sum F_z = 0 : N_A + N_B + N_C - 1600(9.81) = 0$$

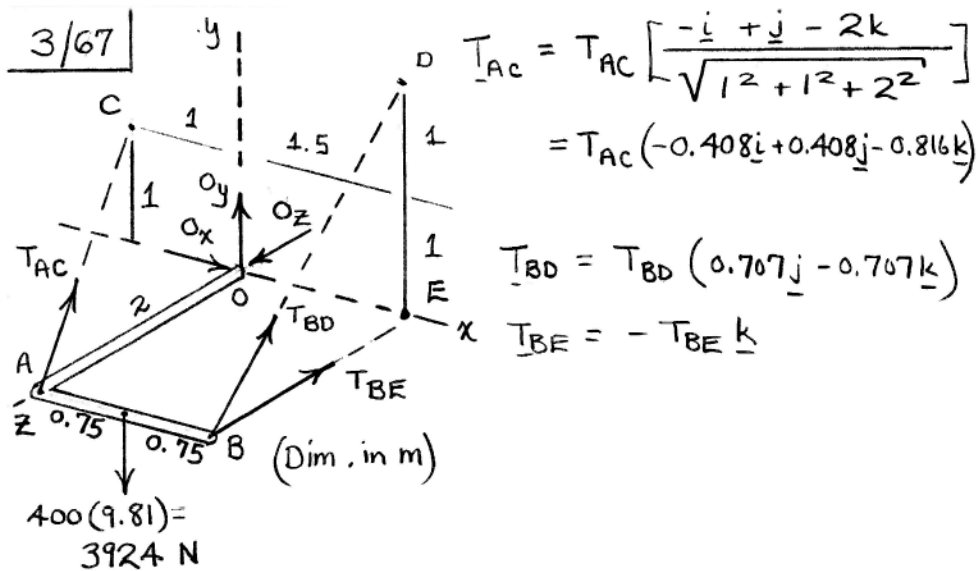
$$\Rightarrow \underline{N_A = 2350 \text{ N}}, \quad \underline{N_B = 5490 \text{ N}}$$

Jacking at D ( $N_C = 0$ ):  $\underline{N_D = 7850 \text{ N}}$  { Same as for  $N_C$

$$\sum M_y = 0 : -1600(9.81)(1680) + N_B(2800) + N_D(1680) = 0$$

$$\sum F_z = 0 : N_A + N_B + N_D - 1600(9.81) = 0$$

$$\Rightarrow \underline{N_A = 3140 \text{ N}}, \quad \underline{N_B = 4710 \text{ N}}$$



$$\sum F_x = 0: \quad O_x - 0.408 T_{AC} = 0$$

$$\sum F_y = 0: \quad O_y + 0.408 T_{AC} + 0.707 T_{BD} - 3924 = 0$$

$$\sum F_z = 0: \quad O_z - 0.816 T_{AC} - 0.707 T_{BD} - T_{BE} = 0$$

$$\sum M_{O_x} = 0: \quad -0.408 T_{AC}(2) - 0.707 T_{BD}(2) + 3924(2) = 0$$

$$\sum M_{O_y} = 0: \quad -0.408 T_{AC}(2) + 0.707 T_{BD}(1.5) + T_{BE}(1.5) = 0$$

$$\sum M_{O_z} = 0: \quad -3924(0.75) + 0.707 T_{BD}(1.5) = 0$$

$$\text{Solution:} \quad O_x = 1962 \text{ N} \quad T_{AC} = 4810 \text{ N}$$

$$O_y = 0 \quad (\text{Note } \sum M_{AB} = 0) \quad T_{BD} = 2770 \text{ N}$$

$$O_z = 6540 \text{ N} \quad T_{BE} = 654 \text{ N}$$

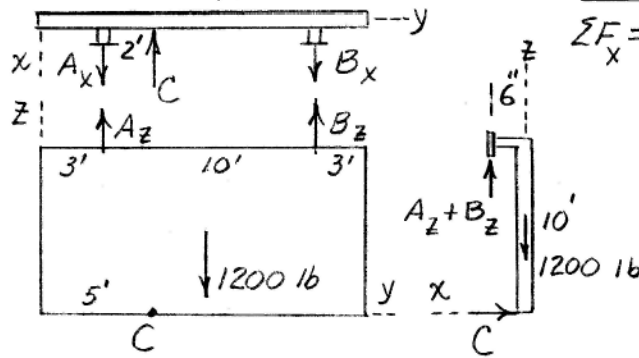


3/68 |  $x-z; \sum M_{AB} = 0; 10C - 1200(6/12) = 0, C = 60 \text{ lb}$

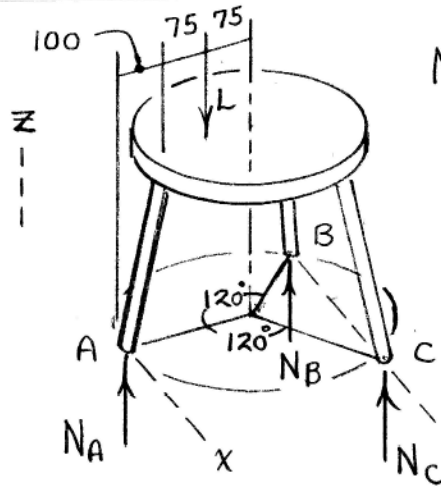
$x-y; \sum M_A = 0; 2(60) - 10B_x = 0; B_x = 12 \text{ lb}$

$\sum F_x = 0; A_x + 12 - 60 = 0$

$A_x = 48 \text{ lb}$



3/69



Note: x-axis is  
 || BC.

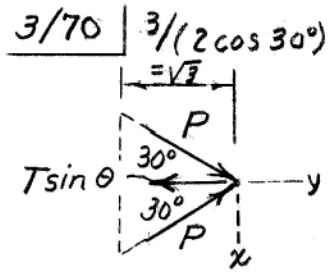
$N_B = N_C = N$ ,  
 by symmetry.

$$\sum M_x = 0 : -L(175) + 2N(250 + 250 \cos 60^\circ) = 0$$

$$N = 0.233L = N_B = N_C$$

$$\sum F_z = 0 : N_A + 2(0.233L) - L = 0$$

$$N_A = 0.533L$$



$$y-z; \Sigma F_z = 0; T \cos 23.4^\circ - 280 = 0$$

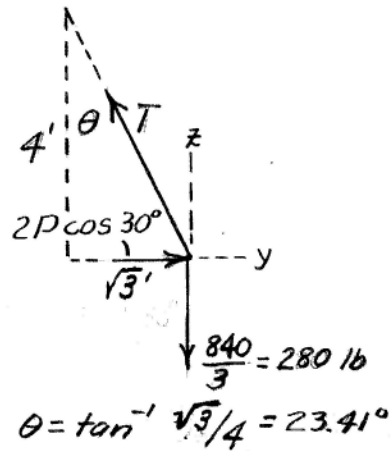
$$T = 280 / 0.9177$$

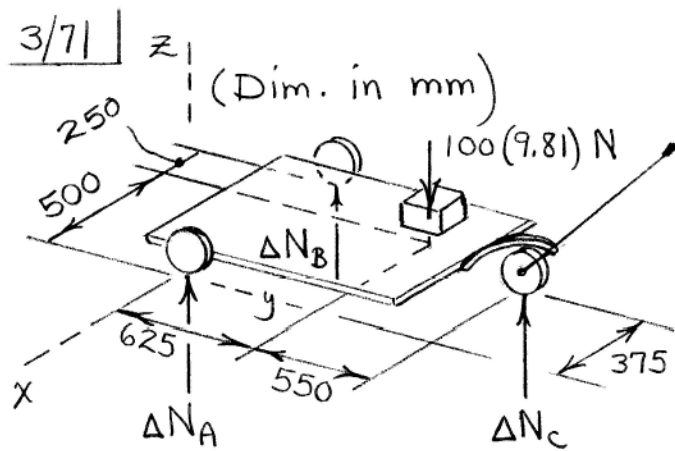
$$= 305 \text{ lb}$$

$$y-z; \Sigma F_y = 0; 305 \sin 23.4^\circ - 2P \cos 30^\circ = 0$$

$$P = \frac{305 (0.3973)}{2 (0.8660)}$$

$$= \underline{70.0 \text{ lb}}$$





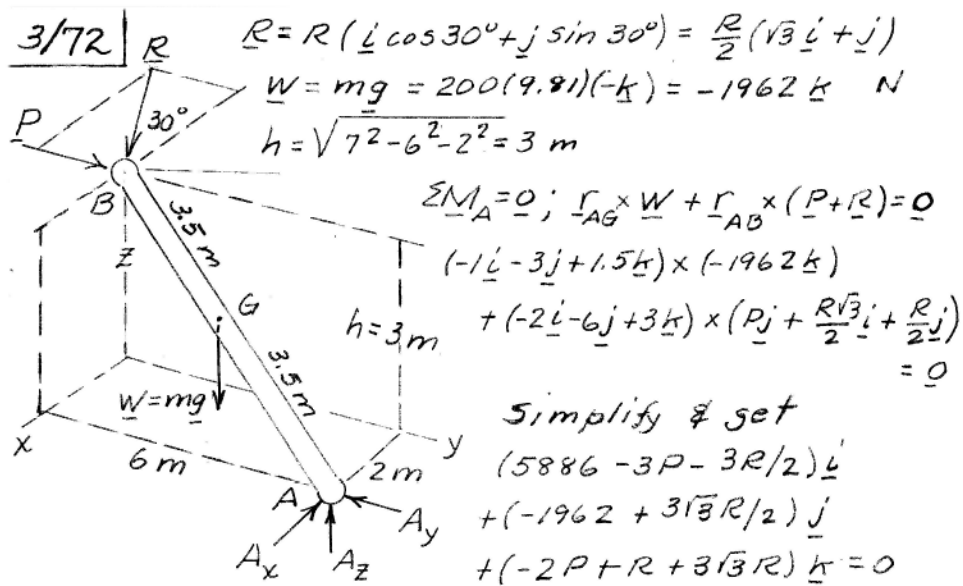
$$\sum M_x = 0: -100(9.81)(625) + \Delta N_c (1175) = 0$$

$$\Delta N_c = 522 \text{ N}$$

$$\sum M_y = 0: \Delta N_B (750) - 100(9.81)(500) + 522(375) = 0, \quad \Delta N_B = 393 \text{ N}$$

$$\sum F_z = 0: 522 + 393 + \Delta N_A - 100(9.81) = 0$$

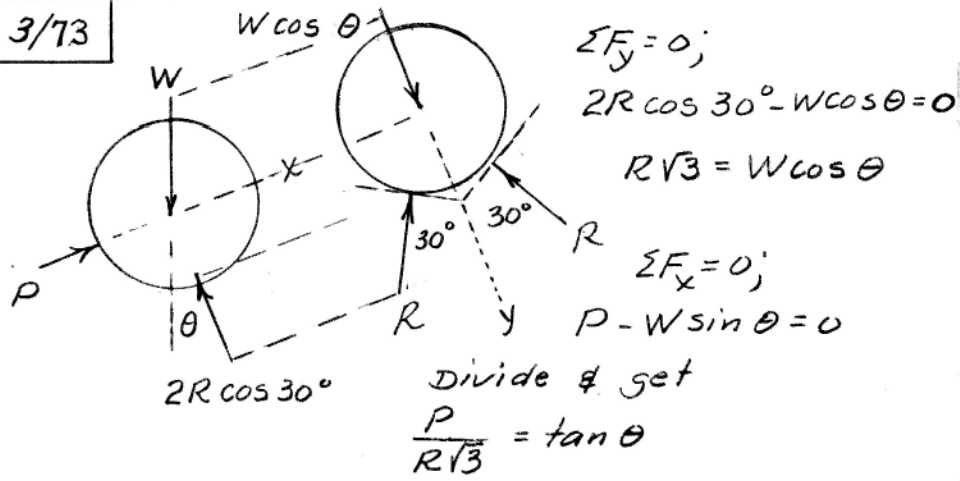
$$\Delta N_A = 66.1 \text{ N}$$



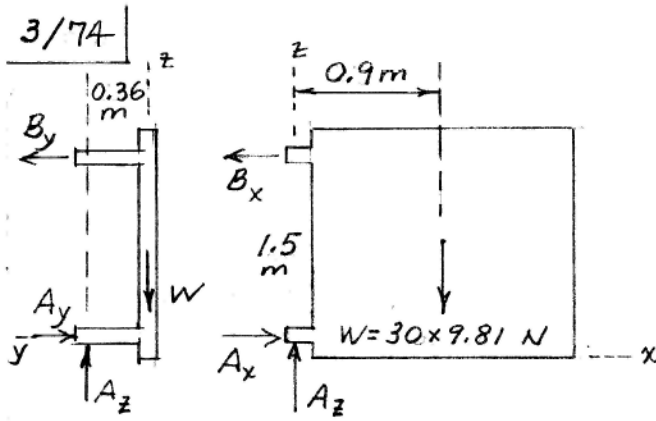
$$R = \frac{2(1962)}{3\sqrt{3}} = 755 \text{ N}$$

$$3P = 5886 - \frac{3}{2}755, \quad P = 1584 \text{ N}$$

3/73



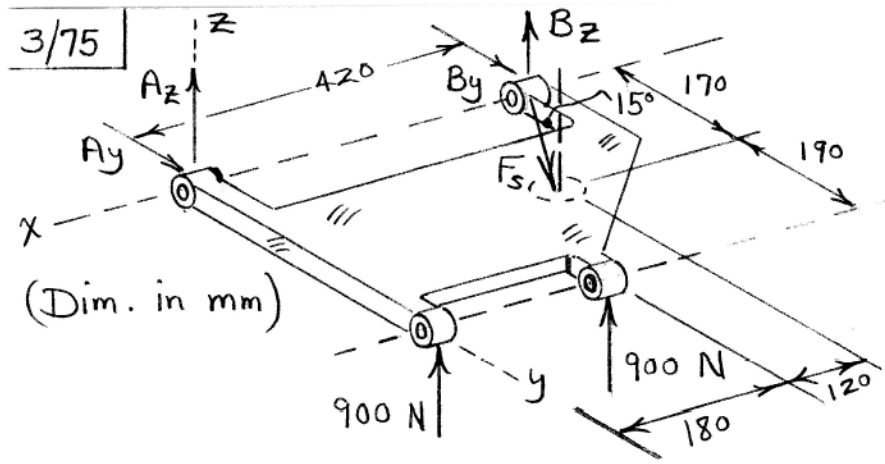
So with  $P=R$ ,  $\theta = \tan^{-1} 1/\sqrt{3} = \underline{30^\circ}$



$$x-z; \sum M_A = 0; 1.5 B_x - 0.9(30)(9.81) = 0, B_x = 176.6 \text{ N}$$

$$y-z; \sum M_A = 0; 1.5 B_y - 0.36(30)(9.81) = 0, B_y = 70.6 \text{ N}$$

$$B = \sqrt{176.6^2 + 70.6^2} = \underline{190.2 \text{ N}}$$



$$\sum M_x = 0: 2(900)(360) - F_s \cos 15^\circ (170) = 0, \underline{F_s = 3950 \text{ N}}$$

$$\sum M_y = 0: B_z (420) + 900(180) - 3950 \cos 15^\circ (300) = 0$$

$$B_z = 2340 \text{ N}$$

$$\sum M_z = 0: -B_y (420) - 3950 \sin 15^\circ (300) = 0, B_y = -730 \text{ N}$$

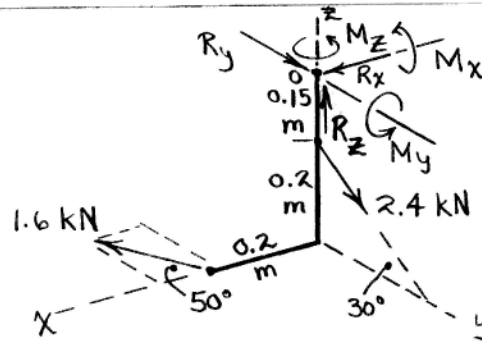
$$\sum F_y = 0: A_y - 730 + 3950 \sin 15^\circ = 0, A_y = -292 \text{ N}$$

$$\sum F_z = 0: A_z + 2340 - 3950 \cos 15^\circ + 1800 = 0, A_z = -325 \text{ N}$$

$$F_A = \sqrt{A_y^2 + A_z^2} = \underline{437 \text{ N}}$$

$$F_B = \sqrt{B_y^2 + B_z^2} = \underline{2450 \text{ N}}$$





$$\sum F_x = 0 : R_x + 1.6 \cos 50^\circ, \quad R_x = -1.028 \text{ kN}$$

$$\sum F_y = 0 : R_y + 2.4 \cos 30^\circ - 1.6 \sin 50^\circ = 0, \quad R_y = -0.853 \text{ kN}$$

$$\sum F_z = 0 : R_z - 2.4 \sin 30^\circ = 0, \quad R_z = 1.2 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{1.796 \text{ kN}}$$

$$\sum M_{O_x} = 0 : M_x + 2.4 \cos 30^\circ (0.15) - 1.6 \sin 50^\circ (0.35) = 0$$

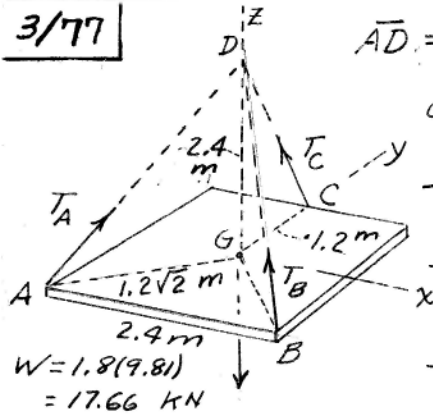
$$M_x = 0.1172 \text{ kN}\cdot\text{m}$$

$$\sum M_{O_y} = 0 : M_y - 1.6 \cos 50^\circ (0.35) = 0, \quad M_y = 0.360 \text{ kN}\cdot\text{m}$$

$$\sum M_{O_z} = 0 : M_z - 1.6 \sin 50^\circ (0.2) = 0, \quad M_z = 0.245 \text{ kN}\cdot\text{m}$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{0.451 \text{ kN}\cdot\text{m}}$$

3/77



$$\overline{AD} = \overline{BD} = \sqrt{2.4^2 + 1.2^2} = 1.2\sqrt{6} \text{ m}$$

$$\overline{CD} = \sqrt{1.2^2 + 2.4^2} = 1.2\sqrt{5} \text{ m}$$

$$\underline{T}_A = \frac{T_A}{1.2\sqrt{6}} (1.2\underline{i} + 1.2\underline{j} + 2.4\underline{k})$$

$$\underline{T}_B = \frac{T_B}{1.2\sqrt{6}} (-1.2\underline{i} + 1.2\underline{j} + 2.4\underline{k})$$

$$\underline{T}_C = \frac{T_C}{1.2\sqrt{5}} (-1.2\underline{j} + 2.4\underline{k})$$

$$\sum \underline{F} = \underline{0}; \underline{T}_A + \underline{T}_B + \underline{T}_C + \underline{W} = \underline{0}$$

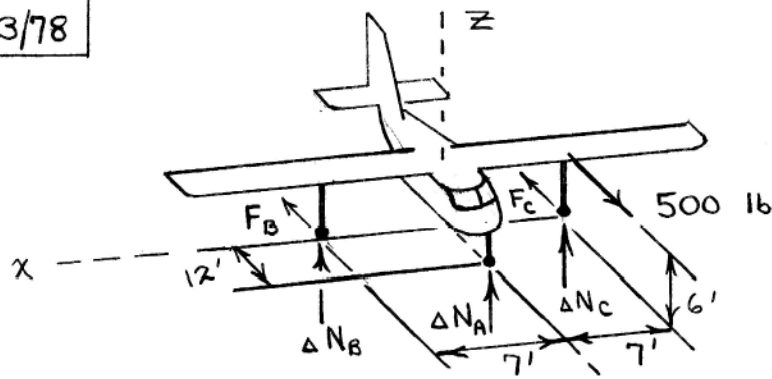
$$\underline{i} \left( \frac{T_A}{\sqrt{6}} - \frac{T_B}{\sqrt{6}} \right) + \underline{j} \left( \frac{T_A}{\sqrt{6}} + \frac{T_B}{\sqrt{6}} - \frac{T_C}{\sqrt{5}} \right) + \underline{k} \left( \frac{2T_A}{\sqrt{6}} + \frac{2T_B}{\sqrt{6}} + \frac{2T_C}{\sqrt{5}} - 17.66 \right) = \underline{0}$$

$$T_A = T_B; 4T_A/\sqrt{6} + 2T_C/\sqrt{5} = 17.66; 2T_A/\sqrt{6} = T_C/\sqrt{5}$$

$$\text{Solve \& get } \underline{T}_A = \underline{T}_B = 5.41 \text{ kN}$$

$$\underline{T}_C = 9.87 \text{ kN}$$

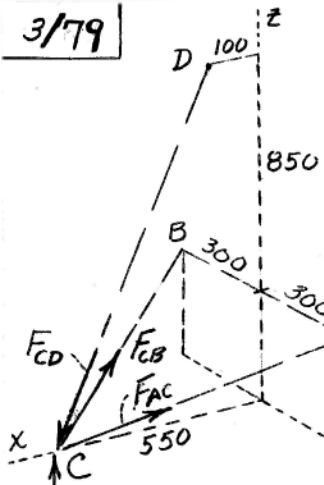
3/78



$$\begin{aligned} \sum M_x = 0 : \Delta N_A (12) - 500(6) &= 0, & \Delta N_A &= 250 \text{ lb} \\ \sum F_z = 0 : \Delta N_A + \Delta N_B + \Delta N_C &= 0 \\ \sum M_y = 0 : \Delta N_C (7) - \Delta N_B (7) &= 0 & \Delta N_B = \Delta N_C &= -125 \text{ lb} \end{aligned}$$

More information would be required to determine  $F_B$  and  $F_C$ . x-components of friction at B and C are possible.

3/79



$$R = \frac{1}{3} mg ; \text{ From Table C2 for Mars } g = 3.73 \text{ m/s}^2$$

$$R = \frac{1}{3} 600(3.73) = 746 \text{ N}$$

$$\bar{AC} = \sqrt{(550)^2 + (300)^2 + (350)^2} = 718 \text{ mm}$$

$$\bar{CD} = \sqrt{(450)^2 + (1200)^2} = 1282 \text{ mm}$$

$$\underline{F}_{AC} = \frac{F_{AC}}{718} (-550\underline{i} + 300\underline{j} + 350\underline{k})$$

$$\underline{F}_{CB} = \frac{F_{CB}}{718} (-550\underline{i} - 300\underline{j} + 350\underline{k})$$

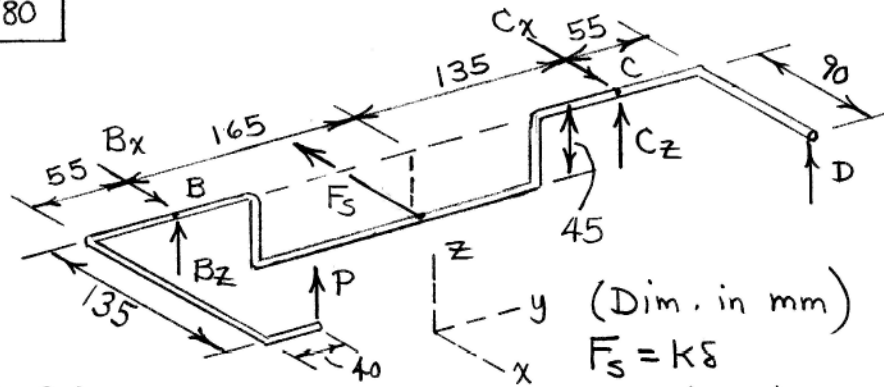
$$\underline{F}_{CD} = \frac{F_{CD}}{1282} (450\underline{i} - 1200\underline{k})$$

$\Sigma \underline{F} = \underline{0}$  with  $F_{CB} = F_{AC}$  gives upon collecting terms

$$\left( -\frac{2F_{AC}}{718} 550 + \frac{F_{CD}}{1282} 450 \right) \underline{i} + \left( 746 + \frac{2F_{AC}}{718} 350 - \frac{F_{CD}}{1282} 1200 \right) \underline{k} = \underline{0}$$

Equate coefficients to zero & solve simultaneously to get  $F_{CD} = 1046 \text{ N}$  compression

$$F_{AC} = F_{CB} = 240 \text{ N tension}$$



$$D=0:$$

$$\sum M_{BC} = 0: -P(135) + 54(45) = 0, \quad P = 18 \text{ N}_{\min}$$

$$\sum M_{B_x} = 0: -18(15) + C_z(300) = 0, \quad C_z = 0.9 \text{ N}$$

$$\sum M_{B_z} = 0: 54(165) - C_x(300) = 0, \quad C_x = 29.7 \text{ N}$$

$$\sum F_x = 0: 29.7 + B_x - 54 = 0, \quad B_x = 24.3 \text{ N}$$

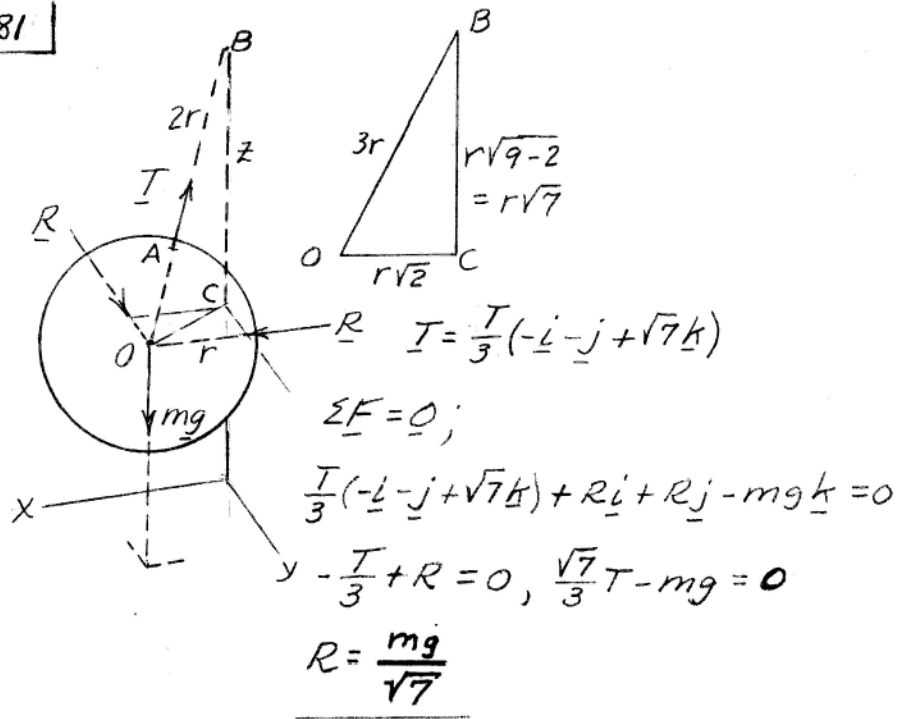
$$\sum F_z = 0: 0.9 + B_z + 18 = 0, \quad B_z = -18.9 \text{ N}$$

$$B = \sqrt{B_x^2 + B_z^2} = 30.8 \text{ N}, \quad C = \sqrt{C_x^2 + C_z^2} = 29.7 \text{ N}$$

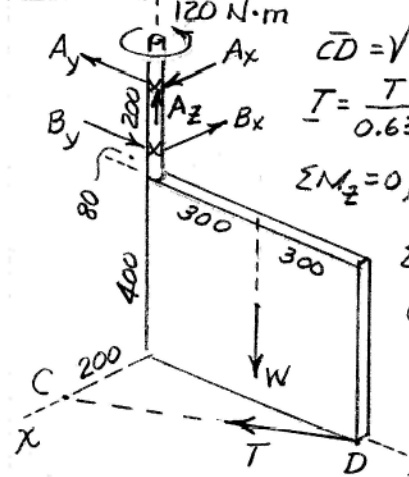
If  $P = P_{\min}/2 = 18/2 = 9 \text{ N}$ , ( $D \neq 0$ ):

$$\sum M_{BC} = 0: -9(135) + 54(45) - D(90) = 0, \quad D = 13.5 \text{ N}$$

3/81



3/82 Dimensions in mm;  $W = 15(9.81)(-k) \text{ N}$



$$CD = \sqrt{0.2^2 + 0.6^2} = 0.632 \text{ m}$$

$$T = \frac{T}{0.632} (0.2\mathbf{i} - 0.6\mathbf{j})$$

$$\Sigma M_z = 0; 120 - \frac{0.2}{0.632} T (0.6) = 0, T = 632 \text{ N}$$

$$\Sigma M_x = 0 \text{ at } A;$$

$$0.2B_y - 147.2(0.3) - 632 \frac{0.6}{0.632} (0.680) = 0$$

$$B_y = 2260 \text{ N}$$

$$\Sigma M_y = 0 \text{ at } A; 0.2B_x - 632 \frac{0.2}{0.632} (0.680) = 0, B_x = 680 \text{ N}$$

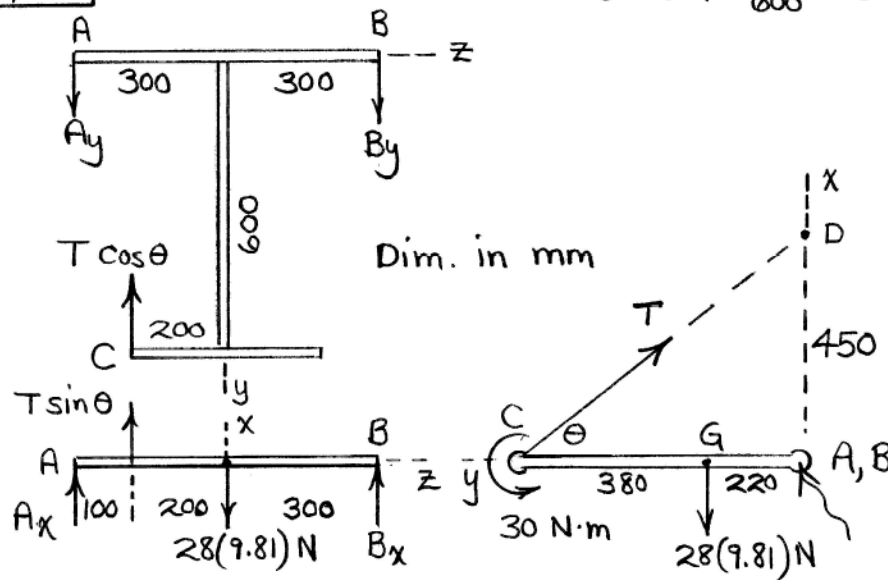
$$B = \sqrt{(2260)^2 + (680)^2} = 2360 \text{ N or } \underline{B = 2.36 \text{ kN}}$$



ارائه دهنده خدمات متنوع مهندسی

3/83

$$\theta = \tan^{-1} \frac{450}{600} = 36.9^\circ$$



$$(x-y) \sum M_z = 0: 28(9.81)(0.220) - T \sin 36.9^\circ (0.600) + 30 = 0, \quad T = 251 \text{ N}$$

$$(x-z) \sum M_B = 0: 28(9.81)(0.300) - 251 \sin 36.9^\circ (0.500) - 0.600 A_x = 0, \quad A_x = 11.74 \text{ N}$$

$$\sum F_x = 0: 11.74 + 251 \sin 36.9^\circ - 28(9.81) + B_x = 0, \quad B_x = 112.2 \text{ N}$$

$$(y-z) \sum M_B = 0: 251 \cos 36.9^\circ (0.500) - 0.6 A_y = 0, \quad A_y = 167.5 \text{ N}$$

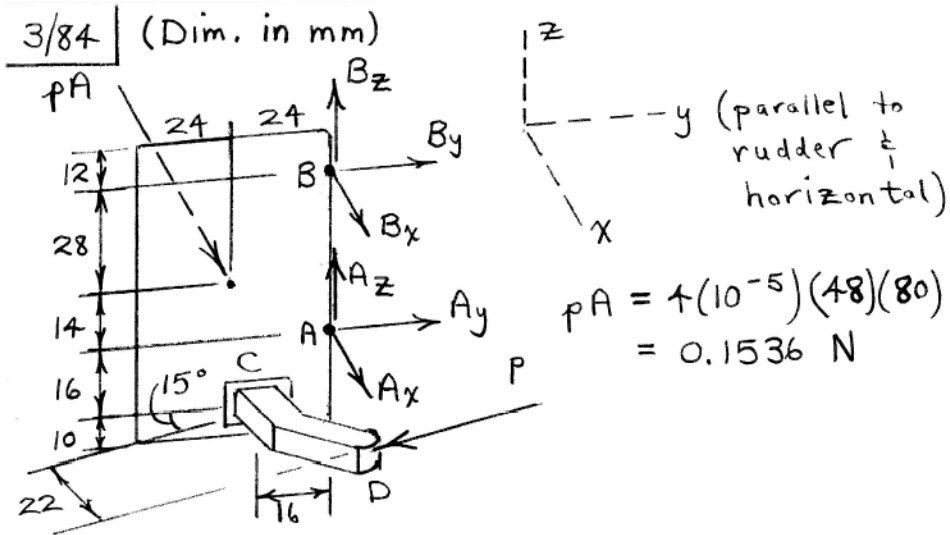
$$\sum F_y = 0: 167.5 + B_y - 251 \cos 36.9^\circ = 0, \quad B_y = 33.5 \text{ N}$$

$$A = \sqrt{11.74^2 + 167.5^2} = 167.9 \text{ N}$$

$$B = \sqrt{112.2^2 + 33.5^2} = 117.1 \text{ N}$$

Couple may be applied at any place on rigid body with the same external effect.





$$\sum M_{AB} = 0 : -P(22 - 16 \sin 15^\circ) + 0.1536(24) = 0$$

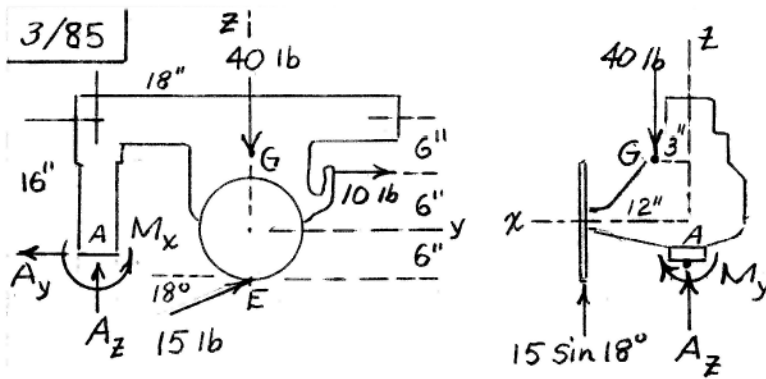
$$P = 0.206 \text{ N}$$

$$\sum M_{Bx} = 0 : A_y(42) - 0.206 \cos 15^\circ(58) = 0$$

$$A_y = 0.275 \text{ N}$$

$$\sum M_{Ax} = 0 : -B_y(42) - 0.206 \cos 15^\circ(16) = 0$$

$$B_y = -0.0760 \text{ N}$$



$$y-z; \sum M_A = 0; 40(18) + 10(16-6) - 15 \sin 18^\circ(18) - 15 \cos 18^\circ(2) - M_x = 0$$

$$M_x = 708 \text{ lb-in.}$$

$$x-z; \sum M_A = 0; 15 \sin 18^\circ(12) + M_y - 40(3) = 0$$

$$M_y = 64.4 \text{ lb-in.}$$

$$M = \sqrt{M_x^2 + M_y^2} = \sqrt{(708)^2 + (64.4)^2} = \underline{711 \text{ lb-in.}}$$

Acceleration of mass center negligible, so equilibrium equations may be used.

3/86

From Prob. 2/95,

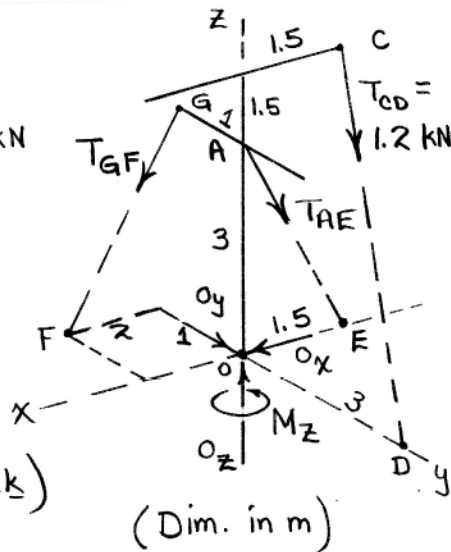
$$\underline{T}_{CD} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k} \text{ kN}$$

$$\underline{T}_{AE} = T_{AE} \frac{-1.5\underline{i} - 3\underline{k}}{\sqrt{1.5^2 + 3^2}}$$

$$= T_{AE} (-0.447\underline{i} - 0.894\underline{k})$$

$$\underline{T}_{GF} = T_{GF} \frac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}}$$

$$= T_{GF} (0.555\underline{i} - 0.832\underline{k})$$



$$\Sigma \underline{M}_O = \underline{0} : \underline{OC} \times \underline{T}_{CD} + \underline{OA} \times \underline{T}_{AE} + \underline{OG} \times \underline{T}_{GF} + M_Z \underline{k} = \underline{0}$$

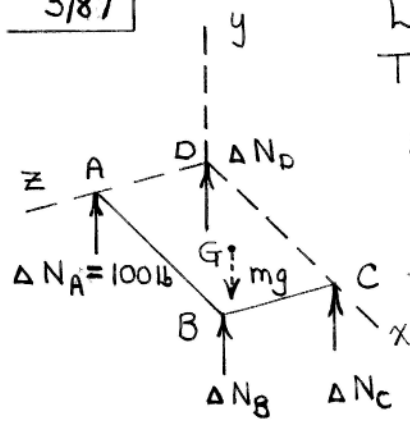
$$(1.5\underline{i} + 4.5\underline{k}) \times (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) + 3\underline{k} \times$$

$$T_{AE} (-0.447\underline{i} - 0.894\underline{k}) + (-\underline{j} + 3\underline{k}) \times T_{GF} (0.555\underline{i} - 0.832\underline{k})$$

$$+ M_Z \underline{k} = \underline{0} \Rightarrow \begin{cases} 0.832 T_{GF} - 2.89 = 0 \\ 1.664 T_{GF} - 1.342 T_{AE} = 0 \\ 0.555 T_{GF} - 0.962 + M_Z = 0 \end{cases}$$

Solve to obtain  $\underline{T}_{GF} = 3.47 \text{ kN}, \underline{T}_{AE} = 4.30 \text{ kN},$   
 $M_Z = -0.962 \text{ kN}\cdot\text{m}$

3/87



Location of  $G$  does not change.

Thus,

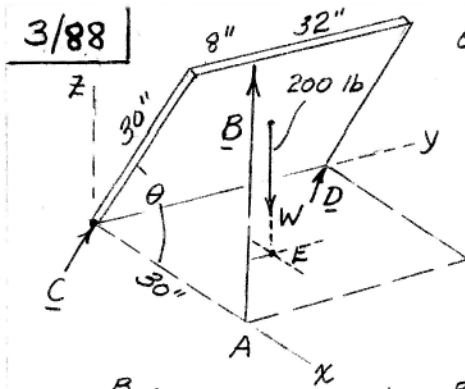
$$\frac{\Delta N_D = -100 \text{ lb}}{\text{total rear-axle loading}}$$

$$\frac{\Delta N_B = -100 \text{ lb}}{\text{total right-side loading}}$$

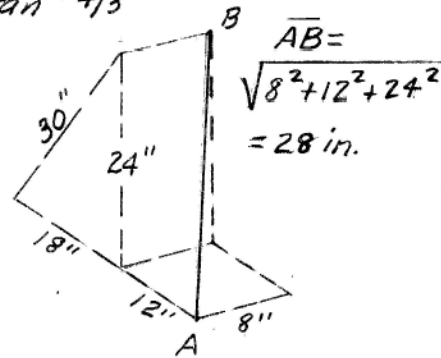
$$\frac{\Delta N_C = 100 \text{ lb}}{\text{total normal force; preserves total front-axle loading}}$$

(Note: The results for  $\Delta N_B$  &  $\Delta N_C$  hold only if the track (distance between tire centers) at the front is equal to that at the rear.)

3/88



$$\theta = \tan^{-1} 4/3$$



$$\underline{B} = \frac{B}{28}(-12\underline{i} + 8\underline{j} + 24\underline{k}) = \frac{B}{7}(-3\underline{i} + 2\underline{j} + 6\underline{k})$$

$$\underline{W} = -200\underline{k} \text{ lb}$$

$$\Sigma M_{CD} = 0; (\underline{r}_{CA} \times \underline{B} + \underline{r}_{CE} \times \underline{W}) \cdot \underline{j} = 0; \text{moment of } \underline{D} = 0$$

$$[30\underline{i} \times \frac{B}{7}(-3\underline{i} + 2\underline{j} + 6\underline{k}) + (9\underline{i} + 20\underline{j}) \times (-200\underline{k})] \cdot \underline{j} = 0$$

$$(-4000\underline{i} + 180(10 - B/7)\underline{j} + 60B/7\underline{k}) \cdot \underline{j} = 0$$

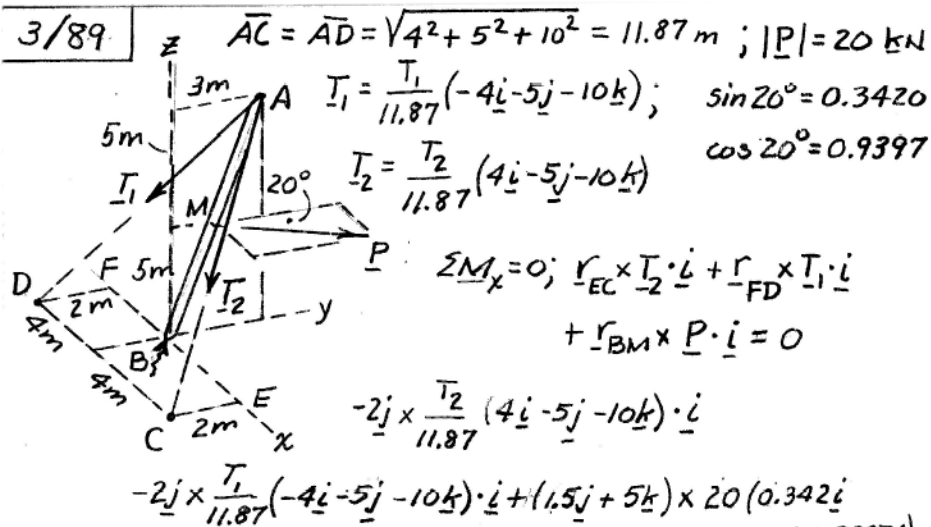
$$B/7 - 10 = 0, \underline{B} = \underline{F}_B = 70 \text{ lb}$$

$$\Sigma M_x = 0; 40D_z - 200(20) = 0, D_z = 100 \text{ lb}$$

$$\Sigma M_z = 0; 30B_y - 40D_x = 0, D_x = \frac{3}{4}20 = 15 \text{ lb}$$

$$\left. \begin{array}{l} D_{\text{normal}} \\ = D_n = \sqrt{100^2 + 15^2} \\ = 101.1 \text{ lb} \end{array} \right\}$$

3/89



Simplify & get  $T_1 + T_2 = 55.79 \dots (1)$

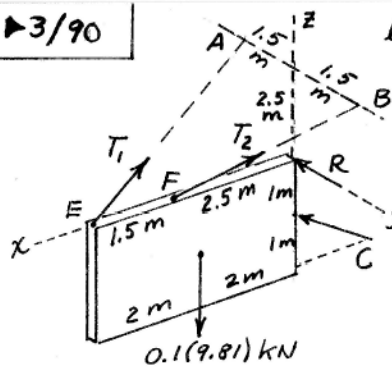
$\sum \underline{M}_z = 0; \underline{r}_{BA} \times (\underline{T}_1 + \underline{T}_2) \cdot \underline{k} + \underline{r}_{BM} \times \underline{P} \cdot \underline{k} = 0$

$$(3\underline{j} + 10\underline{k}) \times \frac{1}{11.87} ([-4T_1 + 4T_2]\underline{i} + [-5T_1 - 5T_2]\underline{j} - [10T_1 + 10T_2]\underline{k}) \cdot \underline{k}$$

Simplify to  $T_1 - T_2 = 10.14 \dots (2)$

Solve (1) & (2) & get  $T_1 = 33.0 \text{ kN}, T_2 = 22.8 \text{ kN}$

3/90



$$l_1 = \overline{AE} = \sqrt{4^2 + 1.5^2 + 2.5^2} = 4.95 \text{ m}$$

$$l_2 = \overline{BF} = \sqrt{2.5^2 + 1.5^2 + 2.5^2} = 3.84 \text{ m}$$

$$\underline{T}_1 = \frac{T_1}{l_1} (-4\underline{i} - 1.5\underline{j} + 2.5\underline{k})$$

$$\underline{T}_2 = \frac{T_2}{l_2} (-2.5\underline{i} + 1.5\underline{j} + 2.5\underline{k})$$

Note:  $\underline{T}_1, \underline{T}_2, \underline{R}$ , and weight all pass through x-axis, so  $\underline{C}$  must also. Thus  $C_y = 0$

$$\sum M_{AB} = 0; C_x(3.5) - 9.81(2) = 0, C_x = 0.561 \text{ kN}$$

$$\sum M_z = 0; 4\underline{i} \times \frac{T_1}{l_1} (-1.5\underline{j}) + 2.5\underline{i} \times \frac{T_2}{l_2} (1.5\underline{j}) = 0, 8T_1/l_1 = 5T_2/l_2$$

$$\sum F_x = 0; -\frac{T_1}{l_1}(4) - \frac{T_2}{l_2}(2.5) + 0.561 = 0, 8T_1/l_1 + 5T_2/l_2 = 1.121 \text{ kN}$$

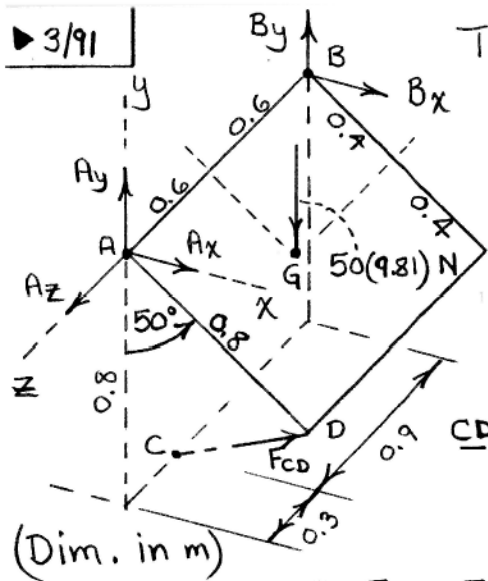
$$\text{solve \& get } T_1 = 1.121 l_1 / 16 = 0.347 \text{ kN}, T_2 = \frac{1.121}{10} l_2 = 0.431 \text{ kN}$$

$$\sum F_y = 0; \frac{1.121}{16} l_1 \frac{1.5}{l_1} - \frac{1.121}{10} l_2 \frac{1.5}{l_2} + R = 0, R = 0.0631 \text{ kN}$$

$$\sum F_z = 0; \frac{1.121}{16} l_1 \frac{2.5}{l_1} + \frac{1.121}{10} l_2 \frac{2.5}{l_2} + C_z - 0.981 = 0, C_z = 0.526 \text{ kN}$$

$$\text{Thus } C = \sqrt{(0.561)^2 + (0.526)^2} = 0.768 \text{ kN}$$

3/91



The coordinates of point D are  $(0.8 \sin 50^\circ, -0.8 \cos 50^\circ, 0)$   
 $= (0.613, -0.514, 0)$

So  $\underline{F}_{CD}$  can be written as  $\underline{F}_{CD} = F_{CD} \underline{n}_{CD}$ ,  
 where  $\underline{n}_{CD} = \frac{\underline{CD}}{CD}$

$$\underline{CD} = 0.613 \underline{i} + (0.8 - 0.514) \underline{j} + 0.3 \underline{k}$$

$$= 0.613 \underline{i} + 0.286 \underline{j} + 0.3 \underline{k} \text{ m}$$

$$\text{So } \underline{F}_{CD} = F_{CD} [0.828 \underline{i} + 0.386 \underline{j} + 0.406 \underline{k}]$$

$$\sum F_x = 0 : A_x + B_x + 0.828 F_{CD} = 0 \quad (1)$$

$$\sum F_y = 0 : A_y + B_y + 0.386 F_{CD} - 50(9.81) = 0 \quad (2)$$

$$\sum F_z = 0 : A_z + 0.406 F_{CD} = 0 \quad (3)$$

Now sum moments about x, y, z axes through A (note change in origin from text illustration):

$$\sum M_{A_x} = 0 : B_y(1.2) - 50(9.81)(0.6) - 0.406 F_{CD}(0.8) + 0.386 F_{CD}(0.3) = 0 \quad (4)$$

$$\sum M_{A_y} = 0 : -B_x(1.2) - 0.828 F_{CD}(0.3) = 0 \quad (5)$$

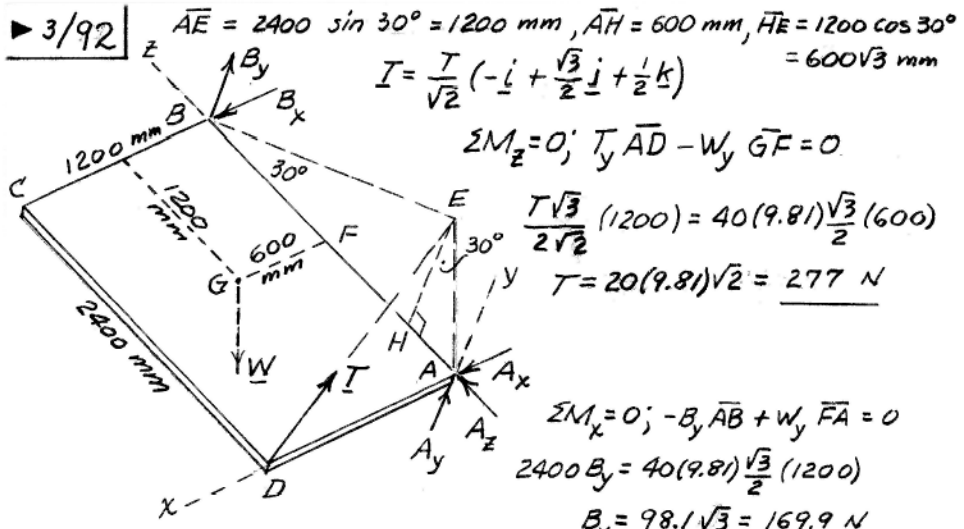
$$\sum M_{A_z} = 0 : -50(9.81)(0.4 \sin 50^\circ) + 0.828 F_{CD}(0.8) = 0 \quad (6)$$

(note:  $\underline{F}_{CD}$  considered as acting at C)

Solution:	$A_x = -140.9 \text{ N}$	$B_x = -47.0 \text{ N}$
	$A_y = 118.2 \text{ N}$	$B_y = 285 \text{ N}$
	$A_z = -92.0 \text{ N}$	$F_{CD} = 227 \text{ N}$



► 3/92



$$\overline{AE} = 2400 \sin 30^\circ = 1200 \text{ mm}, \overline{AH} = 600 \text{ mm}, \overline{HE} = 1200 \cos 30^\circ = 600\sqrt{3} \text{ mm}$$

$$I = \frac{T}{\sqrt{2}} \left( -\frac{1}{2}i + \frac{\sqrt{3}}{2}j + \frac{1}{2}k \right)$$

$$\sum M_z = 0; T_y \overline{AD} - W_y \overline{GF} = 0$$

$$\frac{T\sqrt{3}}{2\sqrt{2}} (1200) = 40(9.81) \frac{\sqrt{3}}{2} (600)$$

$$T = 20(9.81)\sqrt{2} = \underline{277 \text{ N}}$$

$$\sum M_x = 0; -B_y \overline{AB} + W_y \overline{FA} = 0$$

$$2400 B_y = 40(9.81) \frac{\sqrt{3}}{2} (1200)$$

$$B_y = 98.1\sqrt{3} = 169.9 \text{ N}$$

$$\sum M_y = 0; B_x \overline{AB} + W_z \overline{GF} - T_z \overline{AD} = 0$$

$$2400 B_x + 40(9.81)0.5(600) - \frac{277}{2\sqrt{2}} (1200) = 0$$

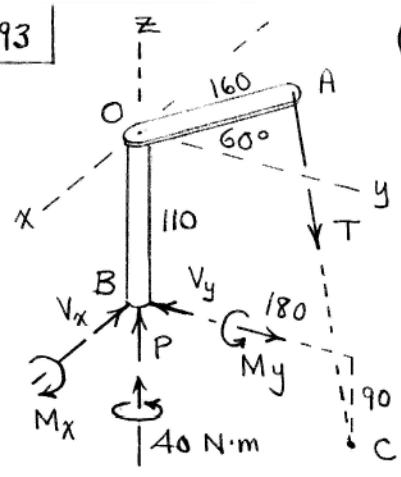
$$B_x = 0$$

$$B = B_y = \underline{169.9 \text{ N}}$$

( $B_x = 0$  can be obtained by inspection by noting  $\sum M_{AE} = 0$  eliminates all terms except  $B_x \overline{BE}$  so  $B_x = 0$ )



ارائه دهنده خدمات متنوع مهندسی



$$(\overline{AC})_x = 0.16 \sin 60^\circ = 0.1386 \text{ m}$$

$$(\overline{AC})_y = (0.18 - 0.16 \cos 60^\circ) = 0.10 \text{ m}$$

$$(\overline{AC})_z = 0.2 \text{ m}$$

$$\overline{AC} = \sqrt{0.1386^2 + 0.1^2 + 0.2^2} = 0.263$$

$$\vec{BC} = 0.18\mathbf{j} - 0.090\mathbf{k} \text{ m}$$

$$\underline{T} = \frac{T}{0.263} (0.1386\mathbf{i} + 0.1\mathbf{j} - 0.2\mathbf{k})$$

$$\sum \underline{M}_B = \underline{0} : \vec{BC} \times \underline{T} - M_x\mathbf{i} + M_y\mathbf{j} + 40\mathbf{k} = \underline{0}$$

$$\Rightarrow (-0.1026T - M_x)\mathbf{i} + (-0.0474T + M_y)\mathbf{j} + (-0.0948T + 40)\mathbf{k} = \underline{0}$$

$$\Rightarrow T = 422 \text{ N}, \quad M_x = -43.3 \text{ N}\cdot\text{m}, \quad M_y = 20.0 \text{ N}\cdot\text{m}$$

$$\text{Bending moment } M = \sqrt{M_x^2 + M_y^2} = \underline{47.7 \text{ N}\cdot\text{m}}$$

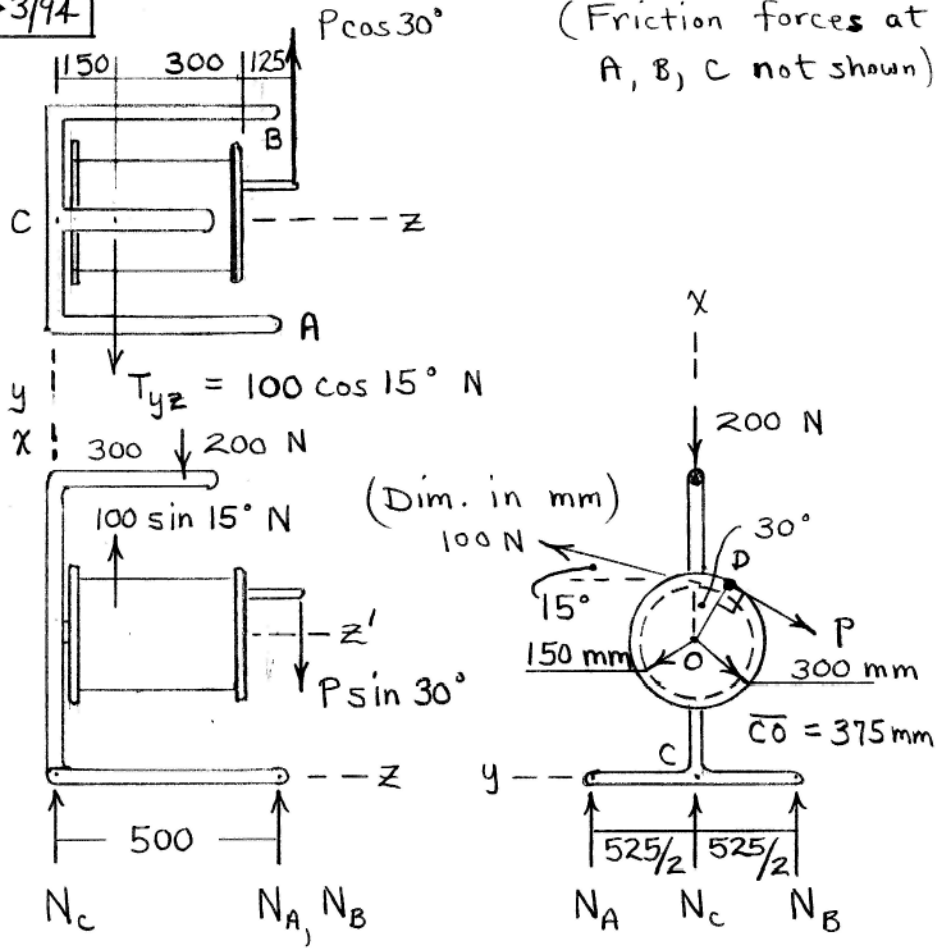
$$\sum F_x = 0 : 422 \frac{0.1386}{0.263} - V_x = 0, \quad V_x = 222 \text{ N}$$

$$\sum F_y = 0 : 422 \frac{0.1}{0.263} - V_y = 0, \quad V_y = 160 \text{ N}$$

$$V = \sqrt{V_x^2 + V_y^2} = \underline{274 \text{ N}}$$

$$\sum F_z = 0 : -422 \left(\frac{0.2}{0.263}\right) + P = 0, \quad P = \underline{320 \text{ N}}$$

► 3/94



$\sum M_{z'} = 0$  for reel alone:

$$100(150) - P(300) = 0, \quad \underline{P = 50 \text{ N}}$$

$$\begin{aligned} \sum M_{AB} = 0 : N_c(500) - 200(200) + 100 \sin 15^\circ(350) \\ + 50 \sin 30^\circ(75) = 0, \quad \underline{N_c = 58.1 \text{ N}} \end{aligned}$$

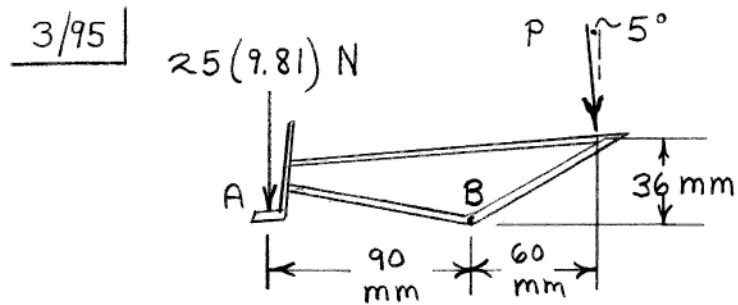
$$\begin{aligned} \sum F_x = 0 : N_A + N_B + N_c - 200 - 50 \sin 30^\circ \\ + 100 \sin 15^\circ = 0 \end{aligned} \quad (1)$$

$$\begin{aligned} \sum M_{Cz} = 0 : (N_B - N_A) \frac{525}{2} + 100(150) - 50(300) \\ + 100 \cos 15^\circ(375) - 50 \cos 30^\circ(375) = 0 \end{aligned} \quad (2)$$

Solve Eqs. (1) & (2) to obtain

$$\begin{cases} \underline{N_A = 108.6 \text{ N}} \\ \underline{N_B = 32.4 \text{ N}} \end{cases}$$

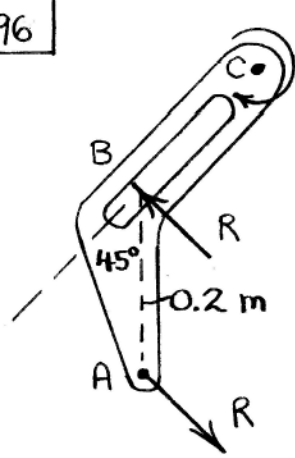




$$\begin{aligned} \curvearrow + \sum M_B = 0: & 25(9.81)(90) - P \cos 5^\circ (60) \\ & - P \sin 5^\circ (36) = 0 \\ & \underline{P = 351 \text{ N}} \end{aligned}$$

Assumptions: The weight of the panel acts at the 90-mm dimension as shown above; panel does not slip.

3/96



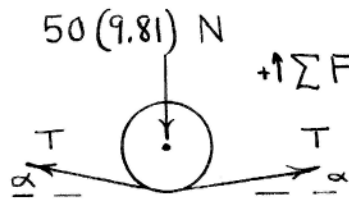
80 N·m

Forces at A and B must constitute a couple.

$$\rightarrow \sum M = 0 : 80 + R(0.2 \cos 45^\circ) = 0$$

$$\underline{R = 566 \text{ N}}$$

3/97 | Isolate wheel of unicycle:  
 $\alpha = \tan^{-1}\left(\frac{0.075}{9}\right) = 0.477^\circ$

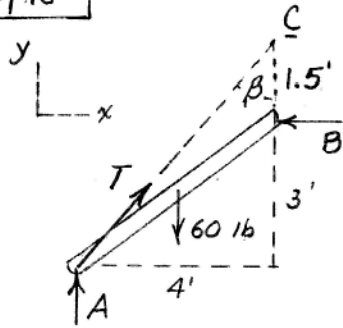


$$+\uparrow \Sigma F = 0: 2T \sin \alpha - 50(9.81) = 0$$

$$T = 29400 \text{ N}$$

$$\text{or } \underline{T = 29.4 \text{ kN}}$$

3/98



$$\sum M_A = 0; 60(2) - 3B = 0$$

$$B = 40 \text{ lb}$$

$$\sum F_x = 0; T \sin \beta - 40 = 0$$

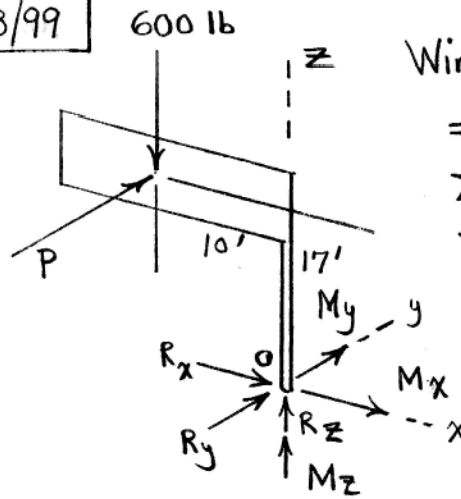
$$T = 40 \frac{\sqrt{4^2 + 4.5^2}}{4} = 60.2 \text{ lb}$$

$$\sum F_y = 0; T \cos \beta + A - 60 = 0$$

$$A = 60 - 60.2 \frac{4.5}{\sqrt{4^2 + 4.5^2}}$$

$$A = 15 \text{ lb}$$

3/99

Wind force  $P = pA$ 

$$= 17.5(6)(12) = 1260 \text{ lb}$$

$$\sum F_x = 0 \Rightarrow R_x = 0$$

$$\sum F_y = 0 \Rightarrow R_y = -1260 \text{ lb}$$

$$\sum F_z = 0 \Rightarrow R_z = 600 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

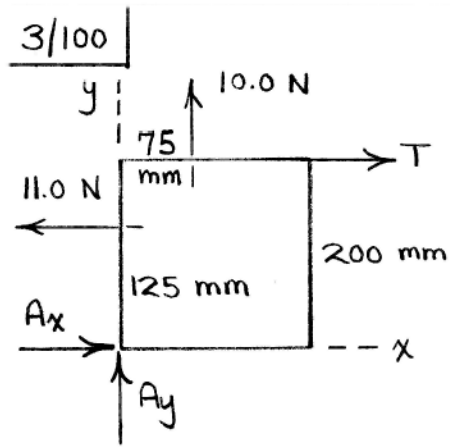
$$= 1396 \text{ lb}$$

$$\sum \underline{M}_o = \underline{0} : \underline{M} + (-10\underline{i} + 17\underline{k}) \times (1260\underline{j} - 600\underline{k}) = \underline{0}$$

$$\Rightarrow \underline{M} = 21,400 \underline{i} + 6000 \underline{j} + 12,600 \underline{k} \text{ lb-ft}$$

$$\underline{M} = 25,600 \text{ lb-ft}$$



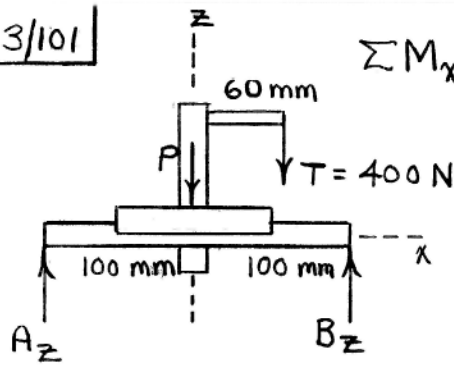


Isolate entire system  
of plate, pulleys, & tape.

$$+\circlearrowleft \sum M_A = 0: T(200) - 10.0(75) - 11.0(125) = 0$$

$$T = \underline{10.62 \text{ N}}$$

3/101



$$\sum M_x = 0: P(200) - 400(120 \cos 30^\circ) = 0$$

$$P = 208 \text{ N}$$

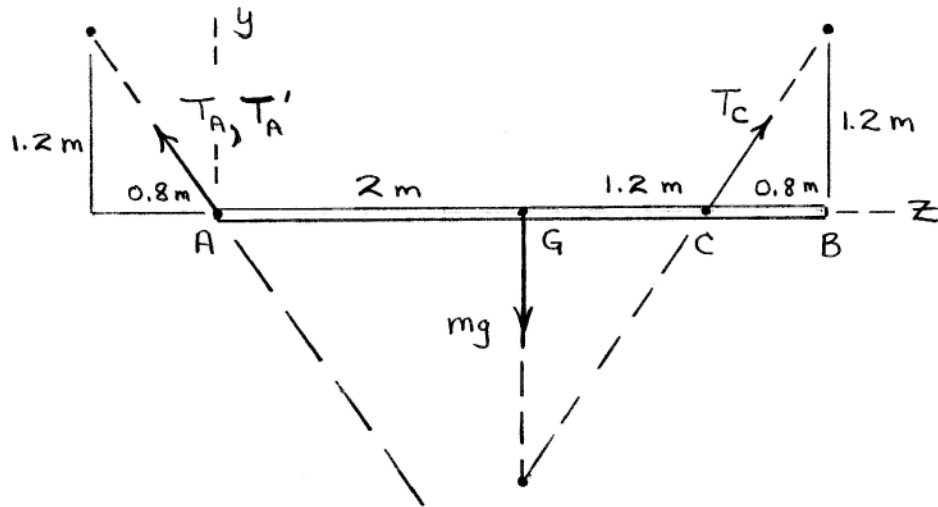
$$\sum M_{Ay} = 0: 208(100) + 400(160) - B_z(200) = 0$$

$$B_z = 424 \text{ N}$$

$$\sum F_z = 0: A_z + 424 - 208 - 400 = 0, A_z = 183.9 \text{ N}$$

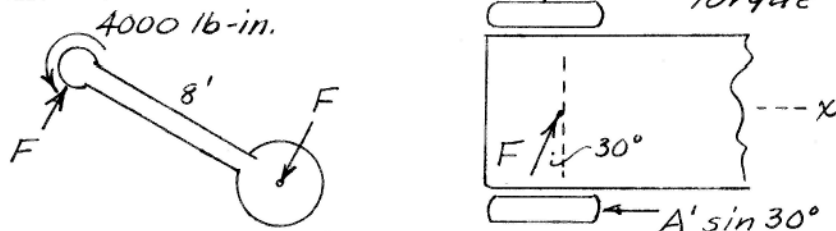
$$\text{Because } A_y = B_y = 0, \underline{A = A_z = 183.9 \text{ N}}, \underline{B = B_z = 424 \text{ N}}$$

3/102 | y-z view :



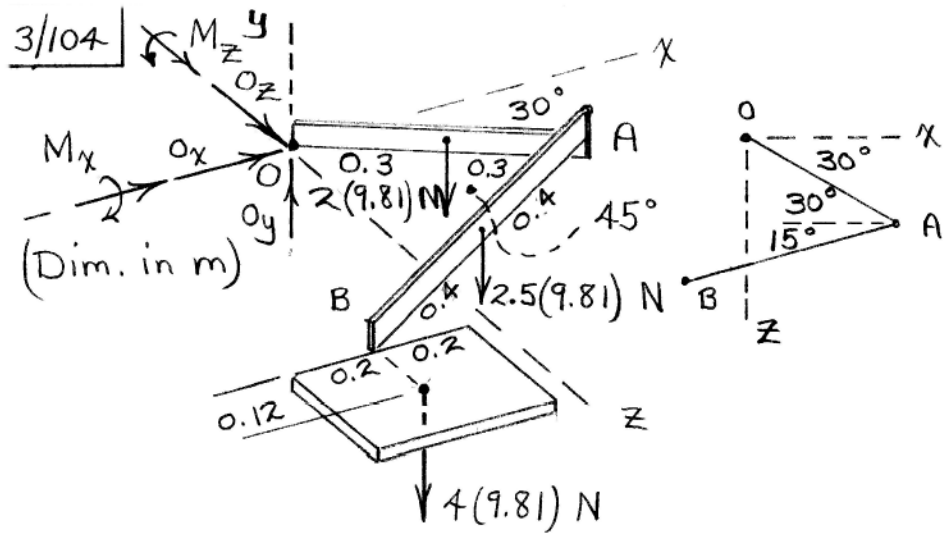
Forces are not concurrent in the y-z plane. Therefore the indicated position is not one of static equilibrium.

3/103 Torque on auger is opposite to applied torque



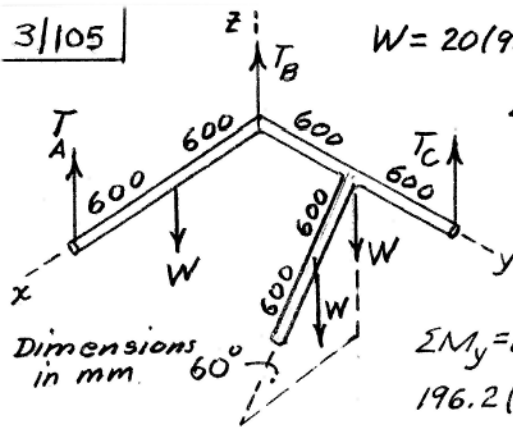
Arm:  $\sum M = 0; 8(12)F - 4000 = 0, F = 41.7 \text{ lb}$

Truck:  $\sum F_x = 0; A' \sin 30^\circ - 41.7 \sin 30^\circ, \underline{A' = 41.7 \text{ lb}}$



$$\begin{aligned} \sum F_x = 0: \quad \underline{O_x = 0} \quad \quad \quad \sum F_z = 0: \quad \underline{O_z = 0} \\ \sum F_y = 0: \quad O_y - 8.5(9.81) = 0, \quad \quad \underline{O_y = 83.4 \text{ N}} \\ \sum M_{O_x} = 0: \quad M_x + 2(9.81)(0.3 \sin 30^\circ) + 2.5(9.81) \times \\ (0.6 \sin 30^\circ + 0.4 \sin 15^\circ) + 4(9.81)(0.6 \sin 30^\circ + \\ 0.8 \sin 15^\circ + 0.12) = 0, \quad \quad \underline{M_x = -37.4 \text{ N}\cdot\text{m}} \\ \sum M_{O_z} = 0: \quad M_z - 2(9.81)(0.3 \cos 30^\circ) - 2.5(9.81) \times \\ (0.6 \cos 30^\circ - 0.4 \cos 15^\circ) - 4(9.81)(0.6 \cos 30^\circ \\ - 0.8 \cos 15^\circ) = 0, \quad \quad \underline{M_z = -1.567 \text{ N}\cdot\text{m}} \end{aligned}$$

3/105



$$W = 20(9.81) = 196.2 \text{ N}$$

$$\begin{aligned} \sum M_x &= 0; \\ 1200 T_C - 2(196.2)600 &= 0 \end{aligned}$$

$$\underline{T_C = 196.2 \text{ N}}$$

$$\begin{aligned} \sum M_y &= 0; \\ 196.2(600 + 600 \cos 60^\circ) - 1200 T_A &= 0 \end{aligned}$$

$$\underline{T_A = 147.2 \text{ N}}$$

$$\sum F_z = 0; T_B + 147.2 + 196.2 - 3(196.2) = 0, \underline{T_B = 245 \text{ N}}$$

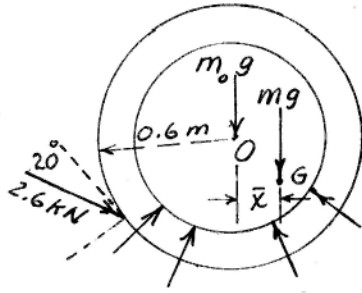
3/106

$$mg = 750 (9.81)(10^{-3}) = 7.358 \text{ kN}$$

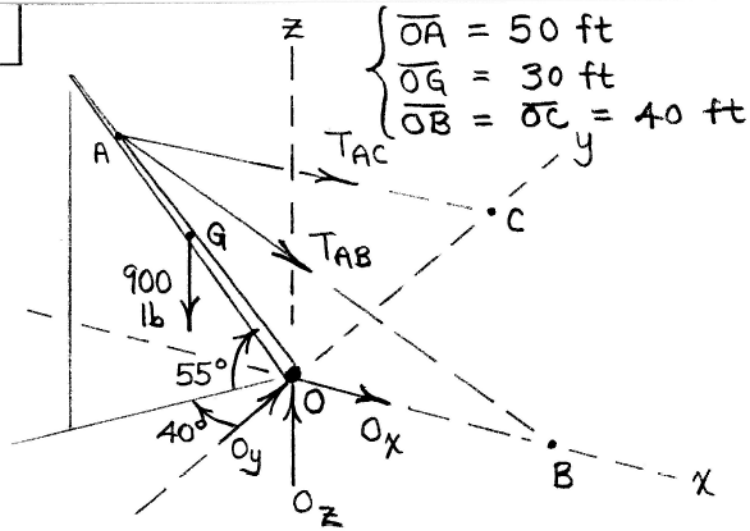
$$\Sigma M_O = 0; 2.6 \cos 20^\circ (600)$$

$$- 7.358 \bar{x} = 0$$

$$\bar{x} = 199.2 \text{ mm}$$



3/107



Coordinates of A:  $50(-\cos 55^\circ \sin 40^\circ, -\cos 55^\circ \cos 40^\circ, \sin 55^\circ) = (-18.43, -22.0, 41.0)$  ft

Coordinates of G:  $30(-\cos 55^\circ \sin 40^\circ, -\cos 55^\circ \cos 40^\circ, \sin 55^\circ) = (-11.06, -13.18, 24.6)$  ft

$$\mathbf{T}_{AB} = T_{AB} \left[ \frac{(18.43 + 40)\mathbf{i} + 22.0\mathbf{j} - 41.0\mathbf{k}}{\sqrt{(18.43 + 40)^2 + 22.0^2 + 41.0^2}} \right]$$

$$= T_{AB} [0.783\mathbf{i} + 0.294\mathbf{j} - 0.549\mathbf{k}]$$

$$\mathbf{T}_{AC} = T_{AC} \left[ \frac{18.43\mathbf{i} + (22.0 + 40)\mathbf{j} - 41.0\mathbf{k}}{\sqrt{18.43^2 + (22.0 + 40)^2 + 41.0^2}} \right]$$



$$= T_{AC} [0.241\mathbf{i} + 0.810\mathbf{j} - 0.535\mathbf{k}]$$

$$\Sigma F_x = 0: 0.783T_{AB} + 0.241T_{AC} + O_x = 0 \quad (1)$$

$$\Sigma F_y = 0: 0.294T_{AB} + 0.810T_{AC} + O_y = 0 \quad (2)$$

$$\Sigma F_z = 0: -0.549T_{AB} - 0.535T_{AC} + O_z - 900 = 0 \quad (3)$$

$$\Sigma M_{BC} = 0: \Sigma \underline{M}_B \cdot \underline{n}_{BC} = 0:$$

$$\left\{ -40\mathbf{i} \times 0_z\mathbf{k} + [(-40 - 11.06)\mathbf{i} - 13.18\mathbf{j}] \times [-900\mathbf{k}] \right\} \cdot$$

$$\left( -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} \right) = 0$$

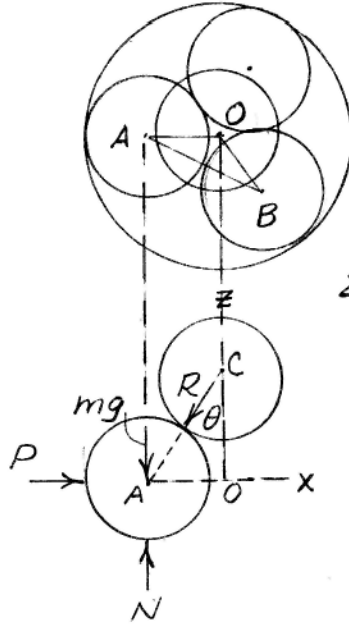
$$\text{or } -(40 \cdot 0_z - 46,000) \frac{\sqrt{2}}{2} + 11,860 \frac{\sqrt{2}}{2} = 0 \quad (4)$$

$$\Sigma M_{O_y} = 0: 0.549T_{AB} (40) - 900(11.06) = 0 \quad (5)$$

Solve Eqs. (1)-(5) in reverse order to

$$\text{obtain } \begin{cases} O_x = -489 \text{ lb} & T_{AB} = 454 \text{ lb} \\ O_y = -582 \text{ lb} & T_{AC} = 554 \text{ lb} \\ O_z = 1445 \text{ lb} \end{cases}$$

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$\angle s$   $OAB$  &  $ABO$  are  $30^\circ$   
also  $\overline{AB} = 2r$  where  
 $r =$  radius of sphere. Thus

$$\overline{AO} = r / \cos 30^\circ = 2r / \sqrt{3}$$

$$\overline{AC} = 2r$$

$$\text{so } \overline{OC} = r \sqrt{2^2 - (2/\sqrt{3})^2} = 2r\sqrt{2/3}$$

Equil. of top ball

$$\sum F_z = 0; 3R \cos \theta - mg = 0$$

$$3R \frac{2r\sqrt{2/3}}{2r} = mg$$

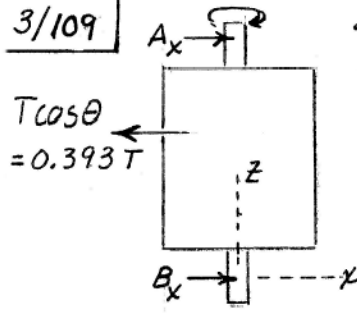
$$R = mg / \sqrt{6}$$

Lower ball :

$$\sum F_x = 0; P - R \sin \theta = 0$$

$$P = \frac{mg}{\sqrt{6}} \frac{2r/\sqrt{3}}{2r} = \frac{mg}{3\sqrt{2}}$$

3/109



$$\sum M_z = 0; 120 - 0.150T = 0, T = 800 \text{ N}$$

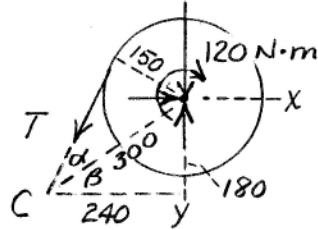
$$x-z; \sum M_B = 0; 0.393(800)(0.360) - 0.700A_x = 0; A_x = 161.6 \text{ N}$$

$$\sum F_x = 0; B_x + 161.6 - 0.393(800) = 0; B_x = 152.6 \text{ N}$$

$$y-z; \sum M_B = 0; 0.7A_y - 0.920(800)(0.360) - 50(9.81)(0.300) = 0; A_y = 588.6 \text{ N}$$

$$\sum F_y = 0; W + T \sin \theta - A_y - B_y = 0;$$

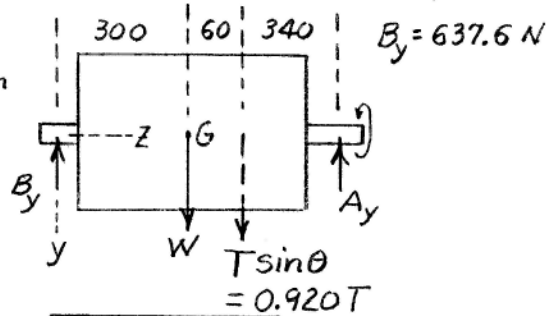
$$W = 50(9.81) \text{ N}$$



$$\beta = \tan^{-1} \frac{180}{240} = 36.9^\circ$$

$$\alpha = \sin^{-1} \frac{150}{300} = 30^\circ$$

$$\theta = \alpha + \beta = 66.9^\circ$$

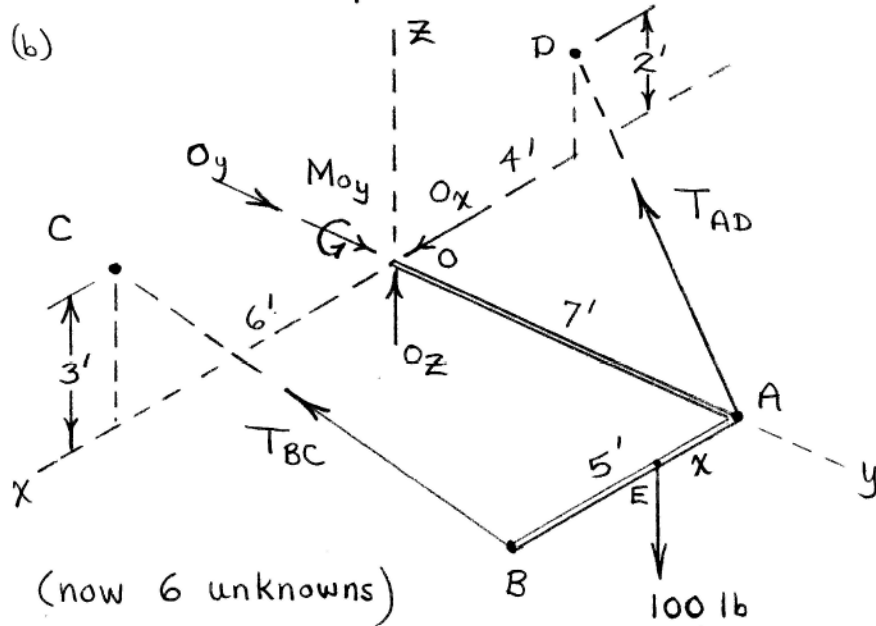


$$A = \sqrt{(161.6)^2 + (588.6)^2} = \underline{610 \text{ N}}$$

$$B = \sqrt{(152.6)^2 + (637.6)^2} = \underline{656 \text{ N}}$$

3/110 (a) There are 5 unknown constraint forces. The bar is free to rotate about a line which passes through point O and through which the lines of action of both tension forces pass.

(b)



$$\underline{T}_{AD} = T_{AD} \left[ \frac{-4\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}}{\sqrt{69}} \right]$$

$$\underline{T}_{BC} = T_{BC} \left[ \frac{\mathbf{i} - 7\mathbf{j} + 3\mathbf{k}}{\sqrt{59}} \right]$$

$$\left\{ \begin{array}{l} \sum F_x = 0: O_x - \frac{4}{\sqrt{69}} T_{AD} + \frac{1}{\sqrt{59}} T_{BC} = 0 \quad (1) \\ \sum F_y = 0: O_y - \frac{7}{\sqrt{69}} T_{AD} - \frac{7}{\sqrt{59}} T_{BC} = 0 \quad (2) \\ \sum F_z = 0: O_z + \frac{2}{\sqrt{69}} T_{AD} + \frac{3}{\sqrt{59}} T_{BC} - 100 = 0 \quad (3) \\ \sum M_{O_x} = 0: 7 \left( \frac{2}{\sqrt{69}} T_{AD} \right) + 7 \left( \frac{3}{\sqrt{59}} T_{BC} \right) - 7(100) = 0 \quad (4) \\ \sum M_{O_y} = 0: -5 \left( \frac{3}{\sqrt{59}} T_{BC} \right) + M_{O_y} + 100x = 0 \quad (5) \\ \sum M_{O_z} = 0: 7 \left( \frac{4}{\sqrt{69}} T_{AD} \right) - 7 \left( \frac{1}{\sqrt{59}} T_{BC} \right) - 5 \left( \frac{7}{\sqrt{59}} T_{BC} \right) = 0 \quad (6) \end{array} \right.$$

$$\sum F_y = 0: O_y - \frac{7}{\sqrt{69}} T_{AD} - \frac{7}{\sqrt{59}} T_{BC} = 0 \quad (2)$$

$$\sum F_z = 0: O_z + \frac{2}{\sqrt{69}} T_{AD} + \frac{3}{\sqrt{59}} T_{BC} - 100 = 0 \quad (3)$$

$$\sum M_{O_x} = 0: 7 \left( \frac{2}{\sqrt{69}} T_{AD} \right) + 7 \left( \frac{3}{\sqrt{59}} T_{BC} \right) - 7(100) = 0 \quad (4)$$

$$\sum M_{O_y} = 0: -5 \left( \frac{3}{\sqrt{59}} T_{BC} \right) + M_{O_y} + 100x = 0 \quad (5)$$

$$\sum M_{O_z} = 0: 7 \left( \frac{4}{\sqrt{69}} T_{AD} \right) - 7 \left( \frac{1}{\sqrt{59}} T_{BC} \right) - 5 \left( \frac{7}{\sqrt{59}} T_{BC} \right) = 0 \quad (6)$$

Solve Eqs. (1)-(6) over  $0.5 \leq x \leq 4.5$  ft

and discover that three of the requested quantities are constant:

$$\left\{ \begin{array}{l} O_x = 83.3 \text{ lb} \\ O_y = 292 \text{ lb} \\ O_z = 0 \end{array} \right.$$

$$\underline{T}_{AD} = 208 \text{ lb} = \text{constant}$$

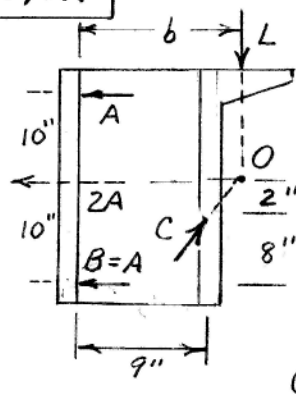
$$\underline{T}_{BC} = 128.0 \text{ lb} = \text{constant}$$

$$\underline{O} = \sqrt{O_x^2 + O_y^2 + O_z^2} = 303 \text{ lb} = \text{constant}$$

and  $\underline{M}_{O_y} = -100x + 250$  (in lb-ft if x in ft)

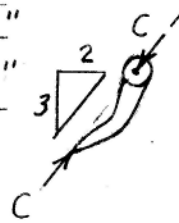
(Note that  $O_y$  could have been obtained from  $\sum M_{CD} = 0$  &  $O_z$  from  $\sum M_{AB} = 0$ )

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Reduce to three forces concurrent at  $O$ .

$$b = 9 + \frac{2}{3}(2) = \underline{10.33 \text{ in.}}$$



Alternative sol.  $\sum M_c = 0; L(b-9) - A(12) + A(8) = 0$

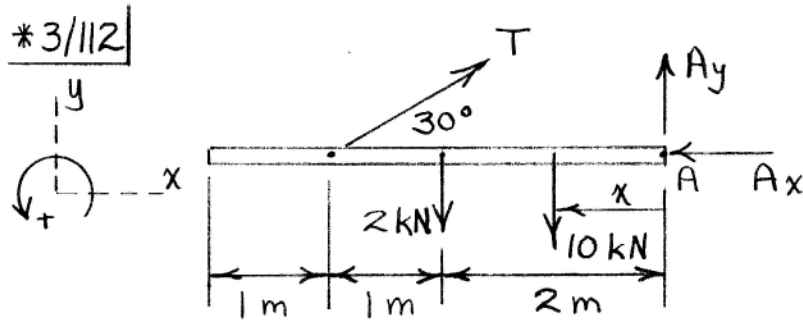
$$L(b-9) = 4A \quad \text{--- (1)}$$

$$\sum F_y = 0; \frac{3}{\sqrt{13}} C = L$$

$$\sum F_x = 0; \frac{2}{\sqrt{13}} C = 2A$$

$$L = 3A \quad \text{--- (2)}$$

(1) & (2) give  $b-9 = \frac{4}{3}, b = 10.33 \text{ in.}$



(Weight of beam =  $200(10)/1000 = 2 \text{ kN}$ )

$$\sum M_A = 0: 10x + 2(2) - T \sin 30^\circ (3) = 0$$

$$T = \frac{2}{3}(10x + 4) \quad (\text{in kN})$$

$$\sum F_x = 0: T \cos 30^\circ - A_x = 0$$

$$A_x = \frac{1}{\sqrt{3}}(10x + 4)$$

$$\sum F_y = 0: T \sin 30^\circ - 2 - 10 + A_y = 0$$

$$A_y = \frac{1}{3}(-10x + 32)$$

$$R = \{A_x^2 + A_y^2\}^{1/2} = \frac{1}{3} \{400x^2 - 400x + 1072\}^{1/2}$$

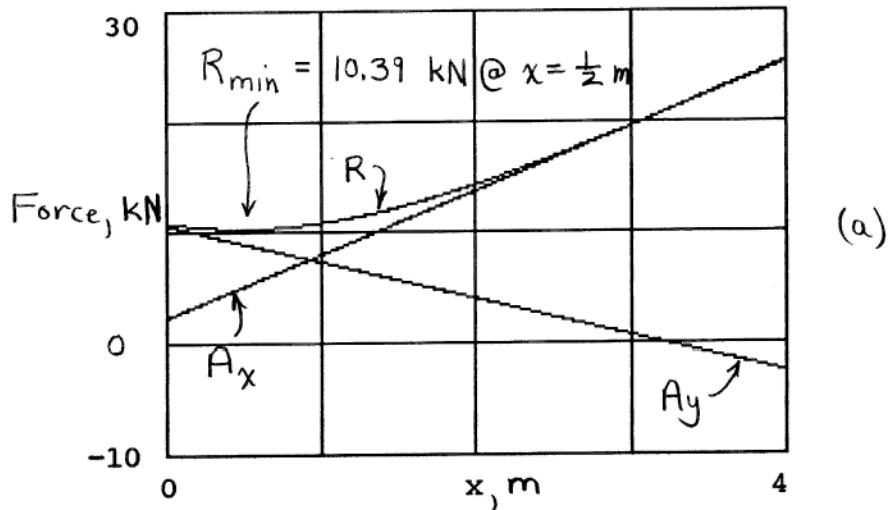
$$\text{Set } \frac{dR^2}{dx} = 0 : \quad 800x - 400 = 0$$

$$x = \frac{1}{2} \text{ m}$$

$$R_{\min} = \frac{1}{3} \left\{ 400 \left( \frac{1}{2} \right)^2 - 400 \left( \frac{1}{2} \right) + 1072 \right\}^{1/2}$$

$$= \underline{10.39 \text{ kN}} \quad (\text{b})$$

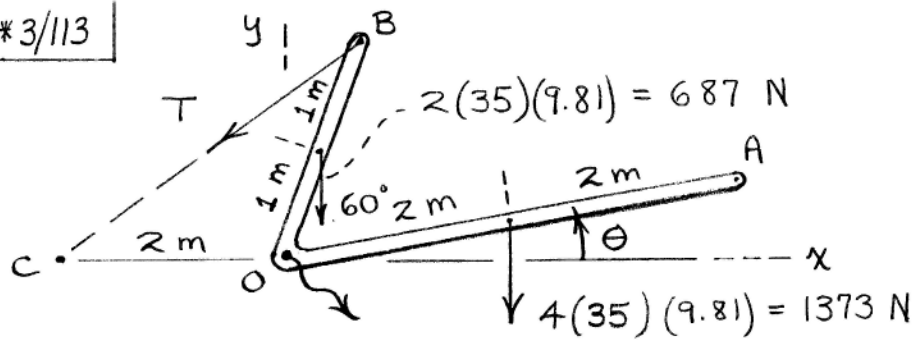
Plot of  $A_x$ ,  $A_y$ , and  $R$ :



(c)  
 $R_{\max} = 24.3 \text{ kN}$  @  $x = 3.8 \text{ m}$  is the value of  $R$  which must be used for the design of the pin at A.



\*3/113



$$\underline{BC} = [-2 - 2 \cos(\theta + 60^\circ)] \underline{i} - [2 \sin(\theta + 60^\circ)] \underline{j}$$

$$\overline{BC}^2 = \{4 + 8 \cos(\theta + 60^\circ) + 4 \cos^2(\theta + 60^\circ) + 4 \sin^2(\theta + 60^\circ)\}$$

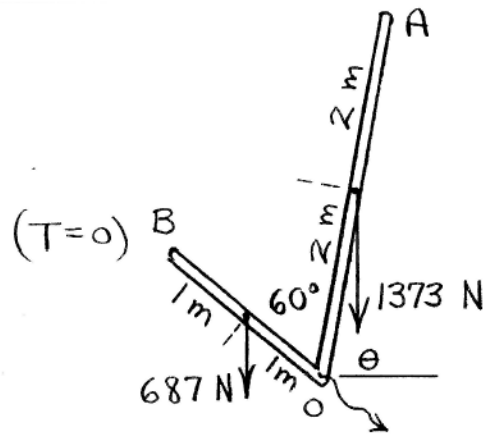
$$\overline{BC} = \{8 + 8 \cos(\theta + 60^\circ)\}^{1/2}$$

$$\underline{n}_{BC} = \frac{[-2 - 2 \cos(\theta + 60^\circ)] \underline{i} - [2 \sin(\theta + 60^\circ)] \underline{j}}{\{8 + 8 \cos(\theta + 60^\circ)\}^{1/2}}$$

Then  $\underline{T} = T \underline{n}_{BC}$

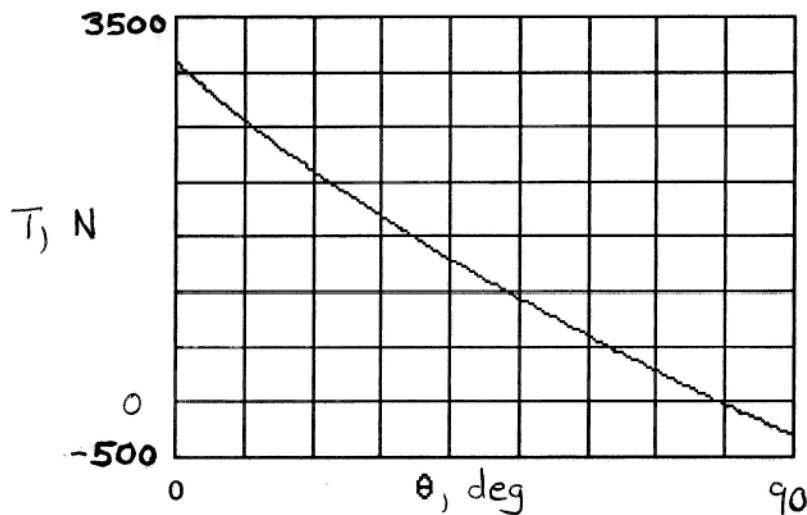
$$\sum M_O = 0: -1373(2 \cos \theta) - 687(1 \cos(\theta + 60^\circ)) + \frac{2T \sin(\theta + 60^\circ)}{\{8 + 8 \cos(\theta + 60^\circ)\}^{1/2}}(2) = 0$$

$$T = \left[ \frac{2750 \cos \theta + 687 \cos(\theta + 60^\circ)}{4 \sin(\theta + 60^\circ)} \right] \left[ 8 + 8 \cos(\theta + 60^\circ) \right]^{1/2}$$



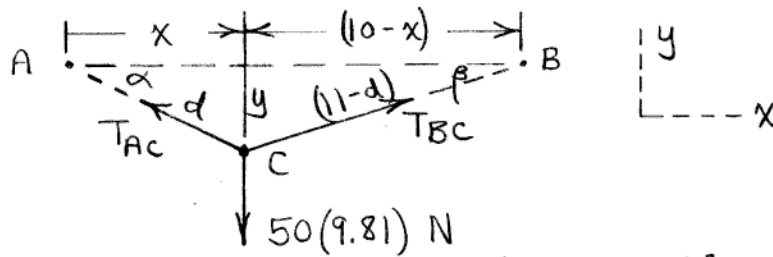
$$\begin{aligned} \sum M_o = 0 &\Rightarrow 687(1 \cos(120^\circ - \theta)) = 1373(2 \cos \theta) \\ &\text{or } \cos(120^\circ - \theta) = 4 \cos \theta \\ \cos 120^\circ \cos \theta + \sin 120^\circ \sin \theta &= 4 \cos \theta \\ -\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta &= 4 \cos \theta \\ \tan \theta = \frac{4.5(2)}{\sqrt{3}} &\Rightarrow \underline{\theta_{\max} = 79.1^\circ} \end{aligned}$$

Plot of T vs.  $\theta$  :



(T goes negative above  $\theta = \theta_{\max} = 79.1^\circ$ )

\*3/114



Geometry:  $y^2 = d^2 - x^2 = (11-d)^2 - (10-x)^2$

Simplify to obtain  $d = \frac{10}{11}x + \frac{21}{22}$

Then  $y = \sqrt{d^2 - x^2} = \left[ -\frac{21}{121}x^2 + \frac{210}{121}x + \frac{441}{484} \right]^{1/2}$

$\tan \alpha = \frac{y}{x} = \frac{\left[ -\frac{21}{121}x^2 + \frac{210}{121}x + \frac{441}{484} \right]^{1/2}}{x}$

$\tan \beta = \frac{y}{10-x} = \frac{\left[ -\frac{21}{121}x^2 + \frac{210}{121}x + \frac{441}{484} \right]^{1/2}}{10-x}$

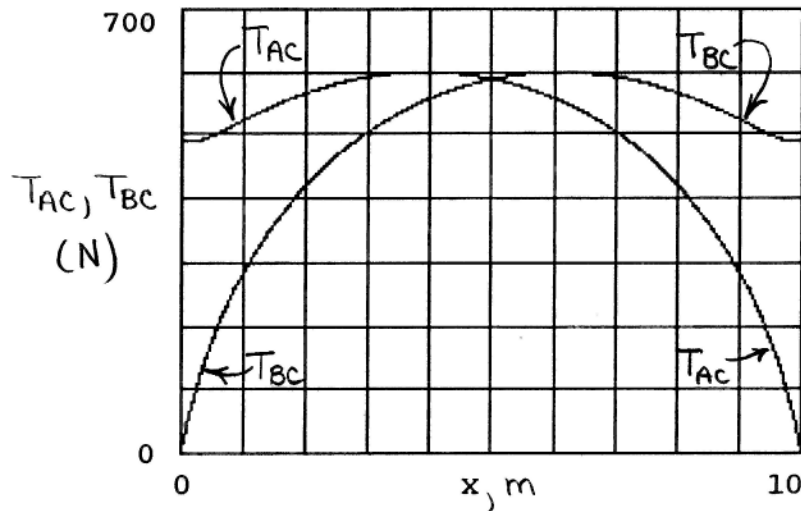
Equilibrium:

$\sum F_x = 0: -T_{AC} \cos \alpha + T_{BC} \cos \beta = 0$

$\sum F_y = 0: T_{AC} \sin \alpha + T_{BC} \cos \beta - 50(9.81) = 0$

Solve the above equations over  $0 \leq x \leq 10$  m

to obtain the following plot:



The maxima are :

$$(T_{AC})_{\max} = 600 \text{ N @ } x = 3.91 \text{ m}$$

$$(T_{BC})_{\max} = 600 \text{ N @ } x = 6.09 \text{ m}$$

As a matter of interest :

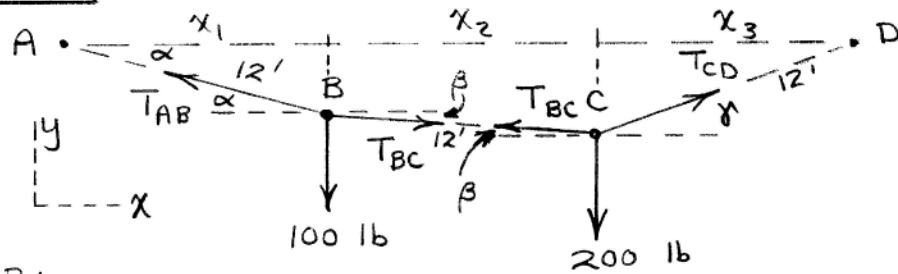
$$(T_{AC})_{\min} = 488 \text{ N @ } x = 0.1287 \text{ m}$$

$$(T_{BC})_{\min} = 488 \text{ N @ } x = 9.87 \text{ m}$$

(These are local, not global, minima; the two global minima are both zero.)

Note that  $x = 0$  does not represent a unique clamping point. For  $x = 0$ , the clamp could be anywhere from zero to  $\frac{21}{22}$  m along the cable from A.

\*3/115|



B:

$$\sum F_x = 0: -T_{AB} \cos \alpha + T_{BC} \cos \beta = 0 \quad (1)$$

$$\sum F_y = 0: T_{AB} \sin \alpha - T_{BC} \sin \beta - 100 = 0 \quad (2)$$

C:

$$\sum F_x = 0: -T_{BC} \cos \beta + T_{CD} \cos \gamma = 0 \quad (3)$$

$$\sum F_y = 0: T_{BC} \sin \beta + T_{CD} \sin \gamma - 200 = 0 \quad (4)$$

$$\cos \alpha = \frac{x_1}{12}, \quad \cos \beta = \frac{x_2}{12}, \quad \cos \gamma = \frac{x_3}{12}$$

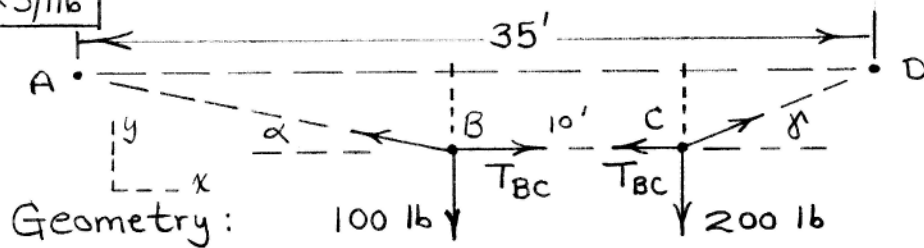
$$\text{So } 12 \cos \alpha + 12 \cos \beta + 12 \cos \gamma = 35 \quad (5)$$

$$\sin \alpha + \sin \beta = \sin \gamma \quad (\text{from figure}) \quad (6)$$

Solve numerically :

$$\begin{cases} \alpha = 14.44^\circ & T_{AB} = 529 \text{ lb} \\ \beta = 3.57^\circ & T_{BC} = 513 \text{ lb} \\ \gamma = 18.16^\circ & T_{CD} = 539 \text{ lb} \end{cases}$$

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Geometry:

$$\overline{AB} + \overline{BC} + \overline{CD} = 36 \text{ ft} \quad (1)$$

$$\overline{AB} \cos \alpha + \overline{BC} + \overline{CD} \cos \delta = 35 \text{ ft} \quad (2)$$

$$\overline{AB} \sin \alpha = \overline{CD} \sin \delta \quad (3)$$

Equilibrium:

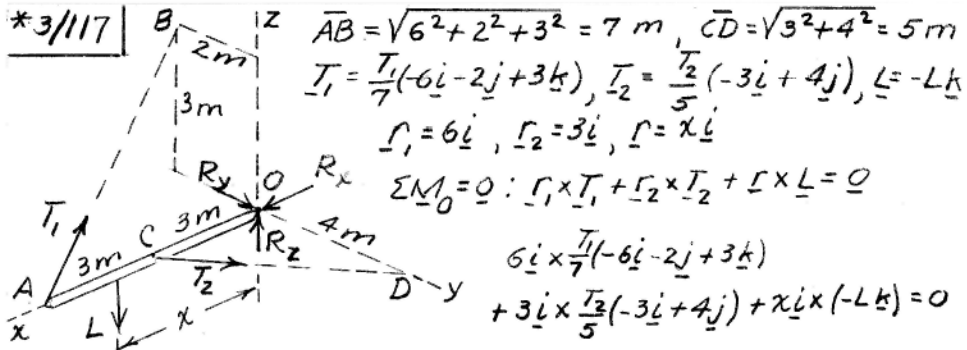
$$\textcircled{B} \begin{cases} \sum F_x = 0: -T_{AB} \cos \alpha + T_{BC} = 0 & (4) \\ \sum F_y = 0: T_{AB} \sin \alpha - 100 = 0 & (5) \end{cases}$$

$$\textcircled{C} \begin{cases} \sum F_x = 0: -T_{BC} + T_{CD} \cos \delta = 0 & (6) \\ \sum F_y = 0: T_{CD} \sin \delta - 200 = 0 & (7) \end{cases}$$

With  $\overline{BC}$  set to 10 ft, solve 7 equations in 7 unknowns & obtain

$\overline{AB} = 17.01 \text{ ft}$	$\alpha = 11.47^\circ$	$T_{AB} = 503 \text{ lb}$
$\overline{CD} = 8.99 \text{ ft}$	$\delta = 22.1^\circ$	$T_{BC} = 493 \text{ lb}$
		$T_{CD} = 532 \text{ lb}$

\*3/117



$$AB = \sqrt{6^2 + 2^2 + 3^2} = 7 \text{ m}, \quad CD = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

$$\underline{T}_1 = \frac{T_1}{7}(-6\hat{i} - 2\hat{j} + 3\hat{k}), \quad \underline{T}_2 = \frac{T_2}{5}(-3\hat{i} + 4\hat{j}), \quad \underline{L} = -L\hat{k}$$

$$\underline{r}_1 = 6\hat{i}, \quad \underline{r}_2 = 3\hat{i}, \quad \underline{r} = x\hat{i}$$

$$\sum \underline{M}_O = 0: \underline{r}_1 \times \underline{T}_1 + \underline{r}_2 \times \underline{T}_2 + \underline{r} \times \underline{L} = 0$$

$$6\hat{i} \times \frac{T_1}{7}(-6\hat{i} - 2\hat{j} + 3\hat{k}) + 3\hat{i} \times \frac{T_2}{5}(-3\hat{i} + 4\hat{j}) + x\hat{i} \times (-L\hat{k}) = 0$$

Expand:  $\frac{6}{7}T_1(-2\hat{k} - 3\hat{j}) + \frac{3}{5}T_2(4\hat{k}) + Lx\hat{j} = 0$

$$-\frac{12}{7}T_1 + \frac{12}{5}T_2 = 0, \quad -\frac{18}{7}T_1 + Lx = 0, \quad \text{so } T_1 = \frac{7}{18}Lx, \quad T_2 = \frac{5}{18}Lx$$

$$\sum F_x = 0; R_x - \frac{3}{5}T_2 - \frac{6}{7}T_1 = 0, \quad R_x = \frac{3}{5} \cdot \frac{5}{18}Lx + \frac{6}{7} \cdot \frac{7}{18}Lx = \frac{1}{2}Lx$$

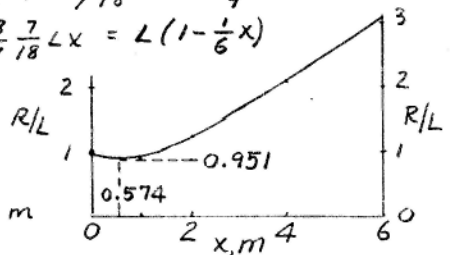
$$\sum F_y = 0; R_y + \frac{4}{5}T_2 - \frac{2}{7}T_1 = 0, \quad R_y = -\frac{4}{5} \cdot \frac{5}{18}Lx + \frac{2}{7} \cdot \frac{7}{18}Lx = -\frac{1}{9}Lx$$

$$\sum F_z = 0; R_z - L + \frac{3}{7}T_1 = 0, \quad R_z = L - \frac{3}{7} \cdot \frac{7}{18}Lx = L(1 - \frac{1}{6}x)$$

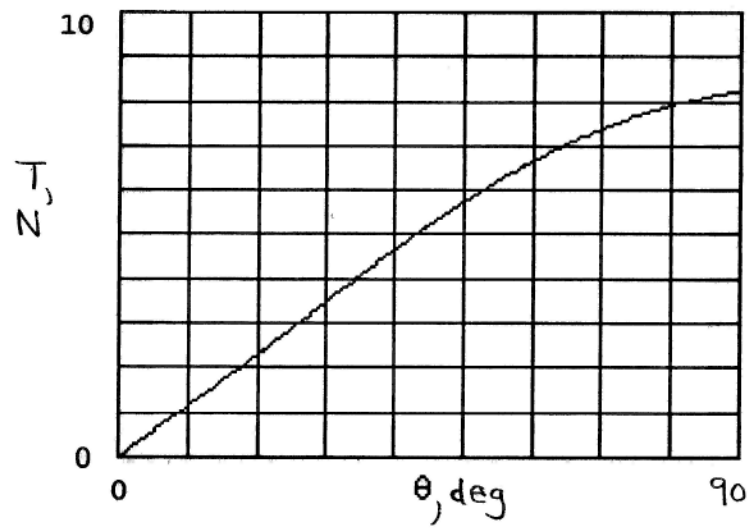
$$R^2 = L^2 \left( \frac{x^2}{4} + \frac{x^2}{81} + 1 - \frac{x}{3} + \frac{x^2}{36} \right)$$

$$R/L = \sqrt{(47x^2/162) - x/3 + 1}$$

$$\frac{dR^2/L^2}{dx} = \frac{47x}{81} - \frac{1}{3} = 0 \text{ for min.}; \quad x = 0.574 \text{ m}$$

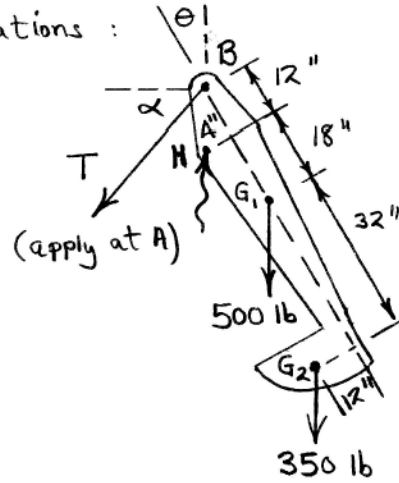
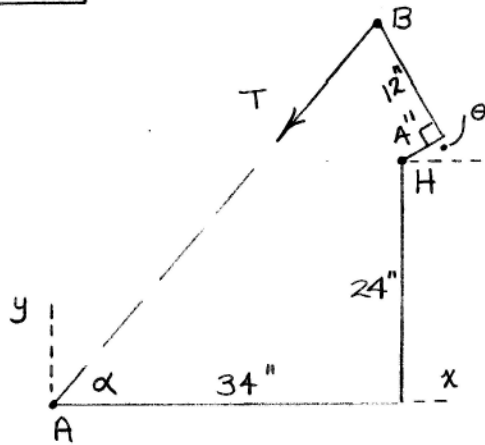


With  $k = 25 \text{ N/m}$ ,  $\delta$  given by (1),  $\alpha$  given by (2),  $\overline{OA} = 0.48 \text{ m}$ ,  $m = 1.5 \text{ kg}$ ,  $g = 9.81 \text{ m/s}^2$ ,  $\overline{OG} = 0.16 \text{ m}$ ,  $\overline{OD} = 0.48 \text{ m}$ , and  $\beta = \frac{90^\circ + \theta}{2}$ , we obtain the following plot:



When  $\theta = 45^\circ$ ,  $T = 5.23 \text{ N}$   
 When  $\theta = 90^\circ$ ,  $T = 8.22 \text{ N}$

\*3/118 Geometrical Considerations :



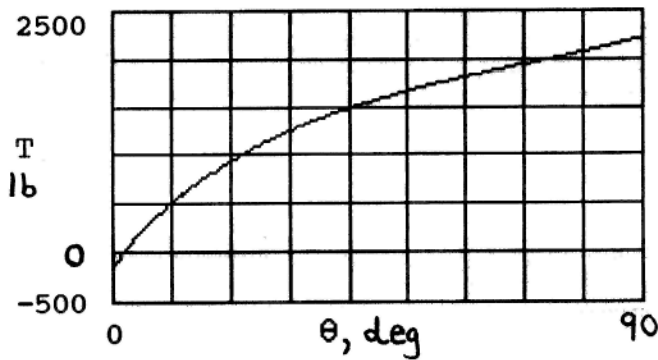
$$\text{Coordinates of B : } \begin{cases} x = 34 + 4 \cos \theta - 12 \sin \theta & (\text{in.}) \\ y = 24 + 4 \sin \theta + 12 \cos \theta & (1) \end{cases}$$

$$\therefore \alpha = \tan^{-1} \frac{y}{x} \quad (\text{aims } \perp) \quad (2)$$

$$\sum M_H = 0 : -(T \cos \alpha)(24) + (T \sin \alpha)(34) - 500(18 \sin \theta + 4 \cos \theta) - 350(4 \cos \theta + (18+32) \sin \theta - 12 \cos \theta) = 0$$

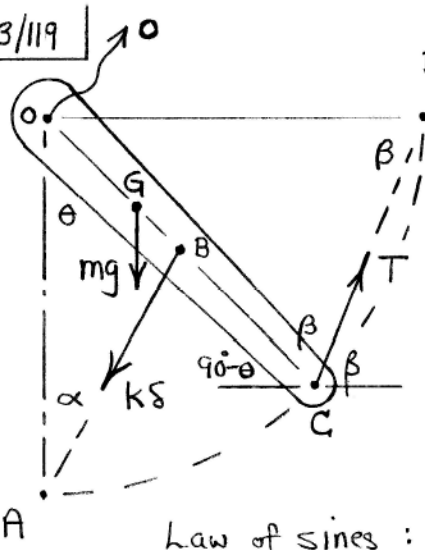
$$\Rightarrow T = \frac{26,500 \sin \theta - 800 \cos \theta}{34 \sin \alpha - 24 \cos \alpha} \quad (\text{lb; positive is tension in AB})$$

Solve (1), (2), & (3) & plot: (Note  $T=0$  @  $\theta = 1.729^\circ$ )





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$$2\beta + (90^\circ - \theta) = 180^\circ$$

$$\beta = \frac{90^\circ + \theta}{2}$$

By law of cosines,  $\overline{AB} = \sqrt{\overline{OA}^2 + \overline{OB}^2 - 2 \cdot \overline{OA} \cdot \overline{OB} \cdot \cos \theta}$ ;

then spring deflection

$$\delta = \overline{AB} - \overline{BC} \quad (1)$$

Law of sines:  $\frac{\sin \alpha}{\overline{OB}} = \frac{\sin \theta}{\overline{AB}}$

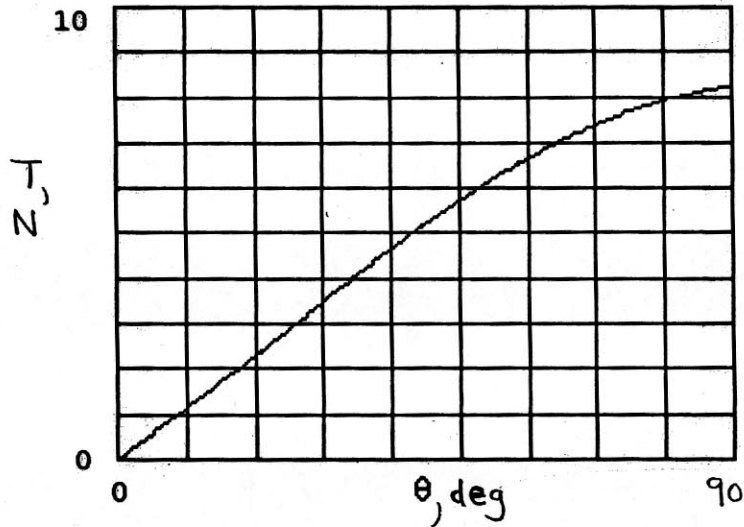
Consider  $k\delta$  applied at A:

$$\alpha = \sin^{-1} \left[ \frac{\overline{OB}}{\overline{AB}} \sin \theta \right] \quad (2)$$

$$\sum M_O = 0: - (k\delta \sin \alpha) (\overline{OA}) - mg (\overline{OG} \sin \theta) + T \sin \beta (\overline{OD}) = 0$$

$$T = \frac{(k\delta \sin \alpha) (\overline{OA}) + mg (\overline{OG} \sin \theta)}{\overline{OD} \sin \beta} \quad (3)$$

With  $k = 25 \text{ N/m}$ ,  $\delta$  given by (1),  $\alpha$  given by (2),  $\overline{OA} = 0.48 \text{ m}$ ,  $m = 1.5 \text{ kg}$ ,  $g = 9.81 \text{ m/s}^2$ ,  $\overline{OG} = 0.16 \text{ m}$ ,  $\overline{OD} = 0.48 \text{ m}$ , and  $\beta = \frac{90^\circ + \theta}{2}$ , we obtain the following plot:

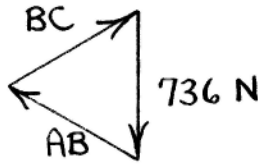
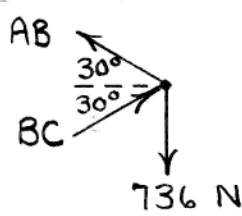


When  $\theta = 45^\circ$ ,  $T = 5.23 \text{ N}$

When  $\theta = 90^\circ$ ,  $T = 8.22 \text{ N}$

4/1 | Load =  $75(9.81) = 736 \text{ N}$

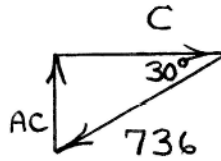
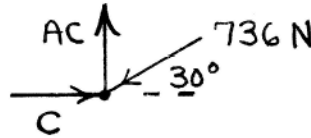
Joint B:



$$\underline{AB = 736 \text{ N T}}$$

$$\underline{BC = 736 \text{ N C}}$$

Joint C:

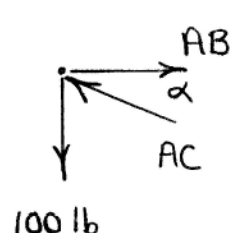


$$AC = 736 \left(\frac{1}{2}\right)$$

$$= \underline{368 \text{ N T}}$$

4/2

Joint A:  $\alpha = \tan^{-1} \frac{2.5}{6} = 22.6^\circ$   
 $(\cos \alpha = \frac{12}{13}, \sin \alpha = \frac{5}{13})$



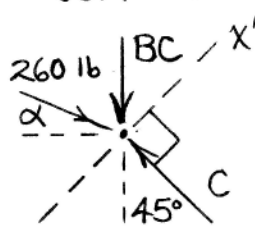
$$\Sigma F_y = 0: AC \sin \alpha - 100 = 0$$

$$AC = 260 \text{ lb C}$$

$$\Sigma F_x = 0: AB - 260 \cos \alpha = 0$$

$$AB = 240 \text{ lb T}$$

Joint C:



$$\Sigma F_{x'} = 0: 260 \left(\frac{12}{13}\right) - C \frac{\sqrt{2}}{2} = 0$$

$$C = 339 \text{ lb (reaction)}$$

$$\Sigma F_{y'} = 0: -260 \frac{5}{13} - BC + 339 \frac{\sqrt{2}}{2} = 0$$

$$BC = 140 \text{ lb C}$$

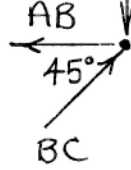
Could use  $\Sigma F_{x'}$  to find BC without involving calculation of C. Nonetheless, observe that changing the  $45^\circ$  support angle would affect BC, but not AB or AC!

4/3 | y | x

Joint B:

5 kN  $\sum F_y = 0: BC \frac{\sqrt{2}}{2} - 5 = 0$

$BC = 5\sqrt{2} \text{ kN C}$

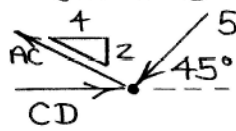


$\sum F_x = 0: -AB + 5\sqrt{2} \frac{\sqrt{2}}{2} = 0, AB = 5 \text{ kN T}$

Joint C:

5 $\sqrt{2}$  kN  $\sum F_y = 0: -5\sqrt{2} \frac{\sqrt{2}}{2} + AC \frac{2}{2\sqrt{5}} = 0$

$AC = 5\sqrt{5} \text{ kN T}$

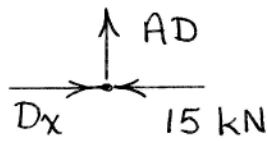


$\sum F_x = 0: CD - 5\sqrt{5} \frac{4}{2\sqrt{5}} - 5\sqrt{2} \frac{\sqrt{2}}{2} = 0$

$CD = 15 \text{ kN C}$

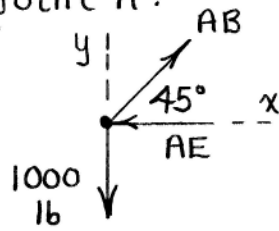
Joint D:

AD From  $\sum F_y = 0, AD = 0$



4/4

Joint A:



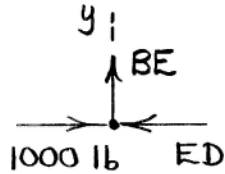
$$\sum F_y = 0: AB \sin 45^\circ - 1000 = 0$$

$$AB = 1414 \text{ lb T}$$

$$\sum F_x = 0: 1414 \cos 45^\circ - AE = 0$$

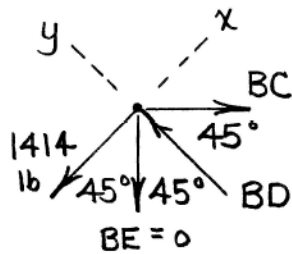
$$AE = 1000 \text{ lb C}$$

Joint E:



$$\sum F_y = 0: \underline{BE = 0}$$

Joint B:

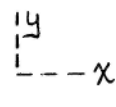


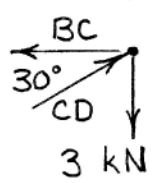
$$\sum F_x = 0: BC \cos 45^\circ - 1414 = 0$$

$$BC = 2000 \text{ lb T}$$

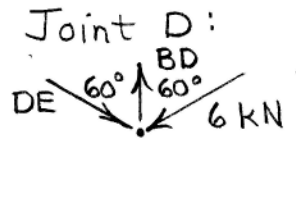
$$\sum F_y = 0: BD - 2000 \cos 45^\circ = 0$$

$$\underline{BD = 1414 \text{ lb C}}$$

4/5 | Joint C: 

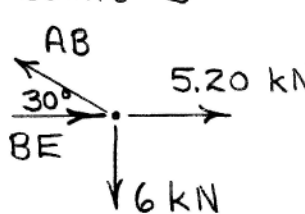


$$\begin{cases} \sum F_y = 0 : CD \left(\frac{1}{2}\right) - 3 = 0, & \underline{CD = 6 \text{ kN C}} \\ \sum F_x = 0 : -BC + 6 \frac{\sqrt{3}}{2} = 0, & \underline{BC = 5.20 \text{ kN T}} \end{cases}$$

Joint D: 

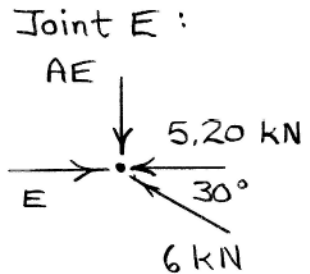
$$\begin{cases} \sum F_x = 0 \Rightarrow & \underline{DE = 6 \text{ kN C}} \\ \sum F_y = 0 : BD - 2(6) \left(\frac{1}{2}\right) = 0 & \\ & \underline{BD = 6 \text{ kN T}} \end{cases}$$

Joint B:



$$\begin{cases} \sum F_y = 0 : AB \left(\frac{1}{2}\right) - 6 = 0, & \underline{AB = 12 \text{ kN T}} \\ \sum F_x = 0 : BE - 12 \frac{\sqrt{3}}{2} + 5.20 = 0 & \\ & \underline{BE = 5.20 \text{ kN C}} \end{cases}$$

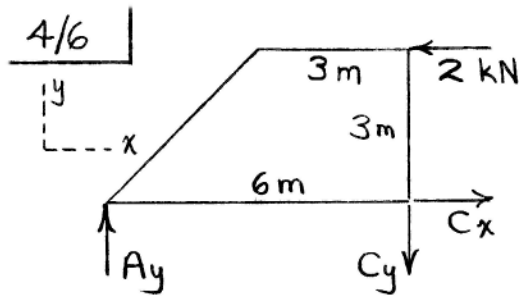
Joint E:



$$\sum F_y = 0 : 6 \left(\frac{1}{2}\right) - AE = 0$$

$$\underline{AE = 3 \text{ kN C}}$$

(Joint A checks after external reactions are determined from the truss as a whole.)



$$+\circlearrowleft \sum M_C = 0 : 6A_y - 2(3) = 0$$

$$A_y = 1 \text{ kN}$$

$$C_x = 2 \text{ kN}, C_y = 1 \text{ kN}$$

Joint A:

$$\left\{ \begin{array}{l} \sum F_y = 0 : 1 - AE \sin 45^\circ = 0 \\ \sum F_x = 0 : AB - 1.414 \cos 45^\circ = 0 \end{array} \right.$$

$$AE = 1.414 \text{ kN C}$$

$$AB = 1 \text{ kN T}$$

Joint E:

$$\left\{ \begin{array}{l} \sum F_x = 0 : 1.414 \sin 45^\circ - DE = 0 \\ \sum F_y = 0 : 1.414 \cos 45^\circ - BE = 0 \end{array} \right.$$

$$DE = 1 \text{ kN C}$$

$$BE = 1 \text{ kN T}$$

Joint B:

$$\left\{ \begin{array}{l} \sum F_y = 0 : 1 - BD \sin 45^\circ = 0 \\ \sum F_x = 0 : BC - 1.414 \cos 45^\circ - 1 = 0 \end{array} \right.$$

$$BD = 1.414 \text{ kN C}$$

$$BC = 2 \text{ kN T}$$

Joint C:

$$\sum F_y = 0 : CD - 1 = 0$$

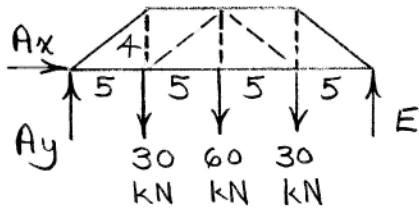
$$CD = 1 \text{ kN T}$$

(Joint D checks)



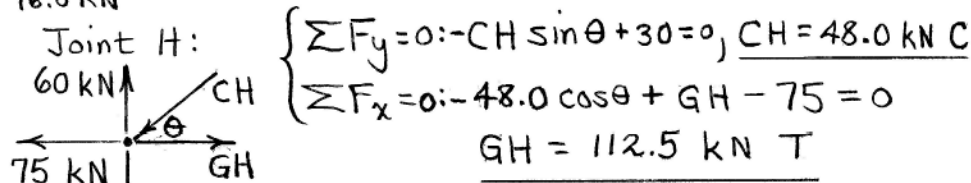
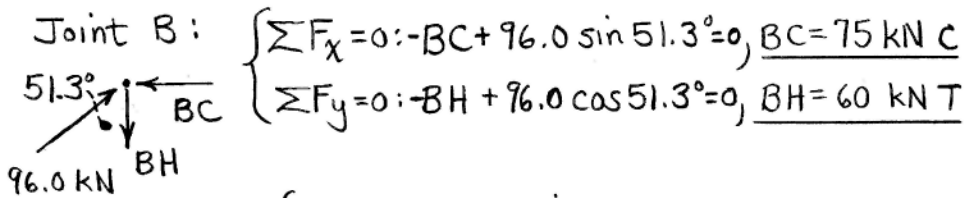
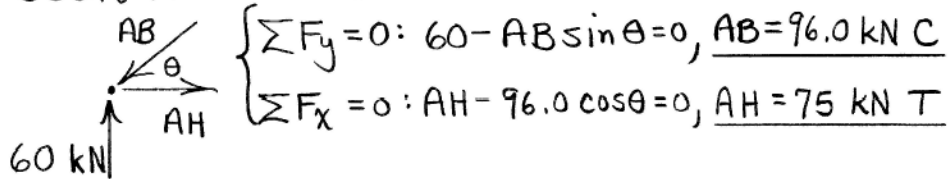
4/7 | As a whole:  $\sum F_x = 0 \Rightarrow A_x = 0$

(Dim. in m)  $\begin{matrix} y \\ | \\ \text{---} x \end{matrix}$   $A_y = E = 60 \text{ kN}$  by

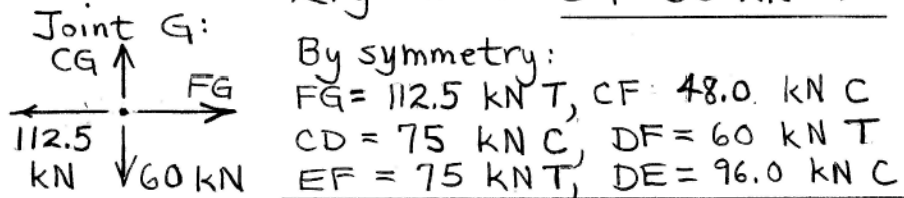


$\sum F_y = 0$  and symmetry.

Joint A:  $(\theta = \tan^{-1}(\frac{4}{5}) = 38.7^\circ)$

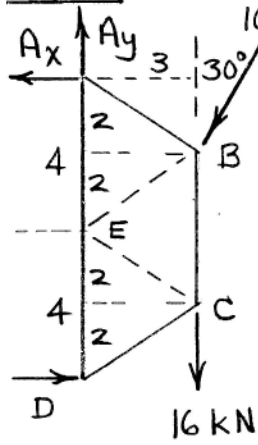


$\sum F_y = 0 \Rightarrow \underline{CG = 60 \text{ kN T}}$



4/8

Entire truss:



$$\uparrow \sum M_A = 0: -10 \cos 30^\circ (3) - 10 \sin 30^\circ (2) - 16(3) + D(8) = 0$$

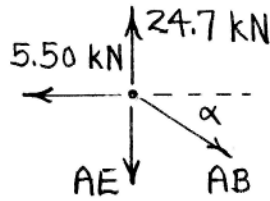
$$D = 10.50 \text{ kN}$$

$$\sum F_x = 0: -A_x - 10\left(\frac{1}{2}\right) + 10.50 = 0$$

$$A_x = 5.50 \text{ kN}$$

$$\sum F_y = 0: -10\frac{\sqrt{3}}{2} - 16 + A_y = 0, A_y = 24.7 \text{ kN}$$

Joint A:



$$(\alpha = \tan^{-1}\left(\frac{2}{3}\right) = 33.7^\circ)$$

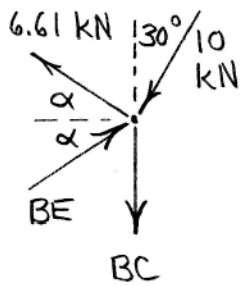
$$\sum F_x = 0: AB \cos 33.7^\circ - 5.50 = 0$$

$$AB = 6.61 \text{ kN T}$$

$$\sum F_y = 0: 24.7 - AE - 6.61 \sin 33.7^\circ = 0$$

$$AE = 21.0 \text{ kN T}$$

Joint B:



$$\sum F_x = 0: -6.61 \cos 33.7^\circ - 10 \sin 30^\circ + BE \cos 33.7^\circ = 0, BE = 12.62 \text{ kN C}$$

$$\sum F_y = 0: 6.61 \sin 33.7^\circ - 10 \cos 30^\circ + 12.62 \sin 33.7^\circ - BC = 0$$

$$BC = 2.00 \text{ kN T}$$

Joint C :  $\sum F_x = 0 : -CE \cos \alpha + CD \cos \alpha = 0$

$CE = CD$

$\sum F_y = 0 : 2 - 16 + (CE + CE) \sin 33.7^\circ = 0$

$CE = 12.62 \text{ kN T}$

$CD = 12.62 \text{ kN C}$

Joint D:

$\sum F_y = 0 : DE - 12.62 \sin 33.7^\circ = 0$

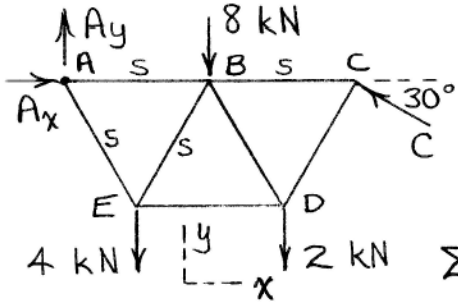
$DE = 7 \text{ kN T}$

Check on joint E :

$\sum F_x = 0 \quad \checkmark$

$\sum F_y = 21 - 7 - 2(12.62) \sin 33.7^\circ = 0 \quad \checkmark$

4/9 As a whole:  $\uparrow \sum M_A = 0: -8s - 4\frac{s}{2} - 2\frac{3s}{2}$



$+ C\frac{1}{2}(2s) = 0, C = 13 \text{ kN}$

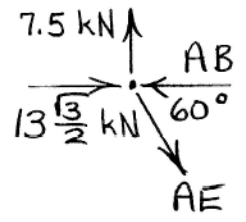
$\sum F_x = 0: A_x - 13\frac{\sqrt{3}}{2} = 0$

$A_x = 13\frac{\sqrt{3}}{2} \text{ kN}$

$\sum F_y = 0: A_y + 13(\frac{1}{2}) - 14 = 0$

$A_y = 7.5 \text{ kN}$

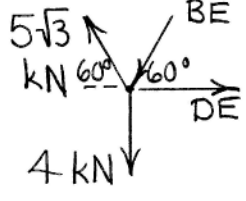
Joint A:



$\left\{ \begin{array}{l} \sum F_y = 0: 7.5 - AE\frac{\sqrt{3}}{2} = 0, AE = 5\sqrt{3} \text{ kN T} \\ \sum F_x = 0: 13\frac{\sqrt{3}}{2} - AB + 5\sqrt{3}(\frac{1}{2}) = 0 \end{array} \right.$

$AB = 9\sqrt{3} \text{ kN C}$

Joint E:

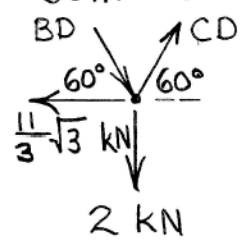


$\left\{ \begin{array}{l} \sum F_y = 0: 5\sqrt{3}\frac{\sqrt{3}}{2} - 4 - BE\frac{\sqrt{3}}{2} = 0 \\ \sum F_x = 0: -5\sqrt{3}(\frac{1}{2}) - \frac{7}{3}\sqrt{3}(\frac{1}{2}) + DE = 0 \end{array} \right.$

$BE = \frac{7}{3}\sqrt{3} \text{ kN C}$

$DE = \frac{11}{3}\sqrt{3} \text{ kN T}$

Joint D:



$\left\{ \begin{array}{l} \sum F_x = 0: BD(\frac{1}{2}) + CD(\frac{1}{2}) - \frac{11}{3}\sqrt{3} = 0 \\ \sum F_y = 0: -BD\frac{\sqrt{3}}{2} + CD\frac{\sqrt{3}}{2} - 2 = 0 \end{array} \right.$

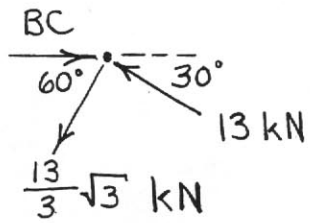
$\Rightarrow \left\{ \begin{array}{l} CD = \frac{13}{3}\sqrt{3} \text{ kN T} \\ BD = 3\sqrt{3} \text{ kN C} \end{array} \right.$



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Joint C:  $\sum F_x = 0: BC - \frac{13}{3}\sqrt{3} \left(\frac{1}{2}\right) - 13\frac{\sqrt{3}}{2} = 0$

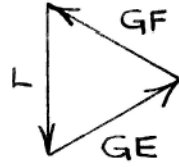
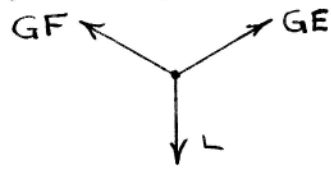
$BC = \frac{26}{3}\sqrt{3} \text{ kN C}$



(Joint B checks)

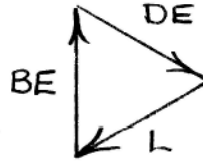
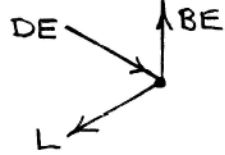
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Joint G:



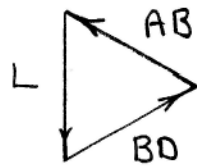
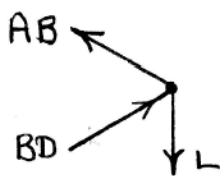
$$GE = GF = L C$$

Joint E:



$$\frac{BE = L T}{DE = L C}$$

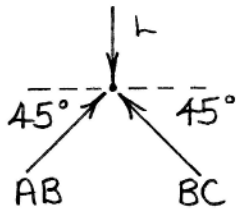
Joint B:



$$\frac{BD = L C}{AB = L T}$$

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Joint B :

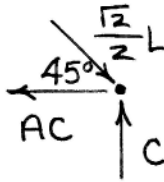


From  $\sum F_x = 0$ ,  $AB = BC$

$$\sum F_y = 0: 2AB \frac{\sqrt{2}}{2} - L = 0$$

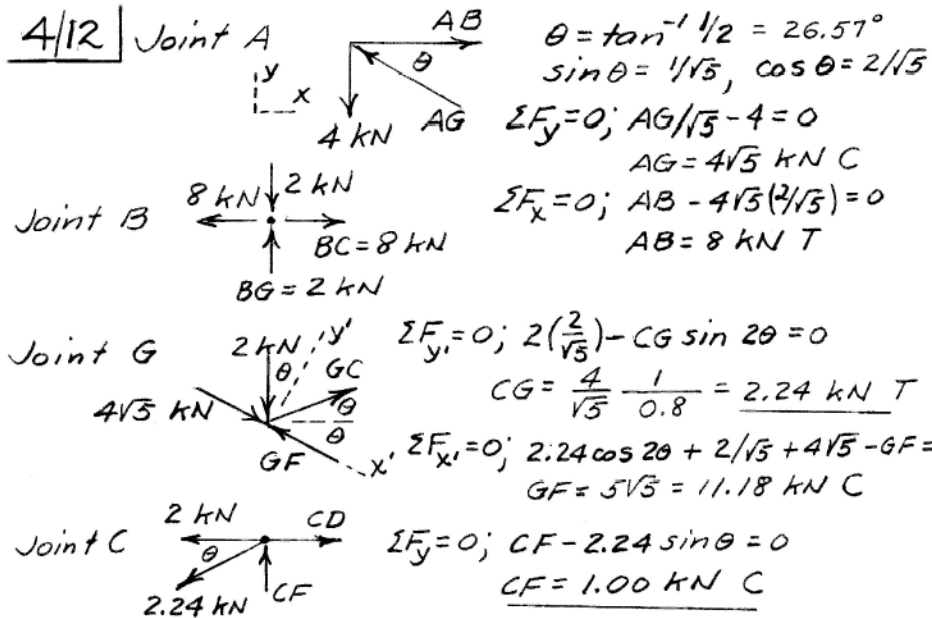
$$AB = \frac{\sqrt{2}}{2} L = BC$$

Joint C :



$$\sum F_x = 0: \frac{\sqrt{2}}{2} L \left( \frac{\sqrt{2}}{2} \right) - AC = 0$$

$$\underline{AC = \frac{L}{2} T}$$

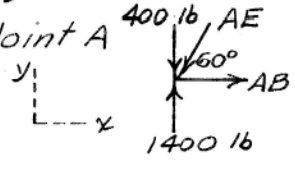




4/13 | Total weight of truss =  $7(400) = 2800$  lb

By symmetry, reactions at A & C are 1400 lb

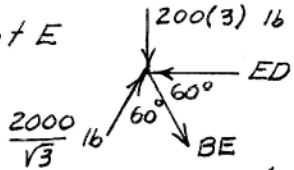
Joint A


$$\sum F_y = 0; AE \cos 30^\circ + 400 - 1400 = 0$$
$$AE = \frac{2000}{\sqrt{3}} \text{ lb C}$$
$$\sum F_x = 0; AB - \frac{2000}{\sqrt{3}} \cos 60^\circ = 0$$
$$AB = \frac{1000}{\sqrt{3}} \text{ lb T}$$

By symmetry  $BC = \frac{1000}{\sqrt{3}} \text{ lb T}$

$CD = \frac{2000}{\sqrt{3}} \text{ lb C}$

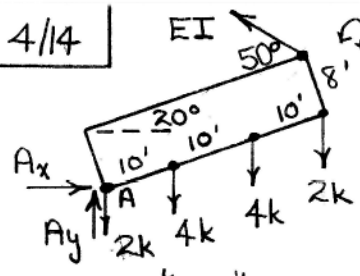
Joint E


$$\sum F_y = 0; BE \sin 60^\circ - \frac{2000}{\sqrt{3}} \sin 60^\circ + 600 = 0$$
$$BE = \frac{800}{\sqrt{3}} \text{ lb T}$$

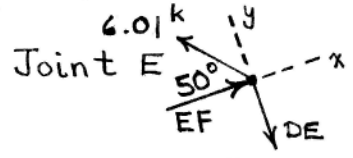
& by symmetry  $BD = \frac{800}{\sqrt{3}} \text{ lb T}$

$$\sum F_x = 0; ED - \frac{2000}{\sqrt{3}} \sin 30^\circ - \frac{800}{\sqrt{3}} \sin 30^\circ = 0$$
$$ED = \frac{1400}{\sqrt{3}} \text{ lb C}$$

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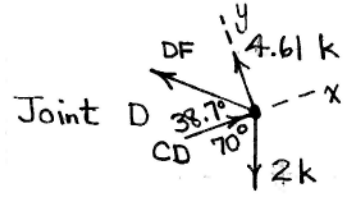


$$\begin{aligned} \sum M_A = 0 &: -4 \cos 20^\circ (10 + 20 + \frac{30}{2}) \\ &+ EI \cos 50^\circ (8) + EI \sin 50^\circ (30) \\ &= 0, \quad EI = 6.01 \text{ kips} \end{aligned}$$



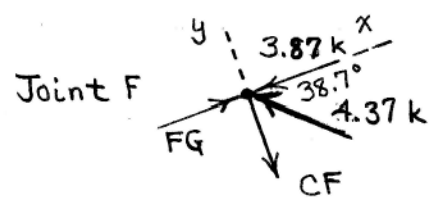
$$\begin{aligned} \sum F_x = 0 &: EF - 6.01 \cos 50^\circ = 0 \\ EF &= 3.87 \text{ kips C} \end{aligned}$$

$$\begin{aligned} \sum F_y = 0 &: -DE + 6.01 \sin 50^\circ = 0 \\ DE &= 4.61 \text{ kips T} \end{aligned}$$



$$\begin{aligned} \sum F_y = 0 &: 4.61 - 2 \sin 70^\circ + DF \sin 38.7^\circ \\ &= 0, \quad DF = -4.37 \text{ kips (C)} \end{aligned}$$

$$\begin{aligned} \sum F_x = 0 &: 4.37 \cos 38.7^\circ - 2 \cos 70^\circ \\ &+ CD = 0, \quad CD = -2.73 \text{ kips (T)} \end{aligned}$$



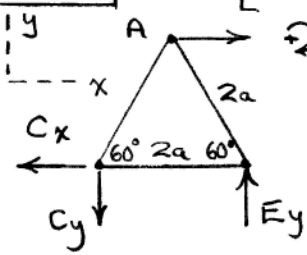
$$\begin{aligned} \sum F_x = 0 &: -3.87 - 4.37 \cos 38.7^\circ \\ &+ FG = 0 \end{aligned}$$

$$FG = 7.28 \text{ kips C}$$



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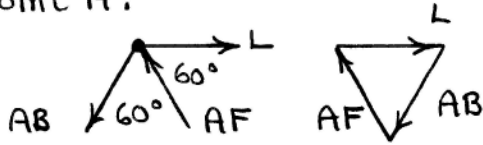
Truss as a whole

$$\sum M_C = 0: L(2a \sin 60^\circ) - E_y(2a) = 0$$

$$E_y = \frac{\sqrt{3}L}{2}$$

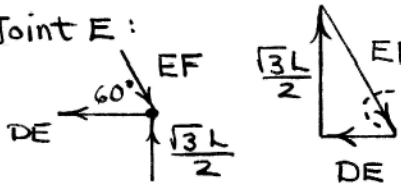
$$\sum F_x, \sum F_y = 0: C_x = L, C_y = \frac{\sqrt{3}L}{2}$$

Joint A:



$$\frac{AB = L \quad T}{AF = L \quad C}$$

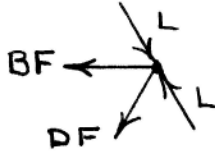
Joint E:



$$EF = \frac{\sqrt{3}L/2}{\cos 30^\circ} = L \quad C$$

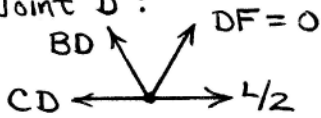
$$DE = EF \sin 30^\circ = \frac{L}{2} \quad T$$

Joint F:



$$\sum F = 0: \underline{BF = DF = 0}$$

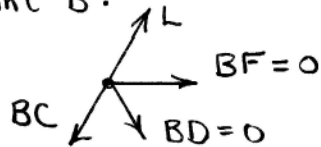
Joint D:



$$\sum F_y = 0: \underline{BD = 0}$$

$$\sum F_x = 0: \underline{CD = \frac{L}{2} \quad T}$$

Joint B:



$$\sum F = 0: \underline{BC = L \quad (T)}$$

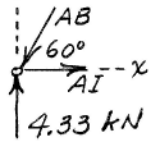
(Joint C checks)

4/16 Truss as a whole

$$\sum M_F = 0; 2(a + \frac{a}{2}) + 4(2a + \frac{a}{2}) - A(3a) = 0$$

$$A = 13/3 = 4.33 \text{ kN}$$

Joint A



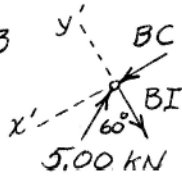
$$\sum F_y = 0; AB \sin 60^\circ - 4.33 = 0$$

$$AB = 5.00 \text{ kN C}$$

$$\sum F_x = 0; AI - 5 \cos 60^\circ = 0$$

$$AI = 2.50 \text{ kN T}$$

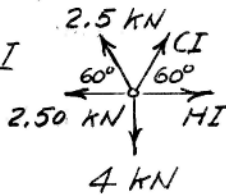
Joint B



$$\sum F_y = 0; 5.00 \cos 60^\circ - BI = 0$$

$$BI = 2.50 \text{ kN T}$$

Joint I



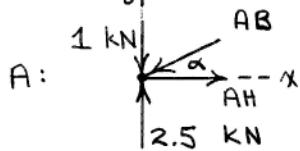
$$\sum F_y = 0; (CI + 2.5) \sin 60^\circ - 4 = 0$$

$$CI = 2.12 \text{ kN T}$$

$$\sum F_x = 0; HI + 2.12 \cos 60^\circ - 2.50 - 2.5 \cos 60^\circ = 0$$

$$HI = 2.69 \text{ kN T}$$

4/17

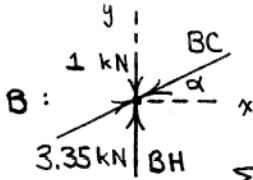
By symmetry,  $A = E = 2.5 \text{ kN}$ ;  $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6^\circ$ 

$$\sum F_y = 0: 2.5 - 1 - AB \sin \alpha = 0$$

$$\underline{AB = 3.35 \text{ kN C}}$$

$$\sum F_x = 0: -3.35 \cos \alpha + AH = 0$$

$$\underline{AH = 3 \text{ kN T}}$$

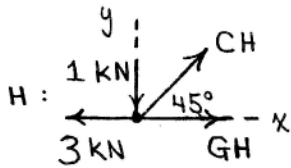


$$\sum F_x = 0: 3.35 \cos \alpha - BC \cos \alpha = 0$$

$$\underline{BC = 3.35 \text{ kN C}}$$

$$\sum F_y = 0: -1 + (3.35 - 3.35) \sin \alpha + BH = 0$$

$$\underline{BH = 1 \text{ kN C}}$$



$$\sum F_y = 0: -1 + CH \sin 45^\circ = 0$$

$$\underline{CH = 1.414 \text{ kN T}}$$

$$\sum F_x = 0: -3 + 1.41 \cos 45^\circ + GH = 0$$

$$\underline{GH = 2 \text{ kN T}}$$

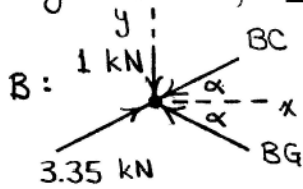
By inspection of joint G and  $\sum F_y = 0$ ,  $\underline{CG = 0}$ .

By symmetry,

$$\left\{ \begin{array}{l} DE = AB = 3.35 \text{ kN C} \\ CD = BC = 3.35 \text{ kN C} \\ EF = AH = 3 \text{ kN T} \\ DF = BH = 1 \text{ kN C} \\ CF = CH = 1.414 \text{ kN T} \\ \underline{FG = GH = 2 \text{ kN T}} \end{array} \right.$$

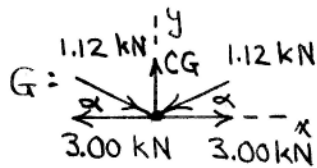
4/18 By symmetry,  $A = E = 2.5 \text{ kN}$ ;  $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6^\circ$

Joint A analysis same as Prob. 4/16:  $\left\{ \begin{array}{l} AB = 3.35 \text{ kN C} \\ AH = 3.00 \text{ kN T} \end{array} \right.$   
 By inspection,  $BH = 0$  and  $GH = AH$ .



$$\begin{aligned} \sum F_y = 0: & -1 + 3.35 \sin \alpha + BG \sin \alpha - BC \sin \alpha = 0 \\ \sum F_x = 0: & 3.35 \cos \alpha - BC \cos \alpha - BG \cos \alpha = 0 \end{aligned}$$

$\Rightarrow BC = 2.24 \text{ kN C}, \quad BG = 1.118 \text{ kN C}$



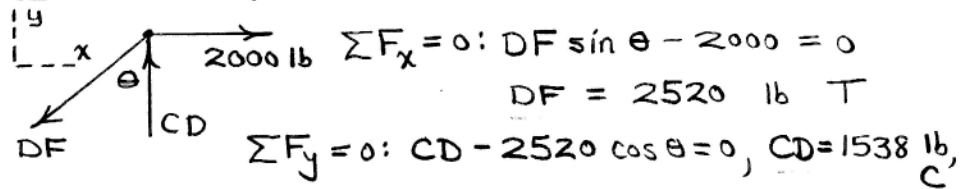
$$\begin{aligned} \sum F_y = 0: & CG - 2(1.12) \sin \alpha = 0 \\ & CG = 1.00 \text{ kN T} \end{aligned}$$

By symmetry,

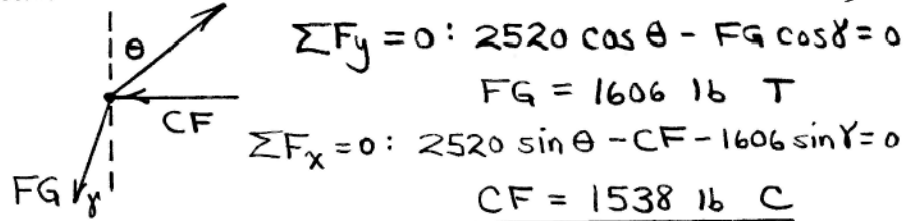
$$\left\{ \begin{array}{l} DE = AB = 3.35 \text{ kN C} \\ CD = BC = 2.24 \text{ kN C} \\ EF = AH = 3.00 \text{ kN T} \\ DF = BH = 0 \\ FG = GH = 3.00 \text{ kN T} \\ DG = BG = 1.118 \text{ kN C} \end{array} \right.$$

4/19 | Joint E :  $DE = EF = 0$

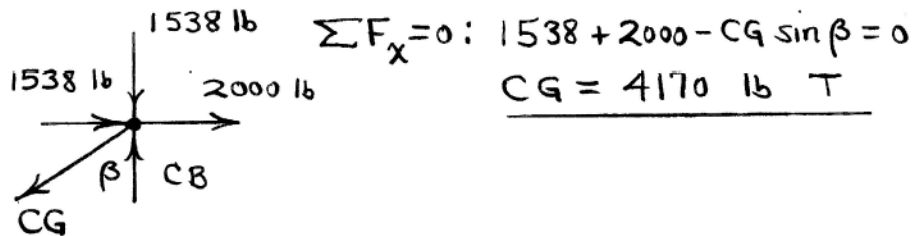
Joint D :  $(\theta = \tan^{-1} \frac{13}{10} = 52.4^\circ)$



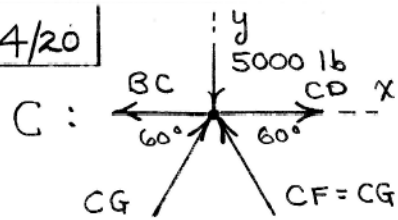
Joint F :  $2520 \text{ lb}$   $(\gamma = \tan^{-1} \frac{3}{10} = 16.7^\circ)$



Joint C :  $(\beta = \tan^{-1} \frac{16}{10} = 58.0^\circ)$

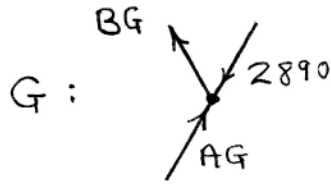


4/20



Arguing symmetry,

$$\sum F_y = 0: 2CG \sin 60^\circ - 5000 = 0, \quad \underline{CG = 2890 \text{ lb C}}$$



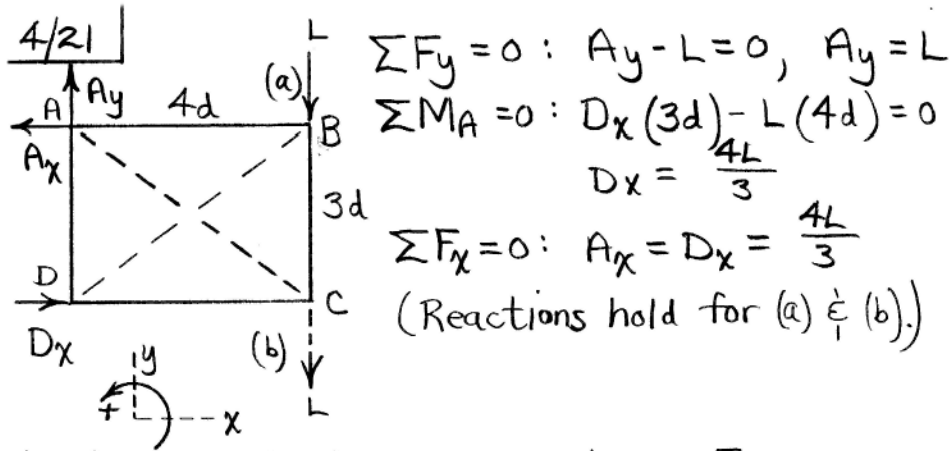
By inspection,

$$\underline{AG = 2890 \text{ lb C}}$$
$$\underline{BG = 0}$$

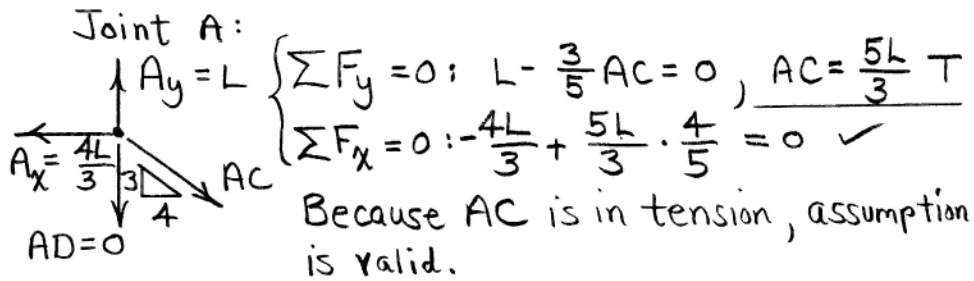
From joint B : AB = 0, BC = 0.

(Right truss symmetric to left one.)

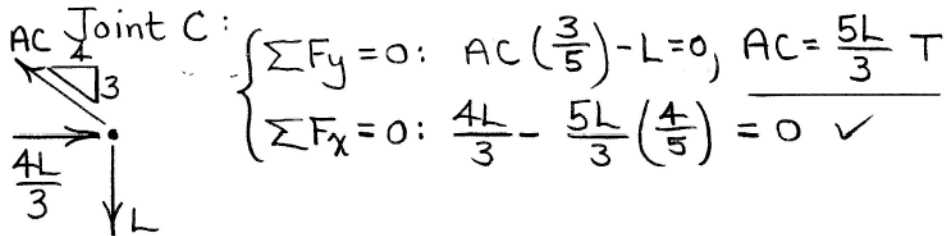


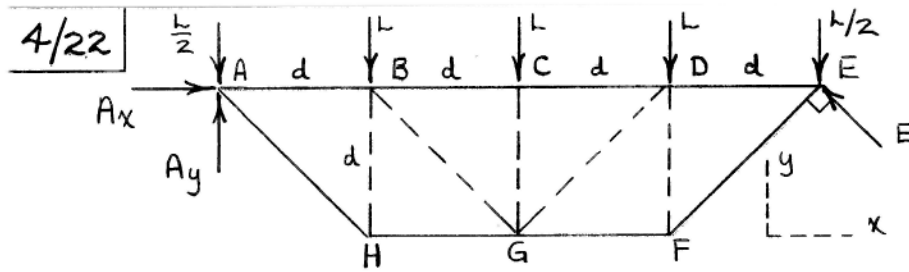


(a) Assume that BD goes slack. From an inspection of joint B,  $AB=0$  and  $BC=L$ . Similarly, from joint D,  $AD=0$  and  $CD=\frac{4L}{3}C$ .



(b) Assume that BD goes slack. From joint B,  $AB=BC=0$ . From joint D,  $AD=0$  &  $CD=\frac{4L}{3}C$ .





Entire truss:

$$\sqrt{\uparrow} \sum M_A = 0: -Ld - L(2d) - L(3d) - \frac{L}{2}(4d) + E \frac{\sqrt{2}}{2}(4d) = 0$$

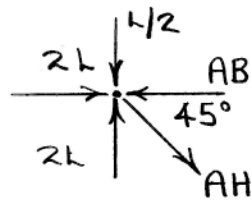
$$E = 2\sqrt{2}L$$

$$\sum F_x = 0: A_x - 2\sqrt{2}L \frac{\sqrt{2}}{2} = 0, \quad A_x = 2L$$

$$\sum F_y = 0: A_y - 4L + 2\sqrt{2}L \frac{\sqrt{2}}{2} = 0, \quad A_y = 2L$$

By inspection of joint C, CG = LC

Joint A:



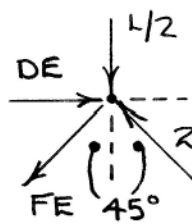
$$\sum F_y = 0: 2L - \frac{L}{2} - AH \frac{\sqrt{2}}{2} = 0$$

$$AH = \frac{3\sqrt{2}}{2}L \quad T$$

$$\sum F_x = 0: 2L + \frac{3\sqrt{2}}{2}L \frac{\sqrt{2}}{2} - AB = 0$$

$$AB = \frac{7}{2}L \quad C$$

Joint E:



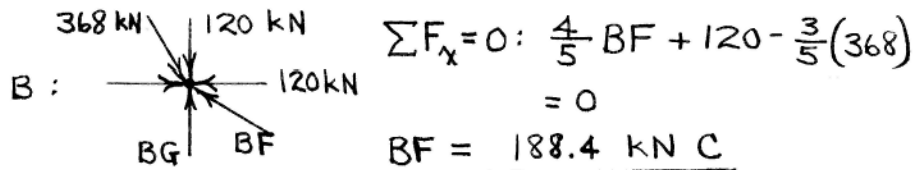
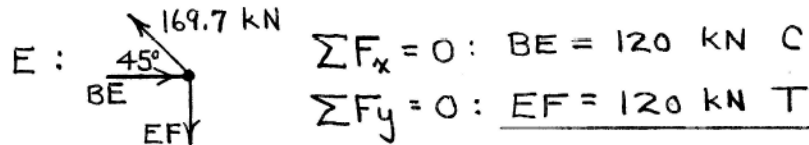
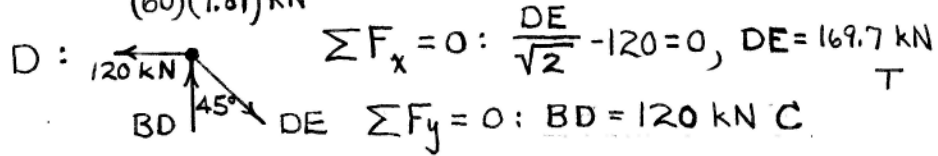
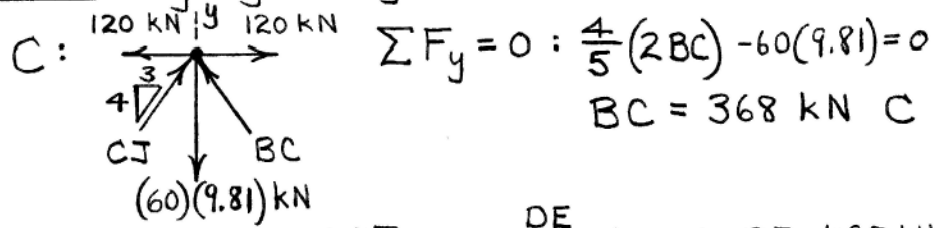
$$\sum F_y = 0: -\frac{L}{2} + 2\sqrt{2}L \frac{\sqrt{2}}{2} - FE \frac{\sqrt{2}}{2} = 0$$

$$FE = \frac{3\sqrt{2}}{2}L \quad T$$

$$\sum F_x = 0: DE - \frac{3\sqrt{2}}{2}L \frac{\sqrt{2}}{2} - 2\sqrt{2}L \frac{\sqrt{2}}{2} = 0$$

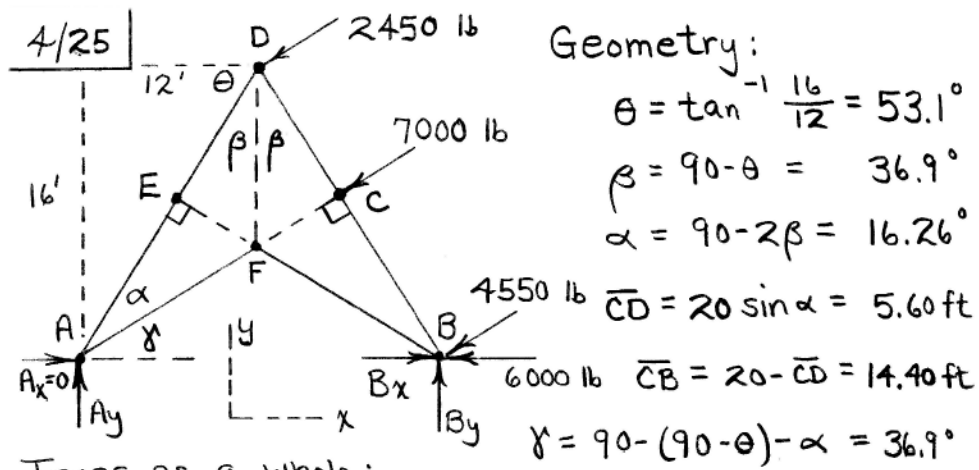
$$DE = \frac{7L}{2} \quad C$$

4/23 | By symmetry,  $AJ = AB$ ,  $CH = CD$ ,  $BC = JC$ .



4/24 |  $m = \text{no. of two-force members}$   
 $j = \text{no. of joints}$

- (a)  $[m+3=13] > [2j=12]$  so redundant members.  
Remove one member connecting B, C, D, and E.
- (b)  $[m+3=12] = [2j=12]$  so sufficient no. of members, but redundancy in external supports. Place A or F on roller.
- (c)  $[m+3=9] > [2j=8]$  so redundant members.  
Supports are also redundant. Remove AE or BE. Supports are then OK.
- (d)  $[m+3=12] = [2j=12]$  so sufficient no. of members, but redundancy in external supports. Place B on roller or remove member CD.



Truss as a whole:

$$\odot \sum M_B = 0: 7000(14.4) + 2450(20) - 24A_y = 0$$

$$A_y = 6240 \text{ lb}$$

$$\sum F_y = 0: B_y + 6240 - (2450 + 7000 + 4550) \sin 36.9^\circ = 0, \quad B_y = 2160 \text{ lb}$$

$$\sum F_x = 0: B_x - (2450 + 7000 + 4550) \cos 36.9^\circ - 6000 = 0, \quad B_x = 17,200 \text{ lb}$$

Joint A:

$$\sum F_x = 0 : -AE \cos 53.1^\circ + AF \cos 36.9^\circ = 0$$

$$\sum F_y = 0 : 6240 - AE \sin 53.1^\circ + AF \sin 36.9^\circ = 0$$

$$\Rightarrow AF = 13,380 \text{ lb T}, AE = 17,830 \text{ lb C}$$

From joint E,  $ED = AE = 17,830 \text{ lb C}$

Joint D:

$$\sum F_x = 0 : 17,830 \sin 36.9^\circ - CD \sin 36.9^\circ - 2450 \cos 36.9^\circ = 0, CD = 14,570 \text{ lb C}$$

$$\sum F_y = 0 : (14,570 + 17,830) \cos 36.9^\circ - FD - 2450 \sin 36.9^\circ = 0, FD = 24,500 \text{ lb T}$$

Joint F:

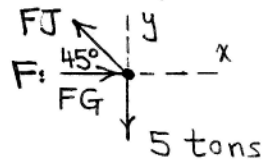
$$\sum F_x = 0 : BF \cos 36.9^\circ - (13,380 + 7000) \cos 36.9^\circ = 0, BF = 20,400 \text{ lb T}$$

(Joint B checks.)

The maximum force occurs in member FD:

$$\underline{FD = 24,500 \text{ lb T}}$$

4/26 | Structure is statically indeterminate externally; member AE in main vertical tower is indeterminate.

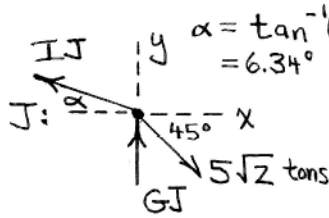


$$\sum F_y = 0: FJ \frac{\sqrt{2}}{2} - 5 = 0$$

$$FJ = 5\sqrt{2} \text{ tons T}$$

$$\sum F_x = 0: FG - 5\sqrt{2} \left( \frac{\sqrt{2}}{2} \right) = 0$$

$$FG = 5 \text{ tons C}$$

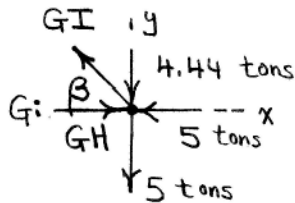


$$\alpha = \tan^{-1} \left( \frac{10}{90} \right) = 6.34^\circ$$

$$\sum F_x = 0: -IJ \cos \alpha + 5\sqrt{2} \left( \frac{\sqrt{2}}{2} \right) = 0$$

$$IJ = 5.03 \text{ tons T}$$

$$\sum F_y = 0: 5.03 \sin \alpha - 5\sqrt{2} \left( \frac{\sqrt{2}}{2} \right) + GJ = 0, \quad GJ = 4.44 \text{ tons C}$$



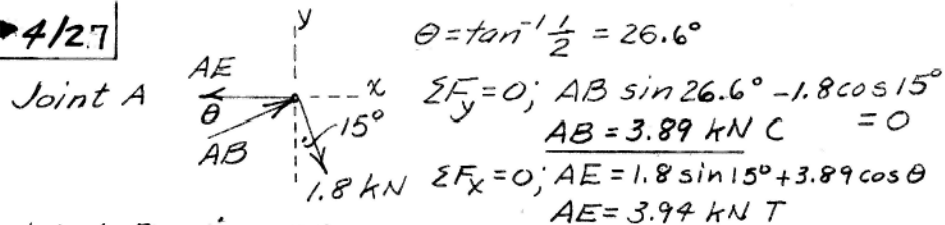
$$\beta = \tan^{-1} \left( \frac{11.67}{15} \right) = 37.9^\circ$$

$$\sum F_y = 0: -5 - 4.44 + GI \sin \beta = 0$$

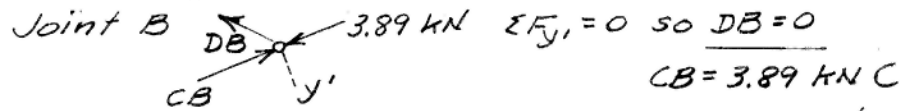
$$GI = 15.38 \text{ tons T}$$

► 4/27

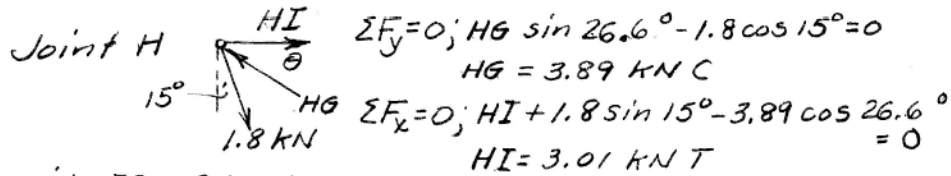
$$\theta = \tan^{-1} \frac{1}{2} = 26.6^\circ$$



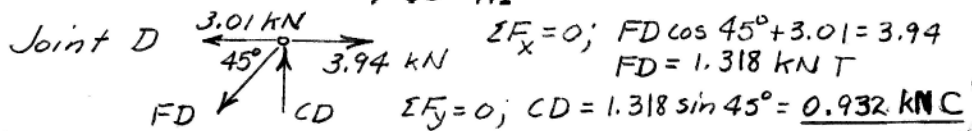
Joint E gives  $EB = 0$



without diagonals, FD would lengthen & CJ shorten, so FD is tension & CJ = 0



with  $IG = GJ = JC = 0, JD = HI$



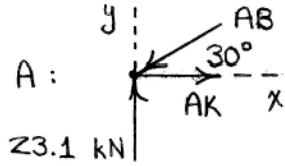


► 4/28 | For entire truss,

$$\sum M_I = 0: 30A - 0.866 [20(5+10+15) + 10(20)] = 0$$

$$A = 23.1 \text{ kN}$$

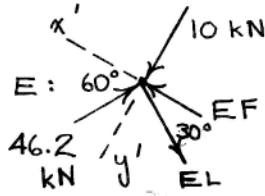
Forces in BP, PC, DN, CN, CO, ON, NE, EM (in this order) are seen to be zero.



$$\sum F_y = 0: -0.5 AB + 23.1 = 0$$

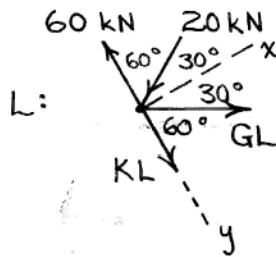
$$AB = 46.2 \text{ kN C}$$

$$DE = AB$$



$$\sum F_{y'} = 0: 0.5 EL + 10 - 0.866(46.2) = 0, \quad EL = 60 \text{ kN T}$$

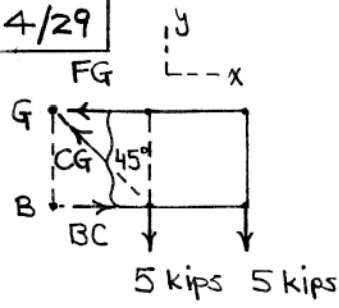
$$\sum F_{x'} = 0: EF - 60(0.866) - 46.2(0.5) = 0, \quad \underline{EF = 75.1 \text{ kN C}}$$



$$\sum F_x = 0: -20 \cos 30^\circ + GL \cos 20^\circ = 0, \quad \underline{GL = 20 \text{ kN T}}$$

$$\sum F_y = 0: KL - 60 + 20(0.5) + 20(0.5) = 0, \quad \underline{KL = 40 \text{ kN T}}$$

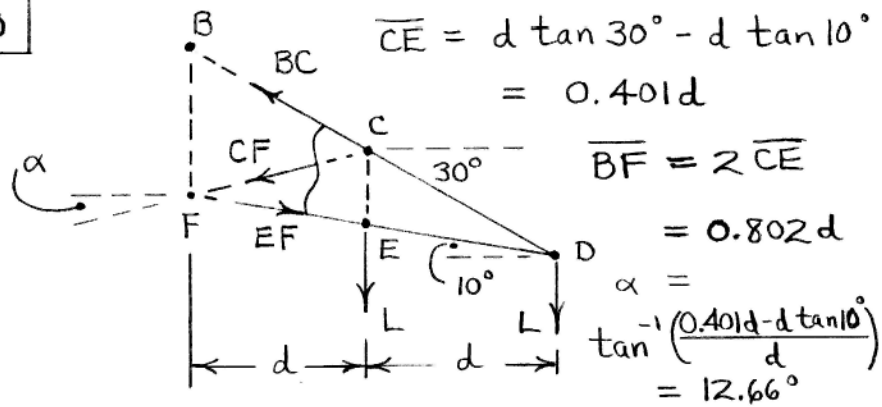
4/29



$$\sum F_y = 0: CG \sin 45^\circ - 5 - 5 = 0$$

$$CG = 14.14 \text{ kips T}$$

4/30



$$\uparrow \sum M_C = 0: -Ld + EF \cos 10^\circ (0.401d) = 0$$

$$\underline{EF = 2.53L \quad C}$$

$$\uparrow \sum M_F = 0: -Ld - L(2d) + BC \cos 30^\circ (0.802d) = 0$$

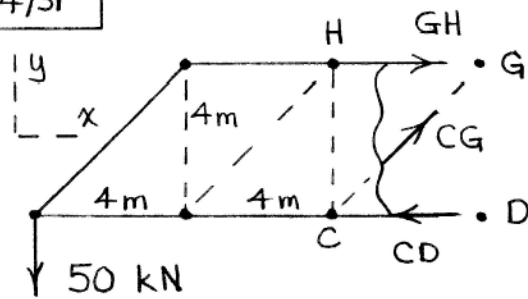
$$\underline{BC = 4.32L \quad T}$$

$$\uparrow \sum M_D = 0: Ld + CF \cos 12.66^\circ (d \tan 30^\circ)$$

$$+ CF \sin 12.66^\circ (d) = 0 \quad CF = -1.278L$$

$$\text{or } \underline{CF = 1.278L \quad C}$$

4/31

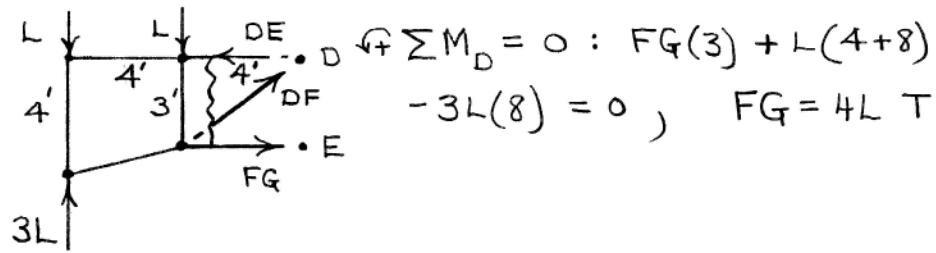


$$\sum F_y = 0 : CG \sin 45^\circ - 50 = 0, \quad \underline{CG = 70.7 \text{ kN T}}$$

$$\sum M_c = 0 : GH(4) - 50(8) = 0, \quad \underline{GH = 100 \text{ kN T}}$$

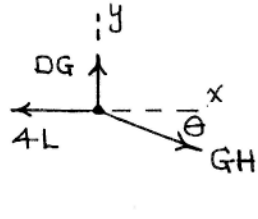
All members except EF are statically determinate, so above solution is unaffected by the redundant support.

4/32 | From entire truss,  $A = B = 3L$ .



Joint G:

$$\theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.04^\circ$$



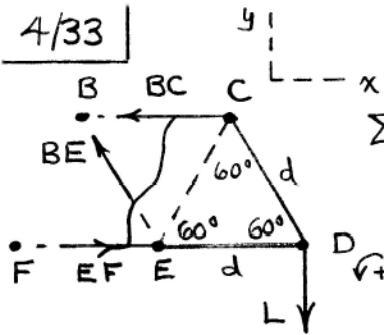
$$\sum F_x = 0 : -4L + GH \cos \theta = 0$$

$$GH = 4.12L \text{ T}$$

$$\sum F_y = 0 : DG - 4.12 \sin \theta = 0$$

$$\underline{DG = 1.000L \text{ T}}$$

4/33

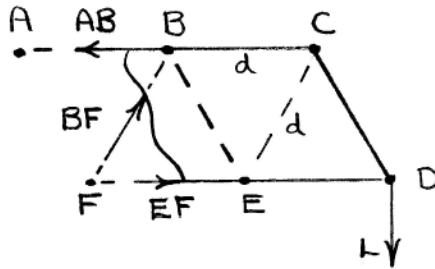


$$\sum F_y = 0: BE \sin 60^\circ - L = 0$$

$$BE = \frac{2L}{\sqrt{3}} T$$

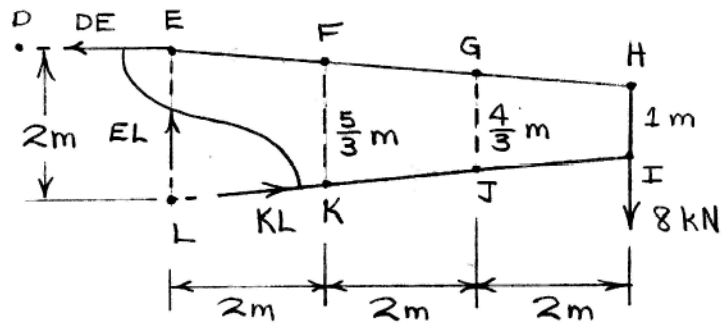
$$\sum M_E = 0: BC(d \cos 30^\circ) - Ld = 0$$

$$BC = \frac{2L}{\sqrt{3}} T$$



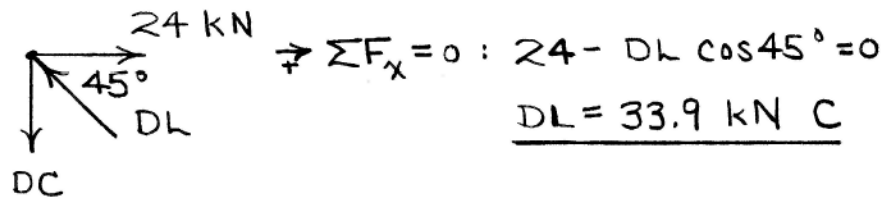
$$\sum F_y = 0: BF \sin 60^\circ - L = 0, \quad BF = \frac{2L}{\sqrt{3}} C$$

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$$\sum M_L = 0: DE(2) - 8(6) = 0, \quad \underline{DE = 24 \text{ kN T}}$$

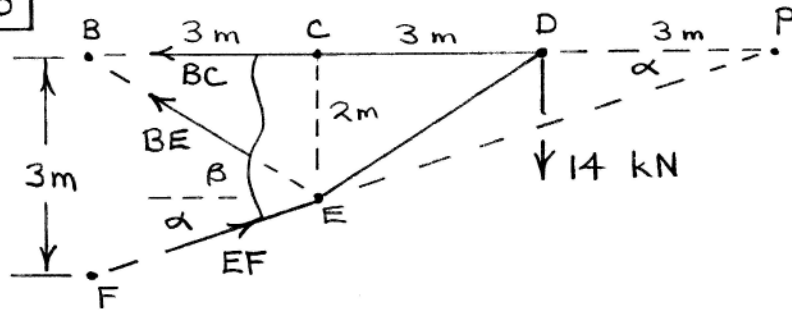
Joint D:



$$\sum F_x = 0: 24 - DL \cos 45^\circ = 0$$

$$\underline{DL = 33.9 \text{ kN C}}$$

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$$\alpha = \tan^{-1}\left(\frac{2}{6}\right) = 18.43^\circ, \quad \beta = \tan^{-1}\frac{2}{3} = 33.7^\circ$$

$$\curvearrowright \sum M_E = 0: BC(2) - 14(3) = 0, \quad \underline{BC = 21 \text{ kN T}}$$

$$\curvearrowright \sum M_P = 0: -BE \sin \beta (9) + 14(3) = 0, \quad \underline{BE = 8.41 \text{ kN T}}$$

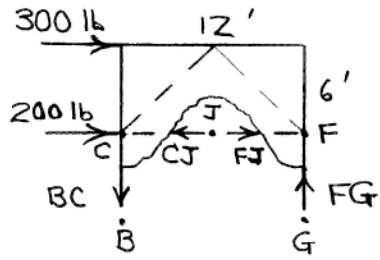
$$\curvearrowright \sum M_B = 0: EF \cos \alpha (3) - 14(6) = 0, \quad \underline{EF = 29.5 \text{ kN C}}$$



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$$\sum M_C = 0: -300(6) + FG(12) = 0$$

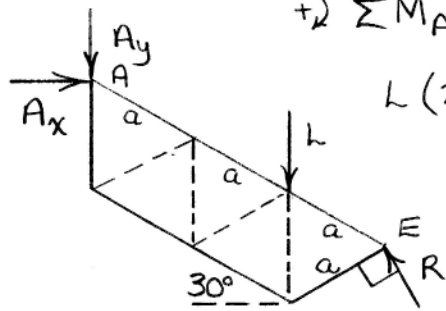
$$\underline{FG = 150 \text{ lb C}}$$



$$\sum M_F = 0: -300(6) + BC(12) = 0$$

$$\underline{BC = 150 \text{ lb T}}$$

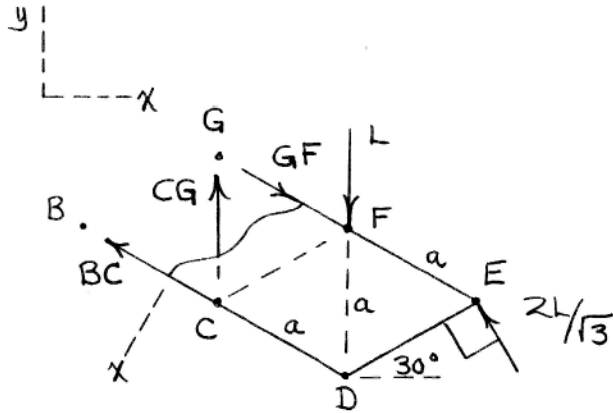
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$$\sum M_A = 0:$$

$$L(2a \cos 30^\circ) - R(3a \sin 30^\circ) = 0$$

$$R = \frac{2L}{\sqrt{3}}$$



$$\sum M_G = 0: L a \cos 30^\circ + BC a \cos 30^\circ - \frac{2L}{\sqrt{3}} 2a \sin 30^\circ = 0$$

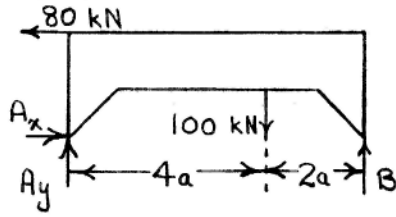
$$BC = \frac{L}{3} \text{ T}$$

$$\sum F_x = 0: CG \cos 30^\circ - L \cos 30^\circ + \frac{2L}{\sqrt{3}} \sin 30^\circ = 0$$

$$CG = \frac{L}{3} \text{ T}$$

$$4/38 \quad \sum M_A = 0: B(6a) - 100(4a) + 80(2a) = 0$$

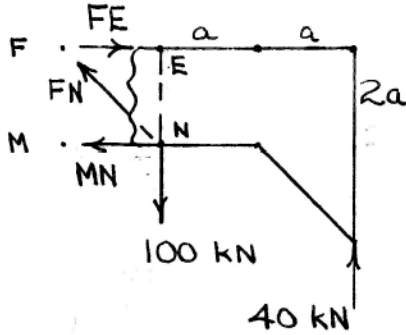
$$B = 40 \text{ kN}$$



For section,

$$\sum F_y = 0: \frac{FN}{\sqrt{2}} + 40 - 100 = 0$$

$$FN = 84.8 \text{ kN T}$$



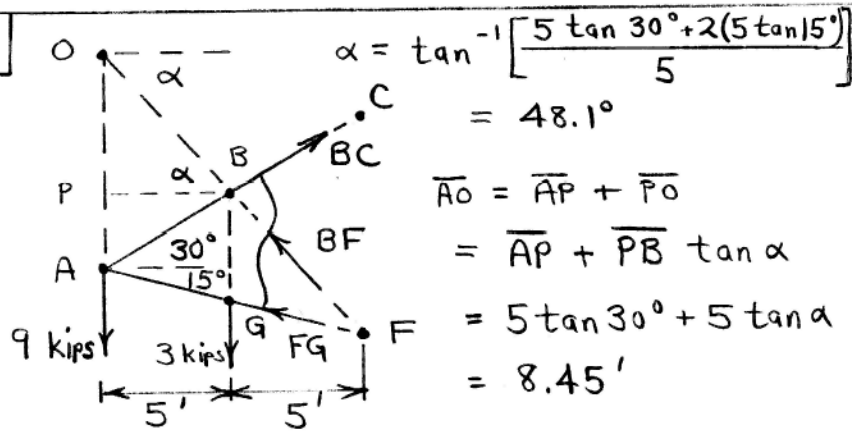
$$\sum M_E = 0: 40(2a) - 84.8 \frac{a}{\sqrt{2}}$$

$$-MN(a) = 0, \quad MN = 20 \text{ kN T}$$

For section through GF & LM,  $\sum F_y = 0$  gives  $GM = 84.8 \text{ kN T}$ .

GF & LM,  $\sum F_y = 0$  gives

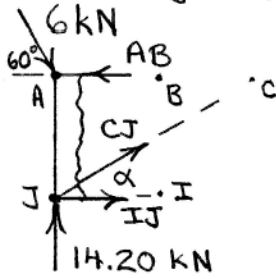
4/39



$$\sum M_A = 0: (BF \cos \alpha) \overline{AO} - 3(5) = 0$$

$$\underline{BF = 2.66 \text{ kips C}}$$

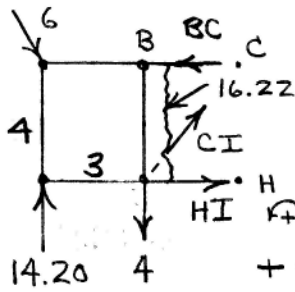
4/40 | From truss as a whole and  $\sum M_F = 0$ ,  
 $J = 14.20 \text{ kN}$ .



$$\sum F_y = 0: 14.20 - 6 \sin 60^\circ + CJ \sin \alpha = 0$$

where  $\alpha = \tan^{-1}\left(\frac{4}{6}\right) = 33.7^\circ$

$$\therefore \underline{CJ = -16.22 \text{ kN C}}$$



$$\sum F_y = 0: -6 \sin 60^\circ + 14.20 - 4$$

$$-16.22 \sin \alpha + CI \frac{4}{5} = 0$$

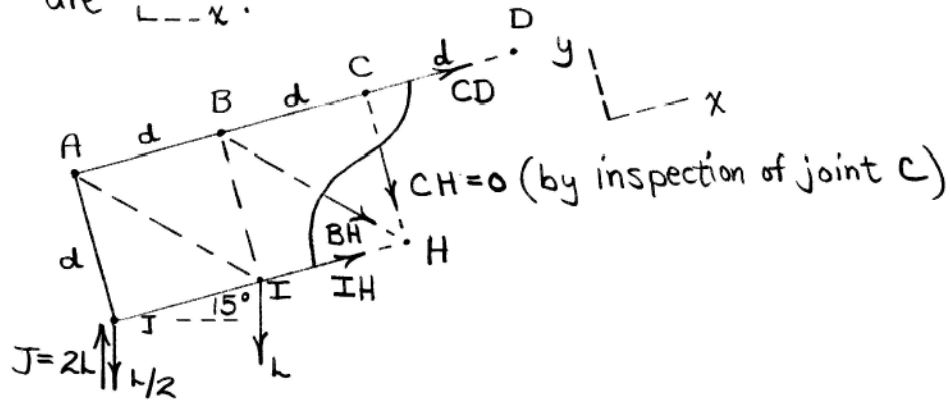
$$\underline{CI = 5.00 \text{ kN T}}$$

$$\sum M_c = 0: (6 \sin 60^\circ) 6 - (14.20) 6 + 4(3) + HI(4) = 0, \underline{HI = 10.50 \text{ kN T}}$$

$$\sum F_x = 0: 6 \cos 60^\circ - 16.22 \cos \alpha + 5\left(\frac{3}{5}\right) + 10.5 - BC = 0,$$

$$\underline{BC = 3.00 \text{ kN C}}$$

4/41 | From the truss as a whole,  $F_x = 0$  and  $J_y = F_y = 2L$  ( $\uparrow$ ), where the coordinates are  $y' \perp x$ .

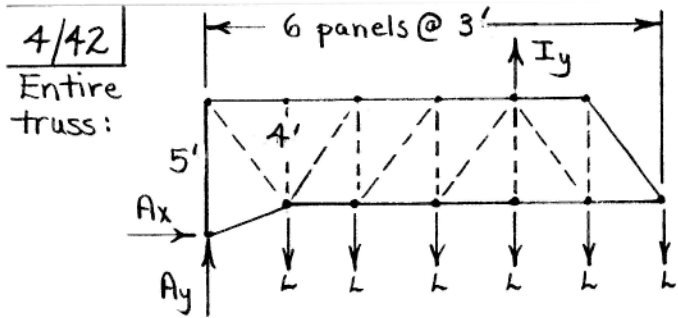


$$\sum F_y = 0: -BH \frac{\sqrt{2}}{2} + (2L - \frac{L}{2} - L) \cos 15^\circ = 0$$

$$\underline{BH = 0.683L \text{ T}}$$

$$\begin{aligned} \sqrt{+} \sum M_H = 0: & \left(\frac{L}{2} - 2L\right)(2d \cos 15^\circ) + L(d \cos 15^\circ) \\ & - CD(d) = 0, \quad CD = -1.932L \end{aligned}$$

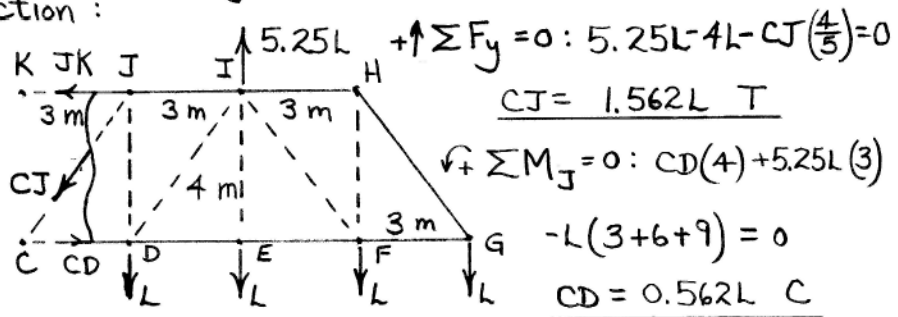
$$\text{So } \underline{CD = 1.932L \text{ C}}$$



$$\sum M_A = 0: I_y (12) - L(3+6+9+12+15+18) = 0$$

$$I_y = 5.25L$$

Section:

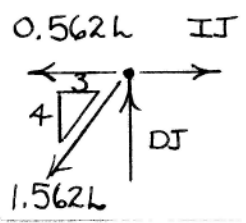


From  $\sum F_x = 0$ ,  $JK = 0.562L \text{ T}$ .

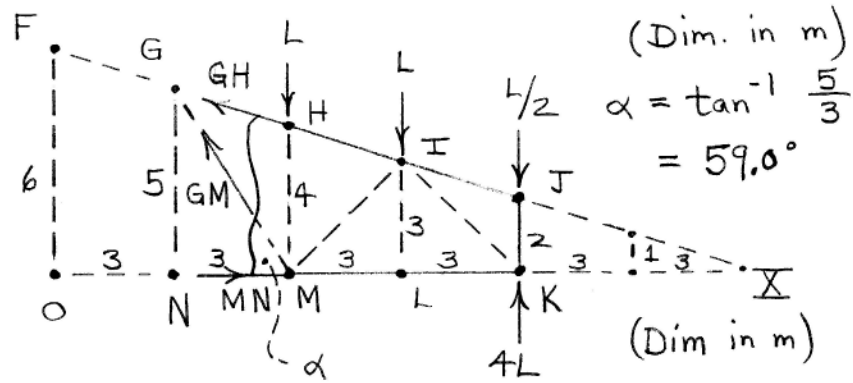
Joint J:

$$\sum F_y = 0: DJ - 1.562L\left(\frac{4}{5}\right) = 0$$

$$DJ = 1.250L \text{ C}$$



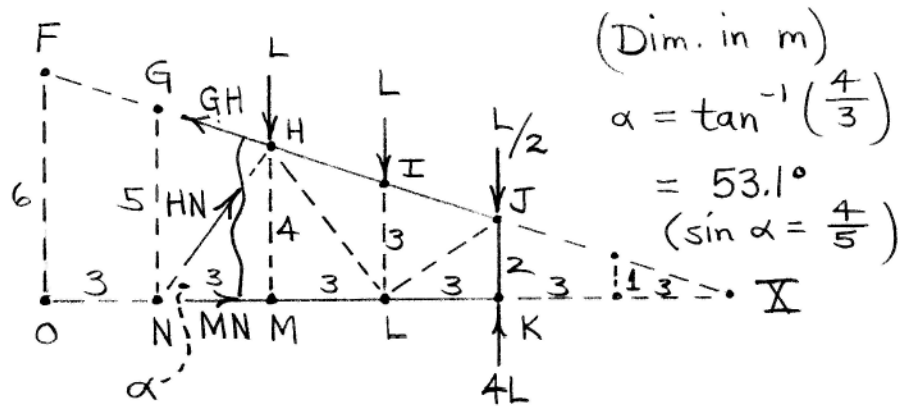
4/43 | From truss as a whole, the reactions at A and K are  $4L$  (up).



$$\begin{aligned} \sum M_X = 0: & \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12) \\ & - GM \sin(59.0^\circ)(12) = 0 \\ & \underline{GM = 0} \end{aligned}$$

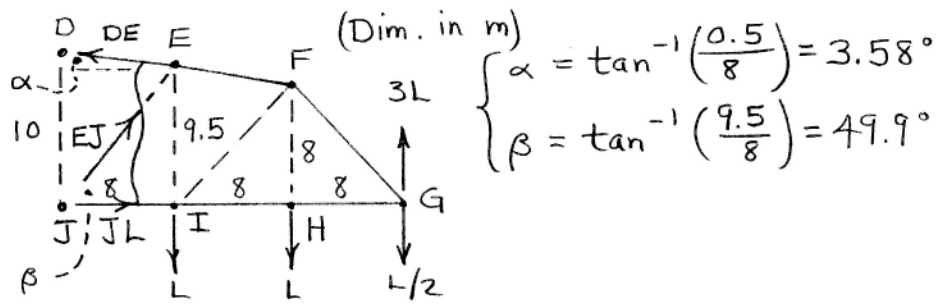


4/44 | From the truss as a whole, the external reactions at A and K are  $4L$  (up)



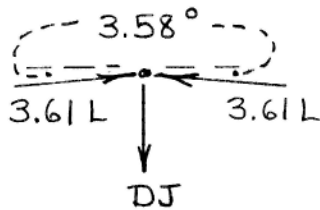
$$\begin{aligned} \sum M_X = 0 : & \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12) \\ & - HN\left(\frac{4}{5}\right)(15) = 0, \quad \underline{HN = 0} \end{aligned}$$

4/45 | From the truss as a whole, the external reactions at A and G are  $3L$  (up)



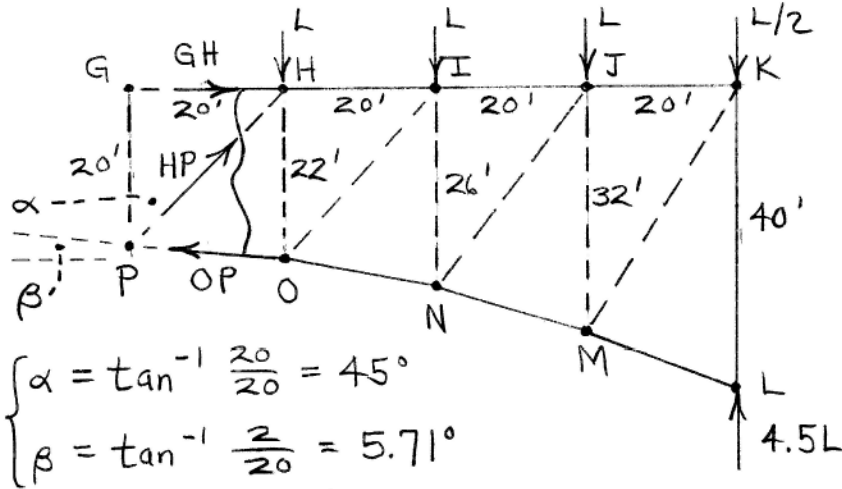
$$\begin{aligned} \curvearrowleft \sum M_J = 0: & -L(8) - L(16) - \frac{L}{2}(24) + 3L(24) \\ & + DE \cos 3.58^\circ (10) = 0, \quad DE = -3.61L \\ \uparrow \sum F = 0: & -3.61L \sin 3.58^\circ - \frac{5}{2}L + 3L + EJ \sin 49.9^\circ = 0 \\ & EJ = -0.360L \text{ or } \underline{EJ = 0.360L T} \end{aligned}$$

Joint D (using symmetry):



$$\begin{aligned} \uparrow \sum F = 0: & 2(3.61L \sin 3.58^\circ) - DJ = 0 \\ & \underline{DJ = 0.45L T} \end{aligned}$$

4/46 | From the truss as a whole, the external reactions at A and L are  $4.5L$  (up).



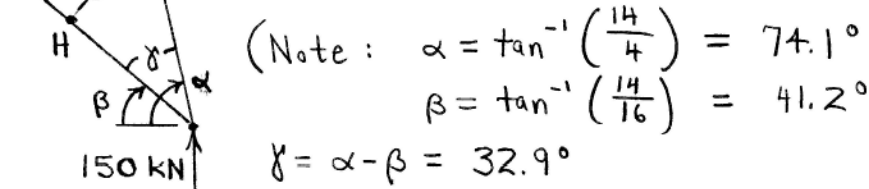
$$\sum M_H = 0: -L(20) - L(40) - \frac{L}{2}(60) + 4.5L(60) - OP \cos 5.71^\circ (22') = 0, \quad OP = 8.22L \text{ T}$$

$$\sum F = 0: -3.5L + 4.5L + 8.22L \sin 5.71^\circ + HP \sin 45^\circ = 0, \quad HP = -2.57L$$

or  $HP = 2.57L \text{ T}$

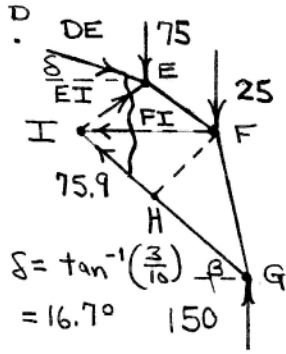
4/47 | By symmetry,  $A = G = 150 \text{ kN}$

$\sum M_F = 0: 150(4) + HI(7.902) = 0$   
 $HI = -75.9 \text{ kN T}$



(Note:  $\alpha = \tan^{-1}\left(\frac{14}{4}\right) = 74.1^\circ$   
 $\beta = \tan^{-1}\left(\frac{14}{16}\right) = 41.2^\circ$   
 $\gamma = \alpha - \beta = 32.9^\circ$

Then  $d_{\perp} = FG \sin \gamma = \sqrt{14^2 + 4^2} \sin \gamma = 7.902 \text{ m}$

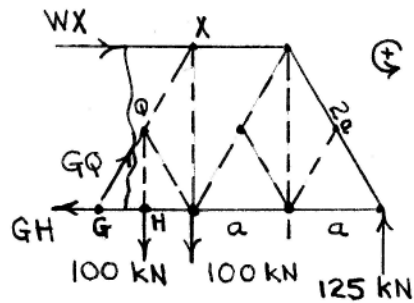


$\sum M_E = 0: -25(6) + 150(10) - FI(4) - (75.9 \sin \beta)(6) - (75.9 \cos \beta)(4) = 0$   
 $FI = 205 \text{ kN T}$

$\sum M_I = 0: -75(6) - 25(12) + 150(16) - (DE \cos \delta)(4) - (DE \sin \delta)(6) = 0$   
 $DE = 297 \text{ kN C}$

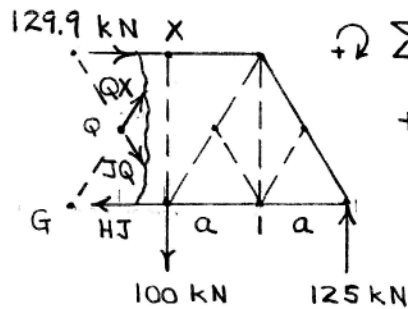
$\sum F_y = 0: -75 - 25 + 150 - 297 \sin \delta + 75.9 \sin \beta + EI \frac{4}{\sqrt{52}} = 0$   
 $EI = -26.4 \text{ kN T}$

4/48 | From truss as a whole,  $\sum M_A = 0$  gives  
 $N = 125 \text{ kN}$ .



$$\sum M_G = 0 : 125(3a) - 100\left(\frac{3a}{2}\right) - WX\left(2a \frac{\sqrt{3}}{2}\right) = 0$$

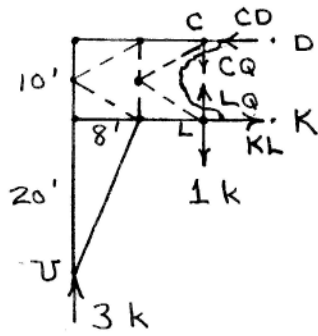
$$WX = 129.9 \text{ kN C}$$



$$\sum M_G = 0 : 100(a) + 129.9(a\sqrt{3}) + JQ\left(a \frac{\sqrt{3}}{2}\right) - 125(3a) = 0$$

$$\underline{JQ = 57.7 \text{ kN C}}$$

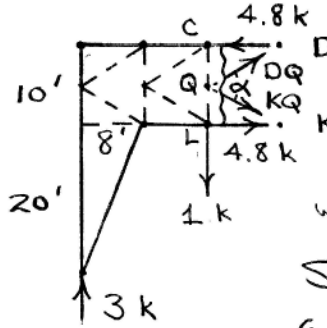
► 4/49 From truss as a whole,  $\begin{cases} U = 3 \text{ kips} \\ V = 4 \text{ kips} \end{cases}$



$$\begin{aligned} \sum M_C = 0: & \quad KL(10) - 3(16) = 0 \\ & \quad KL = 4.8 \text{ kips T} \end{aligned}$$

$$\begin{aligned} \sum M_L = 0: & \quad CD(10) - 3(16) = 0 \\ & \quad CD = 4.8 \text{ kips C} \end{aligned}$$

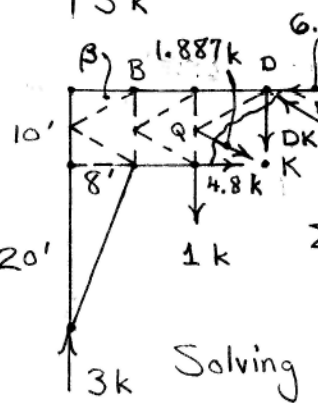
(From a similar right-hand section,  $DE = 6.40 \text{ kips C}$ .)



$$\begin{aligned} \sum M_D = 0: & \quad -3(24) + 1(8) \\ & \quad + KQ(\sqrt{8^2 + 5^2} \sin \alpha) + 4.8(10) \\ & \quad = 0, \end{aligned}$$

where  $\alpha = 180 - 2 \tan^{-1}\left(\frac{8}{5}\right) = 64.0^\circ$

Solving,  $KQ = 1.887 \text{ kips T}$



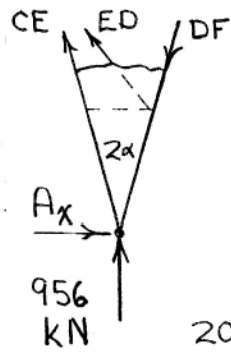
$$\beta = \tan^{-1}\left(\frac{5}{8}\right) = 32.0^\circ$$

$$\begin{aligned} \sum F_x = 0: & \quad -6.40 + 1.887 \cos \beta \\ & \quad + 4.8 - DR \cos \beta = 0, \quad DR = 0 \end{aligned}$$

$$\begin{aligned} \sum M_B = 0: & \quad -3(8) - 1(8) + 4.8(10) \\ & \quad - DK(16) = 0 \end{aligned}$$

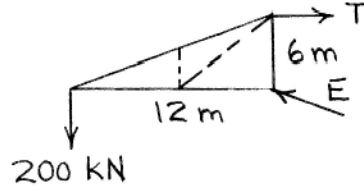
Solving,  $\underline{DK = 1 \text{ kip T}}$

► 4/50 Crane as a whole:  $\sum M_B = 0: 1000(24) + 200(52) = 36 A_y$ ,  $A_y = 956 \text{ kN}$



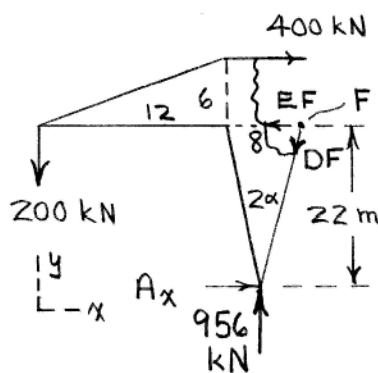
$\sum M_A = 0$  requires  $ED = 0$  (also  $CF = 0$ )

$$\alpha = \tan^{-1}\left(\frac{4}{22}\right) = 10.30, \cos \alpha = 0.984$$



$$\sum M_E = 0: 6T - 12(200) = 0$$

$$T = 400 \text{ kN}$$



$$\sum M_F = 0: 200(20) - 400(6)$$

$$+ A_x(22) - 956(4) = 0$$

$$A_x = 101.1 \text{ kN (to right)}$$

$$\sum M_A = 0: 200(16) + EF(22)$$

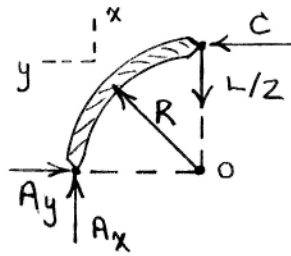
$$- 400(28) = 0$$

$$EF = 364 \text{ kN C}$$

$$\sum F_y = 0: 0.984 DF + 200 - 956 = 0$$

$$DF = 768 \text{ kN C}$$

►4/51 | By symmetry, the force which the right half exerts on the left half at C is horizontal:

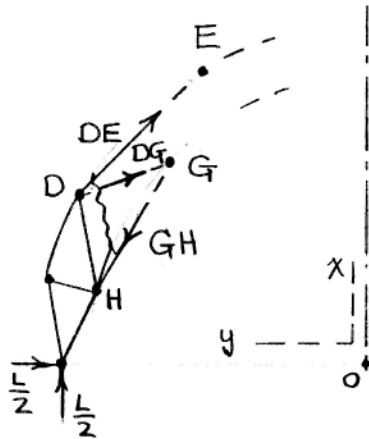


$$\sum M_A = 0: CR - \frac{L}{2}R = 0$$

$$C = L/2$$

$$\sum F_y = 0: -A_y + \frac{L}{2} = 0, A_y = \frac{L}{2}$$

$$\sum F_x = 0: A_x - \frac{L}{2} = 0, A_x = \frac{L}{2}$$



$$\underline{r}_{OD} + \underline{r}_{DE} = \underline{r}_{OE}$$

$$\therefore \underline{r}_{DE} = \underline{r}_{OE} - \underline{r}_{OD}$$

$$= 1.1R(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$

$$- 1.1R(\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j})$$

$$= R(0.403 \underline{i} - 0.403 \underline{j})$$

$$\text{So force } \underline{DE} = DE \frac{\underline{r}_{DE}}{r_{DE}}$$

$$= DE(0.707 \underline{i} - 0.707 \underline{j})$$

Similarly, force  $\underline{GH} = GH(-0.866 \underline{i} + 0.500 \underline{j})$

force  $\underline{DG} = DG(0.264 \underline{i} - 0.965 \underline{j})$

$$\sum F_x = 0: \frac{L}{2} + 0.707 DE - 0.866 GH + 0.264 DG = 0 \quad (1)$$

$$\sum F_y = 0: -\frac{L}{2} - 0.707 DE + 0.500 GH - 0.965 DG = 0 \quad (2)$$

$$\sum M_o = 0: -\frac{L}{2}R \underline{k} + \underline{r}_{OD} \times (\underline{DE} + \underline{DG}) + \underline{r}_{OH} \times \underline{GH},$$

$$\text{where } \underline{r}_{OH} = 0.9R(\cos 75^\circ \underline{i} + \sin 75^\circ \underline{j})$$

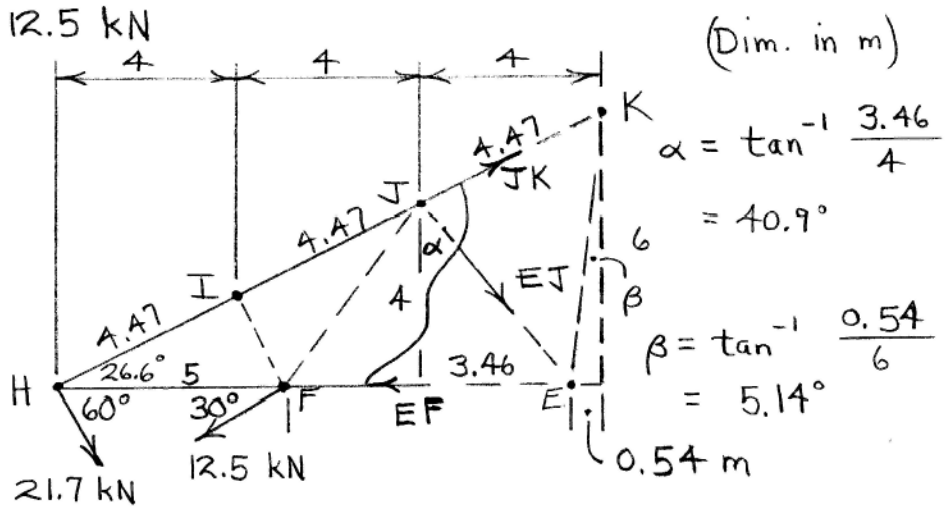
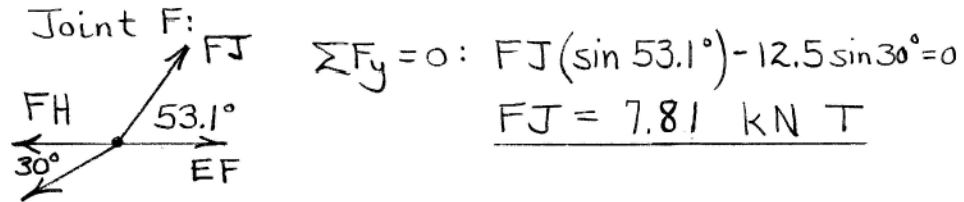
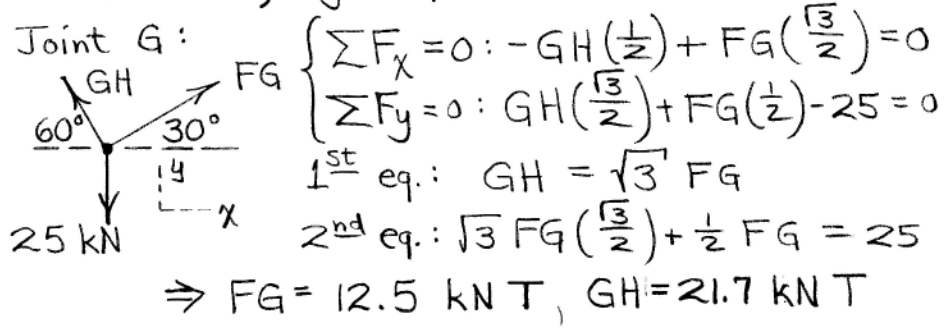
Carrying out the cross products and collecting terms:  $-1.063 DE + 0.869 GH - 0.782 DG = \frac{L}{2}R \underline{k}$  (3)

Simultaneous solution of Eqs. (1)-(3):

$$DE = 0.839LT, GH = 1.090LC, \underline{DG} = -0.569L C$$



► 4/52 |  $F_I = 0$ , by inspection of joint I.



$$+ \sin 40.9^\circ (4)] = 0, \quad EJ = -3.61 \text{ kN}$$

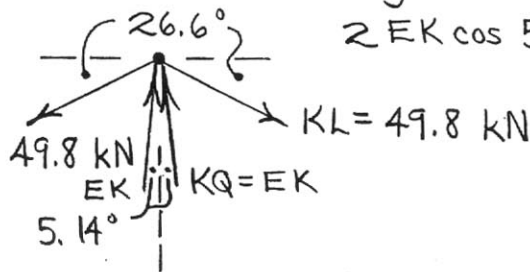
So  $EJ = 3.61 \text{ kN C}$

$$\sum F_y = 0: JK \sin 26.6^\circ + 3.61 \cos 40.9^\circ - 12.5 \left(\frac{1}{2}\right) - 21.7 \left(\frac{\sqrt{3}}{2}\right) = 0, \quad JK = 49.8 \text{ kN T}$$

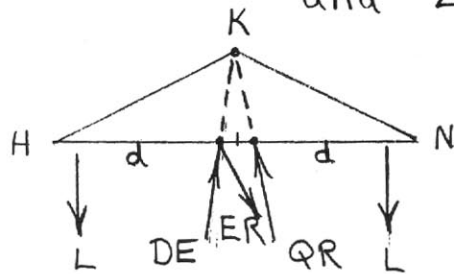
Joint K, using symmetry:

$$\sum F_y = 0: -2(49.8) \sin 26.6^\circ + 2EK \cos 5.14^\circ = 0,$$

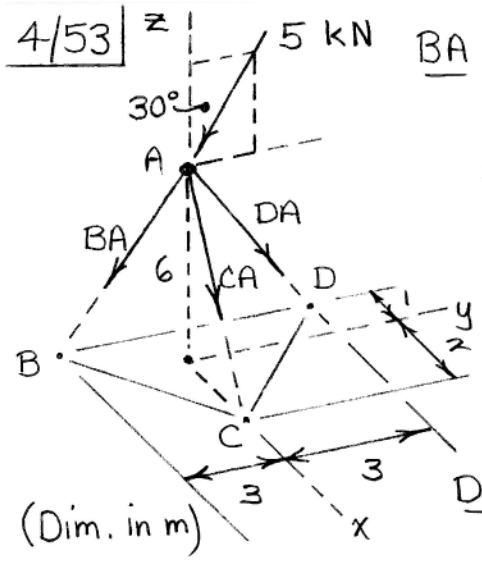
$$\underline{EK = 22.4 \text{ kN C}}$$



By symmetry of loads L and  $\sum M_K = 0, \quad \underline{ER = 0}$



4/53



$$\underline{BA} = BA \left( \frac{-\underline{i} - 3\underline{j} + 6\underline{k}}{\sqrt{1^2 + 3^2 + 6^2}} \right)$$

$$= BA \left( \frac{-\underline{i} - 3\underline{j} - 6\underline{k}}{\sqrt{46}} \right)$$

$$\underline{CA} = CA \left( \frac{+2\underline{i} - 6\underline{k}}{\sqrt{2^2 + 6^2}} \right)$$

$$= CA \left( \frac{+2\underline{i} - 6\underline{k}}{\sqrt{40}} \right)$$

$$\underline{DA} = DA \left( \frac{-\underline{i} + 3\underline{j} - 6\underline{k}}{\sqrt{1^2 + 3^2 + 6^2}} \right)$$

$$= DA \left( \frac{-\underline{i} + 3\underline{j} - 6\underline{k}}{\sqrt{46}} \right)$$

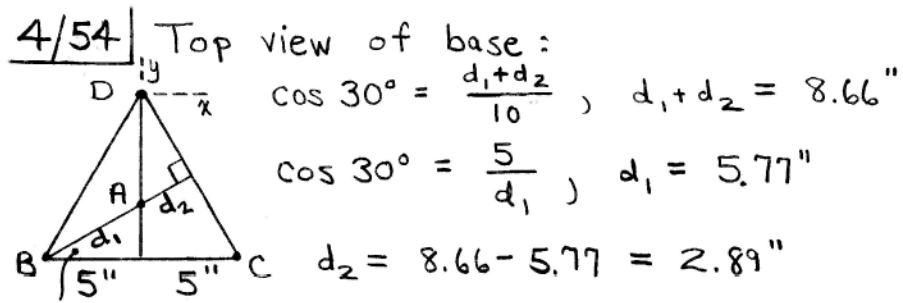
$\Sigma \underline{F} = \underline{0}$  :  $\underline{BA} + \underline{CA} + \underline{DA} - 5(\cos 30^\circ \underline{k} + \sin 30^\circ \underline{j}) = \underline{0}$   
yields

$$\underline{i} : -\frac{1}{\sqrt{46}} BA + \frac{2}{\sqrt{40}} CA - \frac{1}{\sqrt{46}} DA = 0$$

$$\underline{j} : -\frac{3}{\sqrt{46}} BA + \frac{3}{\sqrt{46}} DA - 5\left(\frac{1}{2}\right) = 0$$

$$\underline{k} : -\frac{6}{\sqrt{46}} BA - \frac{6}{\sqrt{40}} CA - \frac{6}{\sqrt{46}} DA - 5\frac{\sqrt{3}}{2} = 0$$

Solution : 
$$\begin{cases} BA = -4.46 \text{ kN} \\ CA = -1.521 \text{ kN} \\ DA = 1.194 \text{ kN} \end{cases}$$



30° For joint A, assuming symmetry:

$$\underline{F}_{BA} = P \left[ \frac{5\underline{i} + 2.89\underline{j} + 16\underline{k}}{(5^2 + 2.89^2 + 16^2)^{1/2}} \right] = P(0.294\underline{i} + 0.1697\underline{j} + 0.941\underline{k})$$

$$\underline{F}_{CA} = P(-0.294\underline{i} + 0.170\underline{j} + 0.941\underline{k}), \underline{F}_{DA} = P(-0.339\underline{j} + 0.941\underline{k})$$

$$\Sigma F_z = 0 \text{ at A: } 3(0.941 P) - 800 = 0, P = 283 \text{ lb}$$

For joint C, assuming symmetry:

$$\underline{F}_{BC} = -Q\underline{i}, \underline{F}_{CD} = Q(-\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j})$$

$$\text{Normal } \underline{N} = 267\underline{k} \text{ lb}$$

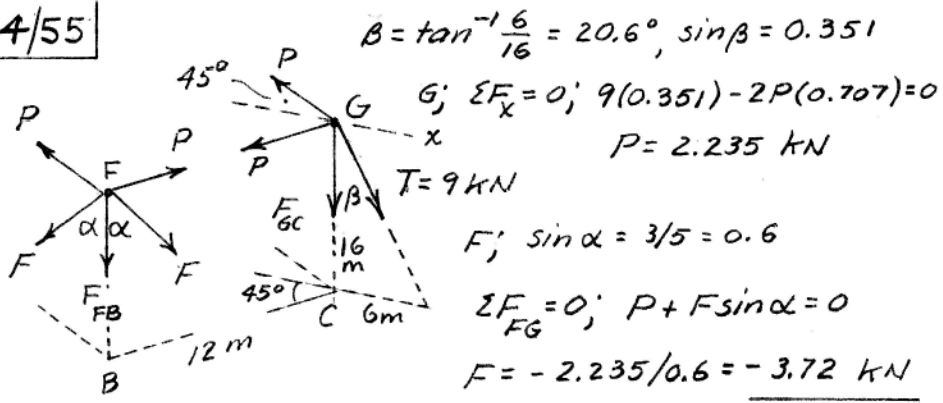
$$\Sigma \underline{F} = \underline{0} \text{ at C: } \underline{N} + \underline{F}_{BC} + \underline{F}_{CD} + \underline{F}_{AC} = \underline{0}$$

$$267\underline{k} - Q\underline{i} + 283(0.294\underline{i} - 0.1697\underline{j} - 0.941\underline{k})$$

$$+ Q(-0.5\underline{i} + 0.866\underline{j}) = \underline{0}$$

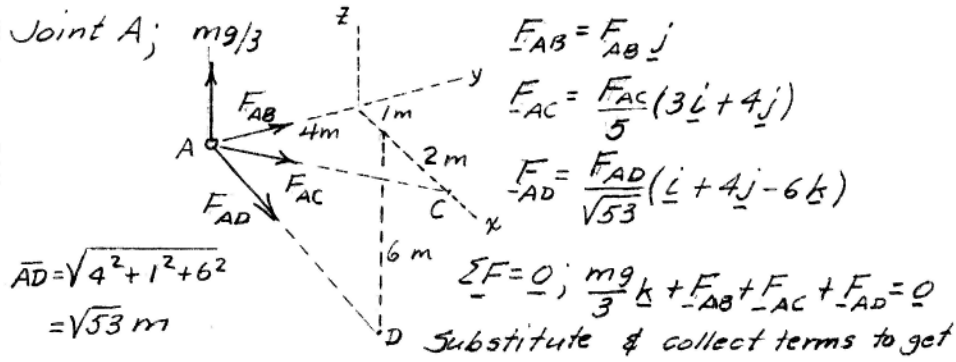
$$\text{Solving, } Q = 55.6 \text{ lb} \Rightarrow \underline{BC = BD = CD = 55.6 \text{ lb T}}$$

4/55



4/56 | From truss as a whole  $\Sigma M = 0$  gives tension in vertical wire at C  $T_C = \frac{1}{3}mg$

By symmetry &  $\Sigma F_z = 0$ ;  $T_A = T_B = \frac{1}{3}mg$

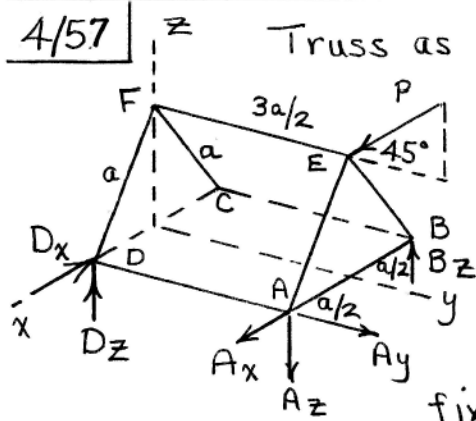


$$\left(\frac{3F_{AC}}{5} + \frac{F_{AD}}{\sqrt{53}}\right) \underline{i} + \left(F_{AB} + \frac{4F_{AC}}{5} + \frac{4F_{AD}}{\sqrt{53}}\right) \underline{j} + \left(\frac{mg}{3} - \frac{6F_{AD}}{\sqrt{53}}\right) \underline{k} = 0$$

Equate coefficients of  $\underline{i}$ ,  $\underline{j}$ , &  $\underline{k}$ -terms to zero & get

$$F_{AD} = \frac{\sqrt{53}}{18} mg, \quad F_{AC} = -\frac{5}{54} mg, \quad F_{AB} = -\frac{4}{27} mg$$

4/57



Truss as a whole is statically determinate with 6 supporting constraints.

$j=6, m=12, 3j=m+6$   
so sufficient members for stability. A, B & D are

fixed so F is fixed. E & C

are also fixed, so truss is a rigid unit.

$$\sum M_{Az} = 0: \frac{P}{\sqrt{2}} \frac{a}{2} - D_x \frac{3a}{2} = 0, D_x = \frac{P}{3\sqrt{2}}, A_x = \frac{P}{3\sqrt{2}}$$

$$\sum M_{AB} = 0: \frac{P}{\sqrt{2}} \frac{a\sqrt{3}}{2} - D_z \frac{3a}{2} = 0, D_z = \frac{P}{\sqrt{6}}$$

$$\sum M_{AD} = 0: \frac{P}{\sqrt{2}} \frac{a}{2} - B_z a = 0, B_z = \frac{P}{2\sqrt{2}}$$

$$\sum F_z = 0 \text{ gives } A_z = \frac{2-\sqrt{3}}{2\sqrt{6}} P$$

Forces at C are all zero. From joint E,

$$\sum F_y = 0 \text{ gives } EF = \frac{P}{\sqrt{2}} \underline{c}$$

Joint F:

$$FC=0 \quad EF = \frac{P}{\sqrt{2}} \begin{cases} \underline{AF} = AF(\underline{i} + 3\underline{j} - \sqrt{3}\underline{k})/\sqrt{13} \\ \underline{BF} = BF(-\underline{i} + 3\underline{j} - \sqrt{3}\underline{k})/\sqrt{13} \\ \underline{DF} = DF(\underline{i} - \sqrt{3}\underline{k})/2, \quad EF = -\frac{P}{\sqrt{2}} \underline{j} \end{cases}$$

$$\sum \underline{F} = 0 = \left( \frac{AF}{\sqrt{13}} - \frac{BF}{\sqrt{13}} + \frac{DF}{2} \right) \underline{i} + \left( -\frac{P}{\sqrt{2}} + \frac{3AF}{\sqrt{13}} + \frac{3BF}{\sqrt{13}} \right) \underline{j} + \left( -\frac{\sqrt{3}}{13} AF - \frac{\sqrt{3}}{13} BF - \frac{\sqrt{3}}{2} DF \right) \underline{k}$$

Solve to obtain  $BF = 0, DF = -\frac{\sqrt{3}}{3} P$

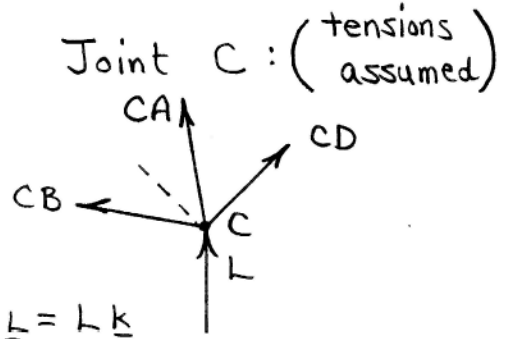
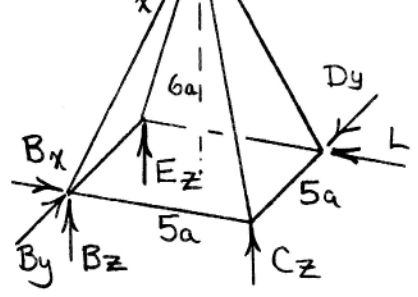
$$\underline{AF} = \frac{\sqrt{13}}{3\sqrt{2}} P$$

4/58 | Number of joints is  $j=7$ , so  $m+b=3j=21$  and  $m=15$ . Since the figure shows only 13 members, 2 more are necessary to ensure stability. Inspection shows that panel ADGE needs a diagonal support AG in order to prevent motion of G toward E. Also, F needs the support of a member DF to fix it in space. Joints E and C then become fixed, and the truss is rigid. (Other possibilities exist for creating stability by adding two members.)



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$$\begin{cases} \sum M_{BE} = 0 \Rightarrow C_z = L \\ \sum M_{Bz} = 0 \Rightarrow D_y = L \end{cases}$$

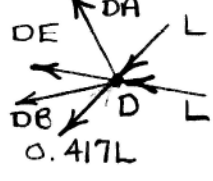


$$\underline{CB} = -CB \underline{i}, \quad \underline{CD} = CD \underline{j}, \quad \underline{L} = L \underline{k}$$

$$\underline{CA} = CA \left( \frac{-2.5a \underline{i} + 2.5a \underline{j} + 6a \underline{k}}{\sqrt{(2.5^2 + 2.5^2 + 6^2)a^2}} \right) = CA(-0.359 \underline{i} + 0.359 \underline{j} + 0.862 \underline{k})$$

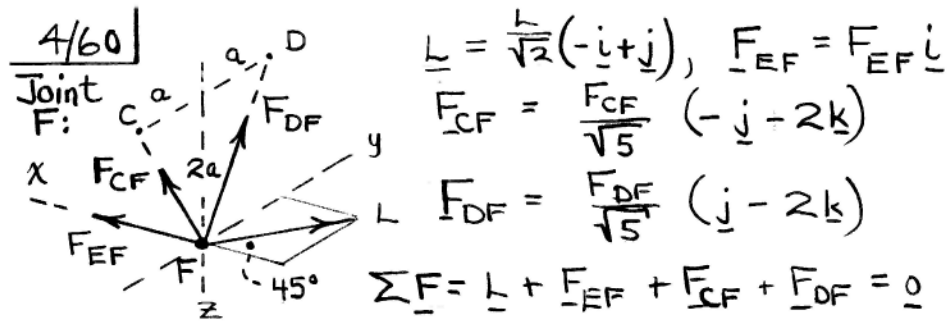
$$\sum \underline{F} = \underline{0} \text{ yields: } \begin{cases} \underline{i}: -CB - 0.359 CA = 0 \\ \underline{j}: CD + 0.359 CA = 0 \\ \underline{k}: L + 0.862 CA = 0 \end{cases} \begin{cases} CA = -0.1667L \\ CD = +0.417L \end{cases}$$

Joint D:

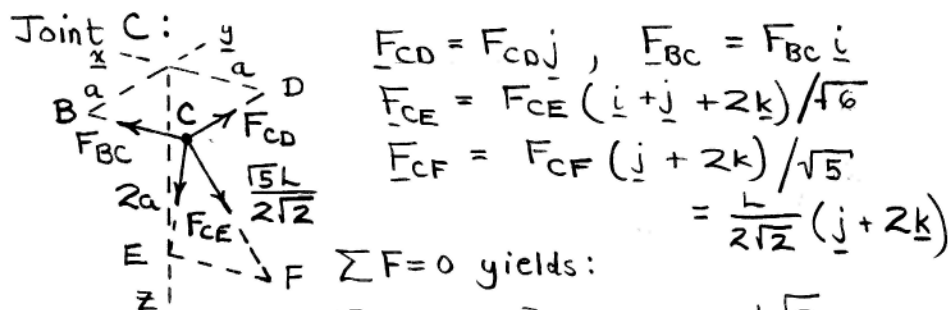


$$\sum \underline{F} = \underline{0} \text{ yields } \begin{cases} \underline{i}: -DE - L - 0.359 DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{j}: -0.417L - L - 0.359 DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{k}: 0.862 DA = 0 \end{cases}$$

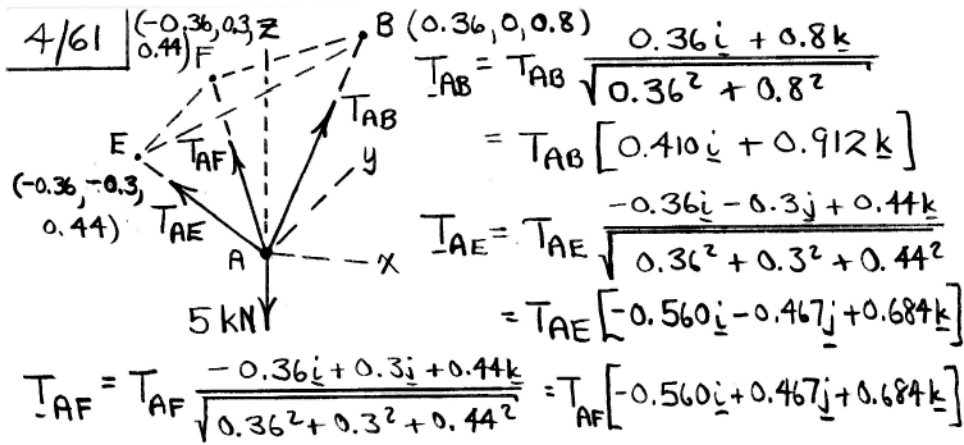
$$DA = 0, \quad \underline{DB} = -2.00L$$



$$\begin{aligned} \underline{i}: & -\frac{L}{\sqrt{2}} + F_{EF} = 0 & F_{EF} &= \frac{L}{\sqrt{2}} \\ \underline{j}: & \frac{L}{\sqrt{2}} - \frac{F_{CF}}{\sqrt{5}} + \frac{F_{DF}}{\sqrt{5}} = 0 \\ \underline{k}: & -\frac{2}{\sqrt{5}}F_{CF} - \frac{2}{\sqrt{5}}F_{DF} = 0 \end{aligned} \left. \vphantom{\begin{aligned} \underline{i}: \\ \underline{j}: \\ \underline{k}: \end{aligned}} \right\} \begin{aligned} F_{CF} &= \frac{\sqrt{5}L}{2\sqrt{2}} \\ F_{DF} &= -\frac{\sqrt{5}L}{2\sqrt{2}} \end{aligned}$$

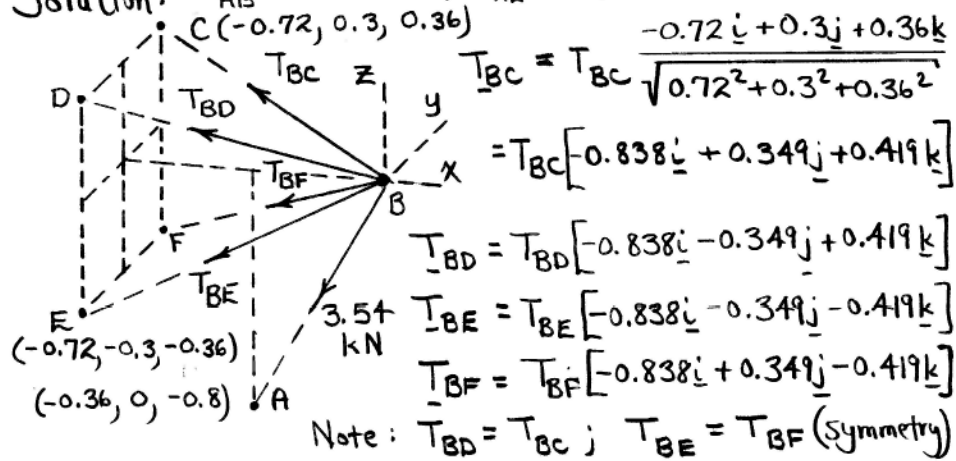


$$\begin{aligned} \underline{i}: & 0 + F_{BC} + \frac{F_{CE}}{\sqrt{6}} = 0 \\ \underline{j}: & F_{CD} + \frac{F_{CE}}{\sqrt{6}} + \frac{L}{2\sqrt{2}} = 0 \\ \underline{k}: & \frac{2F_{CE}}{\sqrt{6}} + \frac{L}{\sqrt{2}} = 0 \end{aligned} \left. \vphantom{\begin{aligned} \underline{i}: \\ \underline{j}: \\ \underline{k}: \end{aligned}} \right\} \begin{aligned} F_{BC} &= \frac{L\sqrt{2}}{4} \\ F_{CD} &= 0 \\ F_{CE} &= -\frac{L\sqrt{3}}{2} \end{aligned}$$



$$\left. \begin{aligned} \sum F_x = 0: & 0.410T_{AB} - 0.560T_{AE} - 0.560T_{AF} = 0 \\ \sum F_y = 0: & -0.467T_{AE} + 0.467T_{AF} = 0 \\ \sum F_z = 0: & 0.912T_{AB} + 0.684T_{AE} + 0.684T_{AF} - 5 = 0 \end{aligned} \right\}$$

Solution:  $T_{AB} = 3.54 \text{ kN}$ ,  $T_{AE} = T_{AF} = 1.296 \text{ kN}$



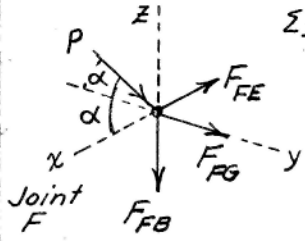
Set  $\sum \underline{F} = 0$  to obtain  $T_{BD} = T_{BC} = 1.491 \text{ kN}$

$T_{BE} = T_{BF} = -2.36 \text{ kN (C)}$

4/62  $\alpha = \cos^{-1} \frac{1}{\sqrt{3}}$ ; By symmetry  $F_{FE} = F_{FG} = F_{FB} = F$

$\Sigma F = 0; F(-i + j - k) + \frac{P}{\sqrt{3}}(-i + j - k) = 0$

$F = -P/\sqrt{3}$  so  $F_{FE} = -P/\sqrt{3}$  (compression)



By inspection, forces on joints A, C, & H are zero, and by symmetry forces at D are

$F_{BD} = F_{GD} = F_{ED} = R$ . Also by symmetry

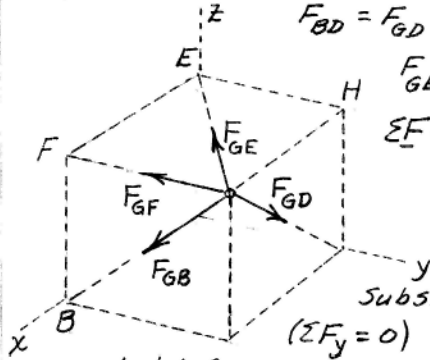
$F_{GE} = F_{GB}; F_{GF} = F_{GF}(-j) = -\frac{P}{\sqrt{3}}(-j)$

$\Sigma F = 0; -\frac{P}{\sqrt{3}}(-j) + \frac{F_{GE}(-i - j)}{\sqrt{2}}$

$+ \frac{F_{GB}(-j - k)}{\sqrt{2}} + \frac{F_{GD}(-i - k)}{\sqrt{2}} = 0$

Substitute  $F_{GB} = F_{GE}$ , collect j-terms

$(\Sigma F_y = 0)$  & get  $(\frac{P}{\sqrt{3}} - \frac{2F_{GE}}{\sqrt{2}}) = 0$



Joint G

No. of members is  $m = 18$

No. of joints is  $j = 8$

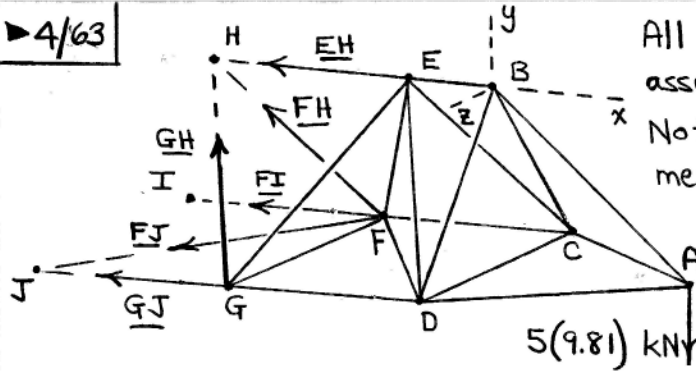
$(m + 6 = 24) = (3j = 24)$  so internally stable

$F_{EG} = P/\sqrt{6}$  (tension)



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► 4/63



All members assumed in tension.  
Note that six members are cut!

$$\underline{GJ} = -GJ \underline{i}, \quad \underline{FI} = -FI \underline{i}, \quad \underline{FJ} = \frac{FJ}{\sqrt{2}} (-\underline{i} + \underline{k})$$

$$\begin{aligned} \sum \underline{M}_H = \underline{0} : & -49.05(5) \underline{k} + (-2 \cos 30^\circ \underline{j} + 2 \sin 30^\circ \underline{k}) \\ & \times (-GJ) \underline{i} + (-2 \cos 30^\circ \underline{j} - 2 \sin 30^\circ \underline{k}) \times (-FI) \underline{i} \\ & + (\underline{i} - 2 \cos 30^\circ \underline{j} - \underline{k}) \times \frac{FJ}{\sqrt{2}} (-\underline{i} + \underline{k}) = \underline{0}. \end{aligned}$$

Equating unit vector coefficients to zero:

$$-1.225 FJ = 0 \Rightarrow \underline{FJ} = 0$$

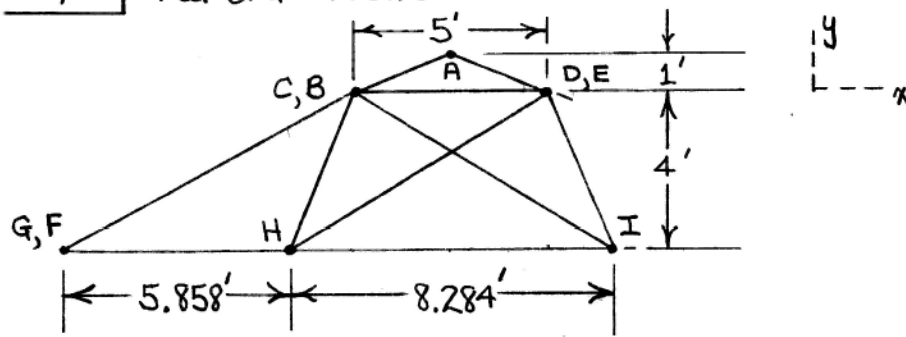
$$-GJ + FI = 0$$

$$-1.732 GJ - 1.732 FI = 245$$

$$\left. \begin{array}{l} FI = GJ = \\ -70.8 \text{ kN} \end{array} \right\}$$

∴ Force in GJ = 70.8 kN C

►4/64 Partial front view:



For equilibrium of joint A, force vectors are

$$\underline{L} = -L\underline{j}$$

$$\underline{BA} = P \left[ \frac{2.5\underline{i} + \underline{j} + 2.5\underline{k}}{\sqrt{2.5^2 + 1^2 + 2.5^2}} \right] = P(0.680\underline{i} + 0.272\underline{j} + 0.680\underline{k})$$

Similarly,

$$\underline{CA} = P(0.680\underline{i} + 0.272\underline{j} - 0.680\underline{k})$$

$$\underline{DA} = P(-0.680\underline{i} + 0.272\underline{j} - 0.680\underline{k})$$

$$\underline{EA} = P(-0.680\underline{i} + 0.272\underline{j} + 0.680\underline{k})$$

where P is the force in the 4 members joined at A, all of which are assumed to be in compression.

$$\sum F_y = 0 \text{ at A: } 4P(0.272) - L = 0, P = 0.919L$$

For equilibrium of joint B, force vectors are

$$\underline{BC} = -Q\underline{k}, \underline{CD} = Q\underline{i}$$

$$\underline{AC} = 0.919L (-0.680\underline{i} - 0.272\underline{j} + 0.680\underline{k})$$

$$\underline{CF} = R \left[ \frac{-(10-2.5)\underline{i} - 4\underline{j} - (8.28/2 + 5/2)\underline{k}}{\sqrt{7.5^2 + 4^2 + 6.64^2}} \right]$$

$$= R (-0.695\underline{i} - 0.371\underline{j} - 0.616\underline{k})$$

Similarly,

$$\underline{CG} = S (-0.866\underline{i} - 0.462\underline{j} + 0.190\underline{k})$$

$$\underline{CH} = S (-0.190\underline{i} - 0.462\underline{j} + 0.866\underline{k})$$

$$\underline{CI} = R (0.616\underline{i} - 0.371\underline{j} + 0.695\underline{k})$$

where Q, R, and S are force magnitudes and where all unknowns are assumed in tension.

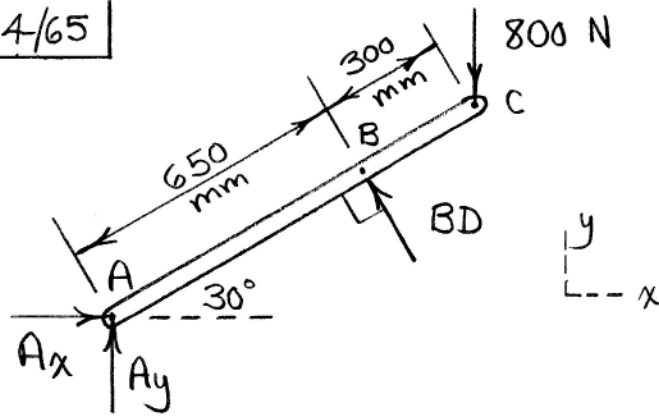
$$\sum \underline{F} = \underline{0} \text{ at joint B: } \underline{AC} + \underline{BC} + \underline{CD} + \underline{CF} + \underline{CG} + \underline{CH} + \underline{CI} = \underline{0}, \text{ or}$$

$$\begin{aligned} & [(0.919L)(-0.680) + Q - 0.695R - 0.866S - 0.190S + 0.616R] \underline{i} \\ & + [(0.919L)(-0.272) - 0.371R - 0.462S - 0.462S - 0.371R] \underline{j} \\ & + [-Q + (0.919L)(0.680) - 0.616R + 0.190S + 0.866S + 0.695R] \underline{k} \\ & = \underline{0} \text{ (note dependency between } \underline{i} \text{ \& } \underline{k} \text{ components!)} \end{aligned}$$

With  $Q = 0.3L$ , solve x- and y-equations to obtain  $R = 0.051L$ ,  $S = -0.312L$

$$\therefore \underline{CF} = 0.051L \text{ T and } \underline{CG} = 0.312L \text{ C}$$

4/65



$$\sum M_A = 0 : BD(650) - 800(950 \cos 30^\circ) = 0$$
$$BD = 1013 \text{ N}$$

So pin-reaction magnitudes at B and D are  $B = D = 1013 \text{ N}$ .

$$\sum F_x = 0 : A_x - 1013 \sin 30^\circ = 0, A_x = 506 \text{ N}$$

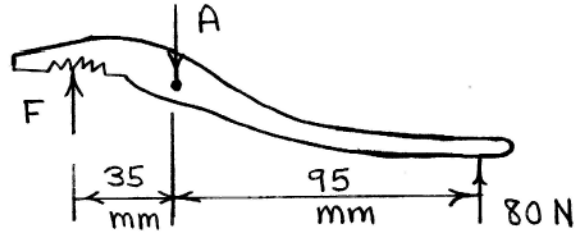
$$\sum F_y = 0 : A_y + 1013 \cos 30^\circ - 800 = 0$$

$$A_y = -76.9 \text{ N}$$

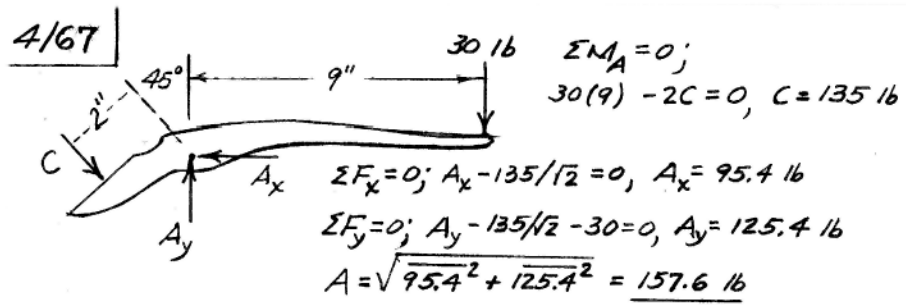
$$A = \sqrt{A_x^2 + A_y^2} = \underline{512 \text{ N}}$$

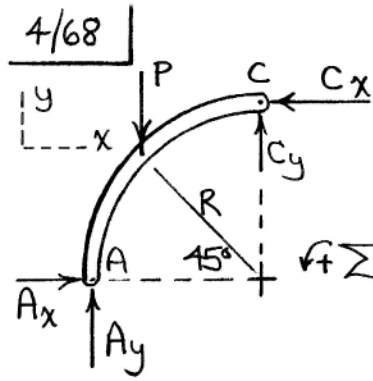


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$$\begin{aligned} \uparrow \sum M_A = 0: & 80(95) - F(35) = 0, \quad \underline{F = 217 \text{ N}} \\ + \uparrow \sum F = 0: & 217 - A + 80 = 0, \quad \underline{A = 297 \text{ N}} \end{aligned}$$





$C_y = 0$  due to symmetry of overall structure.

Then  $\sum F_y = 0$  yields  $A_y = P$

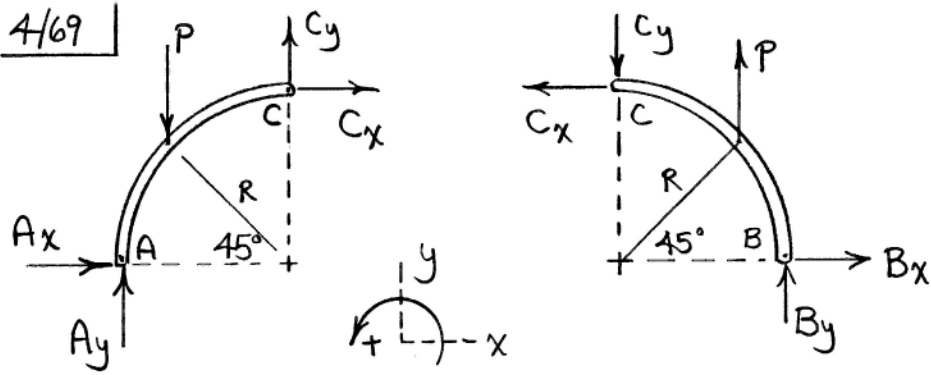
$$\sum M_A = 0: -PR(1 - \cos 45^\circ) + C_x(R) = 0$$

$$C_x = 0.293R$$

Finally,  $\sum F_x = 0$  yields  $A_x = 0.293R$

(Forces on member BC are symmetric)

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AC:

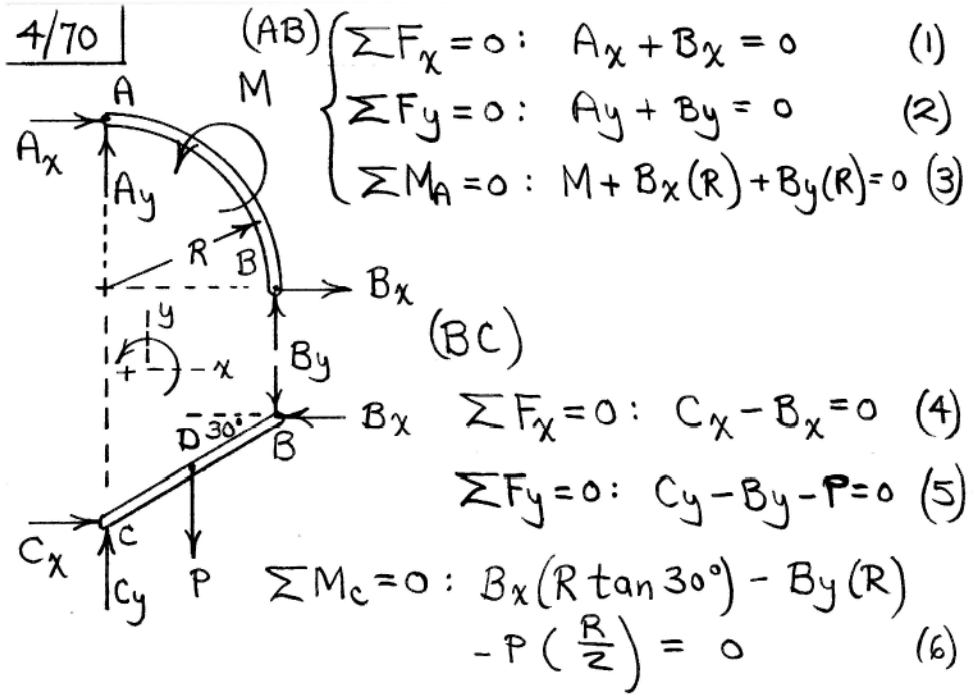
$$\begin{cases} \sum F_x = 0: A_x + C_x = 0 & (1) \\ \sum F_y = 0: A_y + C_y - P = 0 & (2) \\ \sum M_A = 0: C_y(R) - C_x(R) - PR(1 - \cos 45^\circ) = 0 & (3) \end{cases}$$

BC:

$$\begin{cases} \sum F_x = 0: -C_x + B_x = 0 & (4) \\ \sum F_y = 0: -C_y + B_y + P = 0 & (5) \\ \sum M_B = 0: C_y(R) + C_x(R) - PR(1 - \cos 45^\circ) = 0 & (6) \end{cases}$$

Solve Eqs. (1)-(6):  $A_x = C_x = B_x = 0$ 

Unlike Prob. 4/68, this problem is not symmetric.  $A_y = 0.707P$ ,  $B_y = -0.707P$   
 $C_y = 0.293P$

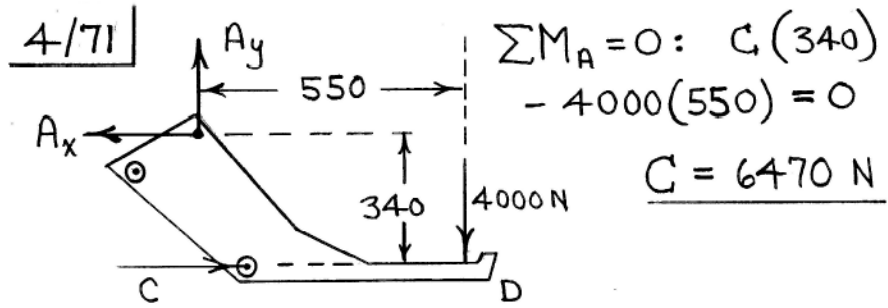


Solve Eqs. (1)-(6) for  $B_x$  and  $B_y$ :

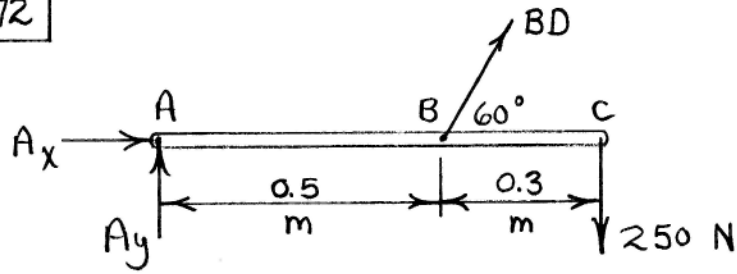
$$B_x = 0.634 \left( \frac{P}{2} - \frac{M}{R} \right), \quad B_y = -0.366 \frac{M}{R} - 0.317P$$

(a) For  $B_x = 0$ ,  $M = \frac{PR}{2}$  (CCW)

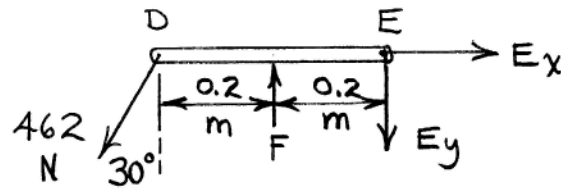
(b) For  $B_y = 0$ ,  $M = -0.866 PR$  (CW)



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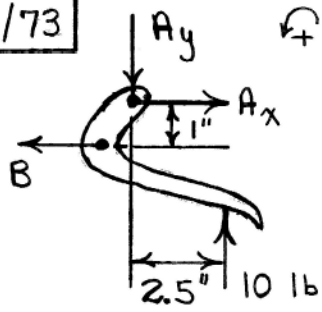


$$\begin{aligned} \curvearrow + \sum M_A = 0 : (BD \sin 60^\circ)(0.5) - 250(0.8) &= 0 \\ BD &= 462 \text{ N} \end{aligned}$$



$$\begin{aligned} \curvearrow + \sum M_E = 0 : 462 \cos 30^\circ (0.4) + F(0.2) &= 0 \\ \underline{F = 800 \text{ N}} \end{aligned}$$

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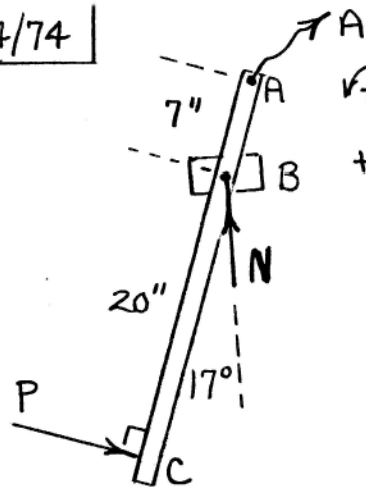
$$\sum M_A = 0: 10(2.5) - B(1) = 0$$

$$B = 25 \text{ lb}$$

$\therefore$  Force F on brad = 25 lb



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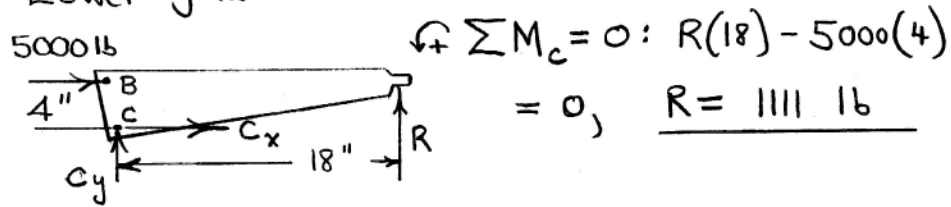
$$\sum M_A = 0:$$

$$+P(27) - N \sin 17^\circ (7) = 0$$

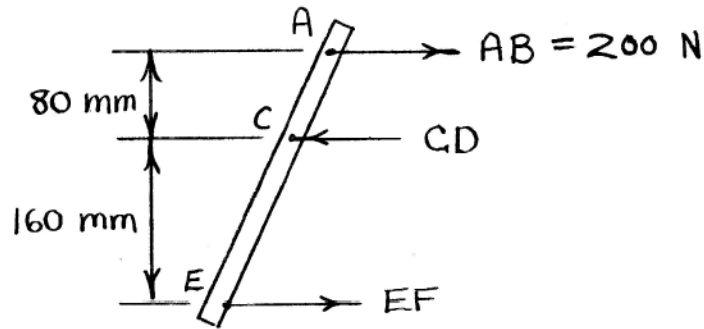
$$\underline{N = 13.19P}$$

4/75 | Piston force =  $(500)(20) = 10,000 \text{ lb}$   
 Force in link AB =  $10,000/2 = 5000 \text{ lb}$

Lower jaw :



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$$\curvearrowleft \sum M_C = 0 : -200(80) + EF(160) = 0$$

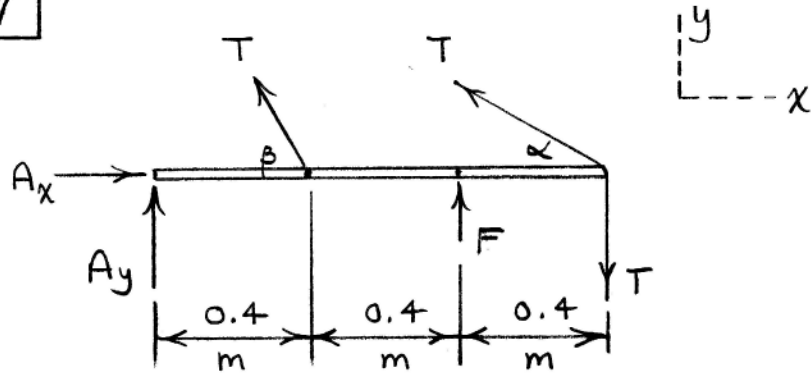
$$EF = 100 \text{ N T}$$

$$\rightarrow \sum F = 0 : 200 - CD + 100 = 0$$

$$CD = 300 \text{ N}$$

So force supported by pin C is F = 300 N

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$$\begin{cases} T = 60(9.81) = 589 \text{ N} \\ \alpha = \tan^{-1}\left(\frac{0.5}{1.2}\right) = 22.6^\circ \\ \beta = \tan^{-1}\left(\frac{0.5}{0.4}\right) = 51.3^\circ \end{cases}$$

$$\begin{aligned} \sum M_A = 0: & T \sin \beta (0.4) + T \sin \alpha (1.2) \\ & - T (1.2) + F (0.8) = 0, \quad \underline{F = 314 \text{ N}} \end{aligned}$$

(Contact at bottom roller)

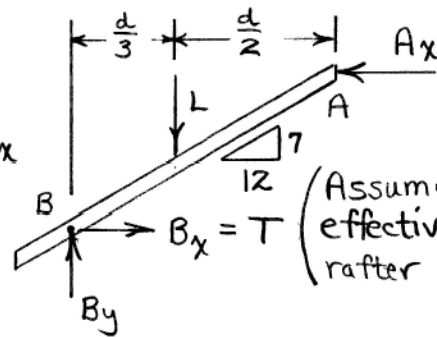
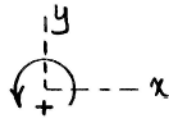
$$\sum F_x = 0: A_x - T \cos \beta - T \cos \alpha = 0, \quad A_x = 911 \text{ N}$$

$$\sum F_y = 0: A_y + T \sin \beta + T \sin \alpha - T + F = 0$$

$$A_y = -411 \text{ N}$$

$$A = \sqrt{A_x^2 + A_y^2} = \underline{999 \text{ N}}$$

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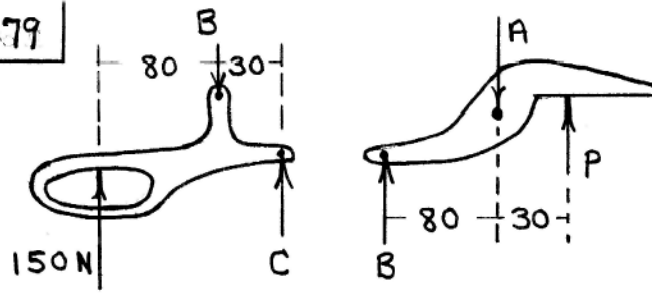
(Assumes cable tension effectively transferred to rafter via proper fastener)

$$\sum F_x = 0 \Rightarrow T = A_x$$

$$\sum M_B = 0 : -L\left(\frac{d}{3}\right) + T\left(\frac{5d}{6} \cdot \frac{7}{12}\right) = 0$$

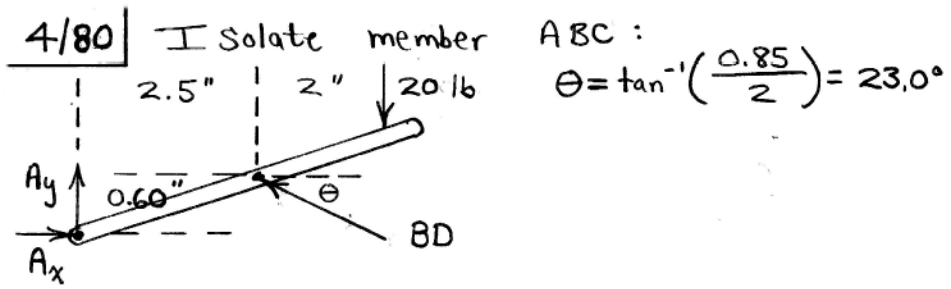
$$T = \frac{24}{35} L$$

4/79



$$\text{(Handle)} \quad \sum M_C = 0: 150(110) - B(30) = 0$$
$$B = 550 \text{ N}$$

$$\text{(Jaw)} \quad \sum M_A = 0: 550(80) - P(30) = 0$$
$$\underline{P = 1467 \text{ N}}$$

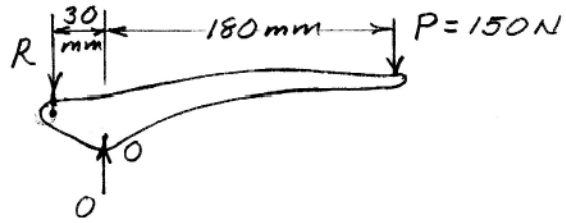
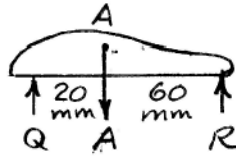


$$\sum M_A = 0 : -20(4.5) + (BD \cos \theta)(0.6) + (BD \sin \theta)(2.5) = 0, \quad BD = 58.8 \text{ lb}$$

$$\sum F_x = 0 : A_x - 58.8 \cos \theta = 0, \quad A_x = 54.1 \text{ lb}$$

Thus squeezing force  $P = 54.1 \text{ lb}$ .

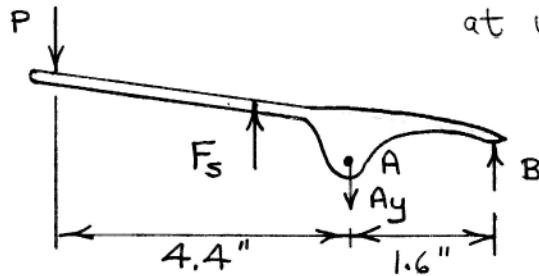
4/81



Handle:  $\sum M_O = 0; 30R - 180(150) = 0, R = 900 \text{ N}$   
Jaw:  $\sum M_A = 0; 20Q - 60(900) = 0, Q = 2700 \text{ N}$   
or  $Q = 2.7 \text{ kN}$



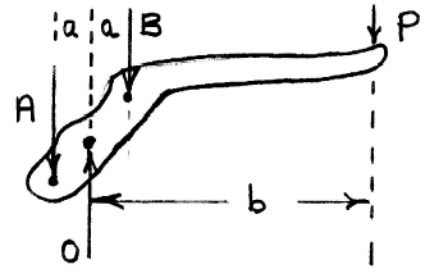
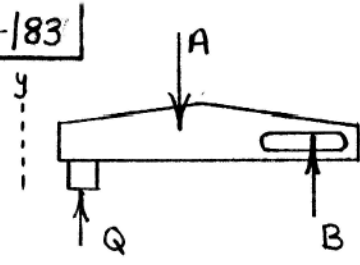
4/82 | Upper handle: (Spring force  $F_s$  acts at unknown location)



When clamp is released,  $B=0$ .  $\sum M_A = 0$ :  
 $P(4.4) - M_{F_s} = 0$ ,  $M_{F_s} = P(4.4) = 6(4.4) = 26.4$   
( $M_{F_s}$  is moment exerted by spring on handle) lb-in.

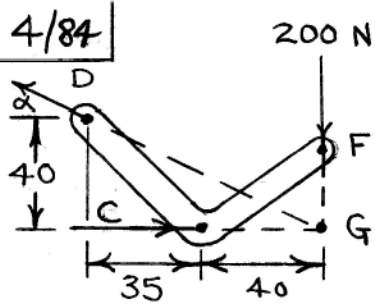
With  $P=0$ ,  $\sum M_A = 0$ :  $B(1.6) - 26.4 = 0$   
 $B = \underline{16.50 \text{ lb}}$

4/83



$$\begin{array}{l}
 \text{(Handle)} \quad \sum M_o = 0: \quad Pb = (A-B)a \\
 \text{(Jaw)} \quad \quad \sum F_y = 0: \quad Q = A-B
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{(Handle)} \\ \text{(Jaw)} \end{array}} \right\} \begin{array}{l} Pb = Qa \\ \underline{Q = P \frac{b}{a}} \end{array}$$

4/84



DCF is a three-force body; forces intersect at G.

$$\alpha = \tan^{-1}\left(\frac{40}{75}\right) = 28.1^\circ$$

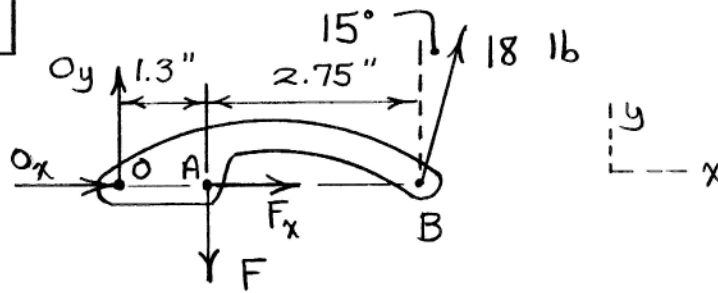
$$\sum F_y = 0: -200 + D \sin \alpha = 0$$

$$D = \underline{425 \text{ N}}$$

$$\sum F_x = 0: -D \cos \alpha + C = 0, \quad C = \underline{375 \text{ N}}$$

(BC in compression)

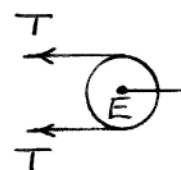
4/85



$$\begin{aligned} \sum M_O = 0 : -F(1.3) + 18 \cos 15^\circ (2.75 + 1.3) &= 0 \\ \underline{F = 54.2 \text{ lb}} \end{aligned}$$

(Note: Treatment of member OC as a three-force body would yield a constraint relationship between  $O_x$  and  $O_y$ .)

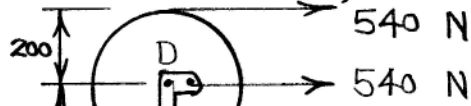
4/86 | T



$$F_{EF} = k\delta = 3600 [0.6 - 0.3]$$

$$= 1080 \text{ N}$$

From  $\sum F = 0$ ,  $T = 540 \text{ N}$



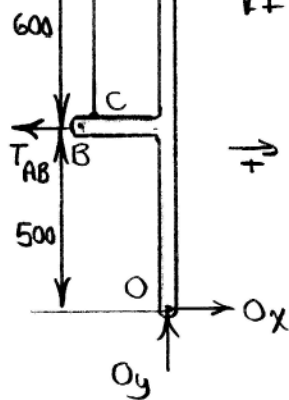
$$\sum M_o = 0: -540(1300) - 540(1100) + T_{AB}(500) = 0, T_{AB} = 2590 \text{ N}$$

$$\sum F = 0: O_x - 2590 + 2(540) = 0$$

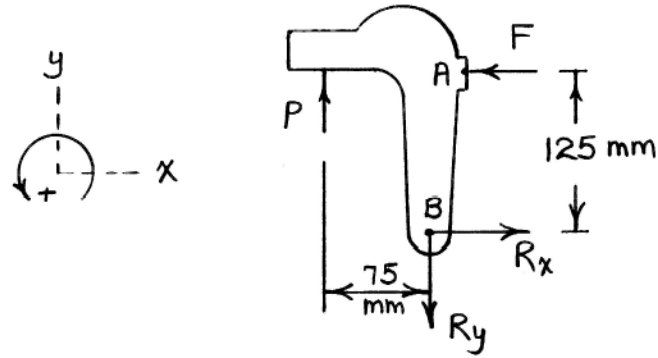
$$O_x = 1512 \text{ N}$$

$$O_y = 0$$

$$\sum_o O = 1512 \text{ N}$$



4/87



For  $P = 3 \text{ kN}$  :

$$\sum M_B = 0 : 125F - 3(75) = 0, \quad F = 1.8 \text{ kN}$$

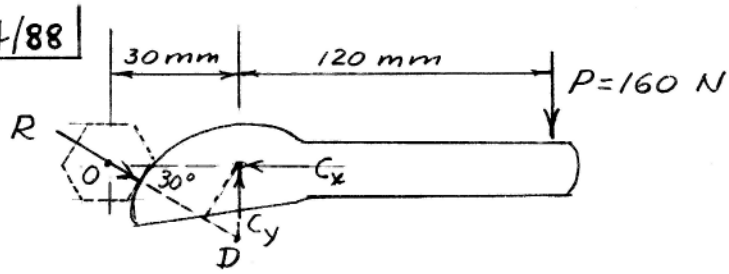
$$\text{For } F = 2(1.8) = 3.6 \text{ kN}, \quad P = 3(2) = 6 \text{ kN}$$

$$\sum F_x = 0 : R_x - 3.6 = 0, \quad R_x = 3.6 \text{ kN}$$

$$\sum F_y = 0 : -R_y + 6 = 0, \quad R_y = 6 \text{ kN}$$

$$R = \sqrt{3.6^2 + 6^2} = \underline{7.00 \text{ kN}}$$

4/88

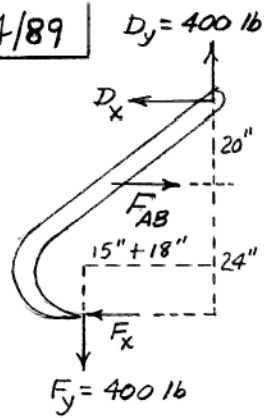


$$\sum M_O = 0; 30 C_y - 150(160) = 0, C_y = 800 \text{ N}$$

$$\sum M_D = 0; (30 \tan 30^\circ) C_x - 120(160) = 0, C_x = 1109 \text{ N}$$

$$C = \sqrt{800^2 + 1109^2} = \underline{1367 \text{ N}}$$

4/89



From tongs as a whole

$$D_y = \frac{1}{2}(800) = 400 \text{ lb} = F_y$$

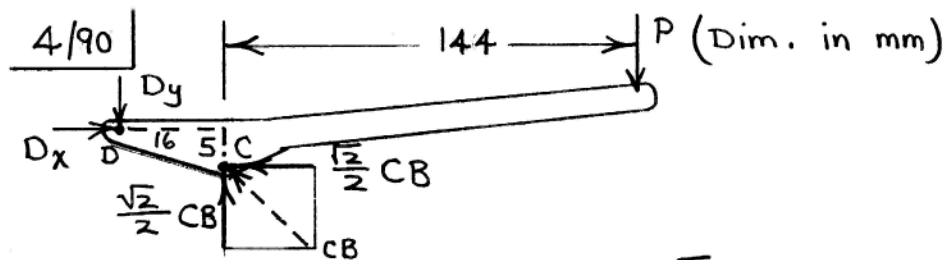
$$\text{From ED, } D_x = \frac{18}{12} D_y = \frac{18}{12}(400) = 600 \text{ lb}$$

From DF,  $\sum M_F = 0$ ;

$$F_{AB}(24) - 600(44) - 400(18 + 15) = 0$$

$$F_{AB} = \underline{1650 \text{ lb tension}}$$

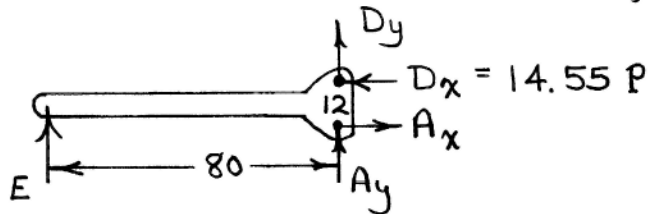




$$\rightarrow \sum M_D = 0: P(160) + \frac{\sqrt{2}}{2} CB (5) - \frac{\sqrt{2}}{2} CB (16) = 0$$

$$CB = 20.6 P$$

$$\rightarrow \sum F = 0: Dx - 20.6 P \frac{\sqrt{2}}{2} = 0, Dx = 14.55 P$$

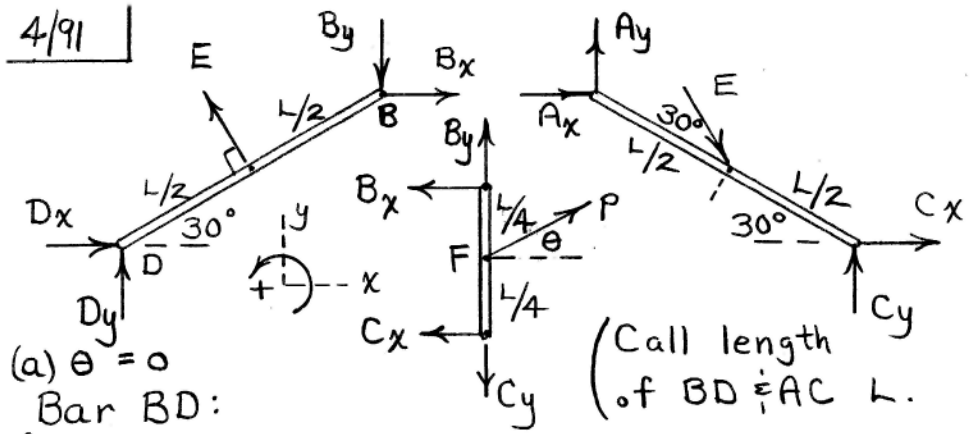


$$\rightarrow \sum M_A = 0: E(80) - 14.55 P (12) = 0$$

$$E = 2.18 P$$

(Note: Mechanical advantage will increase as CB becomes more aligned with CD.)

4/91



(a)  $\theta = 0$

Bar BD:

$$\begin{cases} \sum F_x = 0: B_x + D_x - E\left(\frac{1}{2}\right) = 0 & (1) \\ \sum F_y = 0: -B_y + D_y + E\left(\frac{\sqrt{3}}{2}\right) = 0 & (2) \\ \sum M_B = 0: E\left(\frac{L}{2}\right) - B_y\left(L\frac{\sqrt{3}}{2}\right) - B_x\left(L\frac{1}{2}\right) = 0 & (3) \end{cases}$$

Bar AC:

$$\begin{cases} \sum F_x = 0: A_x + C_x + E\left(\frac{1}{2}\right) = 0 & (4) \\ \sum F_y = 0: A_y + C_y - E\left(\frac{\sqrt{3}}{2}\right) = 0 & (5) \\ \sum M_A = 0: -E\left(\frac{1}{2}\right)\left(\frac{L}{2}\right) + C_y\left(L\frac{\sqrt{3}}{2}\right) + C_x\left(L\frac{1}{2}\right) = 0 & (6) \end{cases}$$

Bar BC:

$$\begin{cases} \sum F_x = 0: -B_x - C_x + P = 0 & (7) \\ \sum F_y = 0: B_y - C_y = 0 & (8) \\ \sum M_F = 0: B_x\left(\frac{L}{4}\right) - C_x\left(\frac{L}{4}\right) = 0 & (9) \end{cases}$$

Solve the above nine equations to obtain:

$$\begin{cases} A_x = 0.289P & C_y = -0.289P & E = 0 \\ B_x = P/2 & D_x = -P/2 \\ B_y = -0.289P & D_y = -0.289P \end{cases}$$


---

(b)  $\theta = 30^\circ$  All equations remain the same except (7) & (8):

$$-B_x - C_x + P \frac{\sqrt{3}}{2} = 0 \quad (7)$$

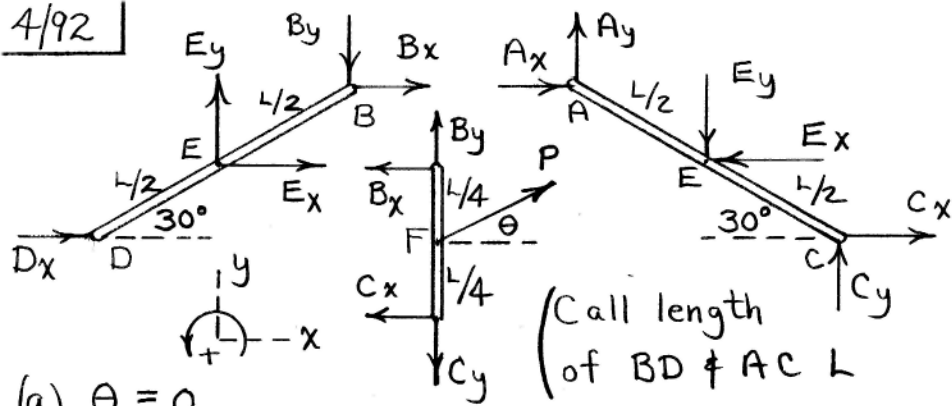
$$B_y - C_y + \frac{P}{2} = 0 \quad (8)$$

Resolve:

$$\begin{cases} A_x = 0.433P & C_x = 0.433P \\ A_y = -0.75P & C_y = -0.75P \\ B_x = 0.433P & D_x = -1.299P \\ B_y = -1.25P & D_y = 0.25P \end{cases}$$

$$E = -1.732P$$

4/92

(a)  $\theta = 0$ 

Bar BD:

$$\left\{ \begin{array}{l} \sum F_x = 0: B_x + D_x + E_x = 0 \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} \sum F_y = 0: -B_y + E_y = 0 \end{array} \right. \quad (2)$$

$$\left\{ \begin{array}{l} \sum M_D = 0: E_y \left( \frac{L}{2} \cdot \frac{\sqrt{3}}{2} \right) - E_x \left( \frac{L}{2} \cdot \frac{1}{2} \right) - B_y \left( L \cdot \frac{\sqrt{3}}{2} \right) - B_x \left( L \cdot \frac{1}{2} \right) = 0 \end{array} \right. \quad (3)$$

Bar AC:

$$\left\{ \begin{array}{l} \sum F_x = 0: A_x + C_x - E_x = 0 \end{array} \right. \quad (4)$$

$$\left\{ \begin{array}{l} \sum F_y = 0: A_y + C_y - E_y = 0 \end{array} \right. \quad (5)$$

$$\left\{ \begin{array}{l} \sum M_A = 0: -E_y \left( \frac{L}{2} \cdot \frac{\sqrt{3}}{2} \right) - E_x \left( \frac{L}{2} \cdot \frac{1}{2} \right) + C_y \left( L \cdot \frac{\sqrt{3}}{2} \right) + C_x \left( L \cdot \frac{1}{2} \right) = 0 \end{array} \right. \quad (6)$$

Bar BC:

$$\left\{ \begin{array}{l} \sum F_x = 0: -B_x - C_x + P = 0 \end{array} \right. \quad (7)$$

$$\left\{ \begin{array}{l} \sum F_y = 0: B_y - C_y = 0 \end{array} \right. \quad (8)$$

$$\left\{ \begin{array}{l} \sum M_F = 0: B_x \left( \frac{L}{4} \right) - C_x \left( \frac{L}{4} \right) = 0 \end{array} \right. \quad (9)$$

Solve the above nine equations to obtain

$$\begin{cases} A_x = -P/2 & C_x = P/2 \\ A_y = 0 & C_y = -0.577P \\ B_x = P/2 & D_x = -P/2 \\ B_y = -0.577P & E_x = 0, E_y = -0.577P \end{cases}$$


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(b)  $\theta = 30^\circ$ . All equations remain the same except (7) & (8):

$$-B_x - C_x + P \frac{\sqrt{3}}{2} = 0 \quad (7)$$

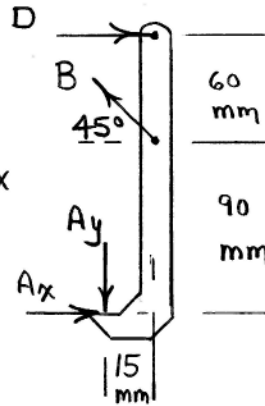
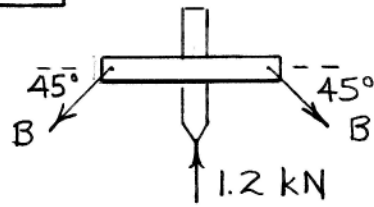
$$B_y - C_y + P/2 = 0 \quad (8)$$

Resolve:

$$\begin{cases} A_x = 0.433P & C_x = 0.433P \\ A_y = -P/2 & C_y = -P/2 \\ B_x = 0.433P & D_x = -1.299P \\ B_y = -P & E_x = 0.866P \quad E_y = -P \end{cases}$$


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(Upper bar &amp; screw)

$$\sum F_y = 0 : -2B \sin 45^\circ + 1.2 = 0, \quad B = 0.849 \text{ kN}$$

(ABD)

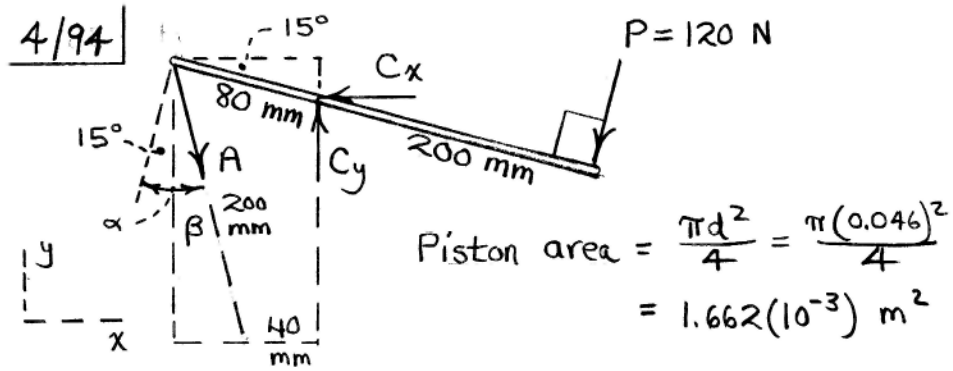
$$\sum M_A = 0 : 150D - 0.849 \cos 45^\circ (90) - 0.849 \sin 45^\circ (15) = 0$$

$$D = 0.420 \text{ kN}$$

$$\sum F_x = 0 : A_x - 0.849 \cos 45^\circ + 0.420 = 0, \quad A_x = 0.1800 \text{ kN}$$

$$\sum F_y = 0 : -A_y + 0.849 \sin 45^\circ = 0, \quad A_y = 0.6 \text{ kN}$$

$$A = \sqrt{A_x^2 + A_y^2} = \underline{0.626 \text{ kN}}$$



$$\sin \beta = (80 \cos 15^\circ - 40) / 200, \quad \beta = 10.74^\circ$$

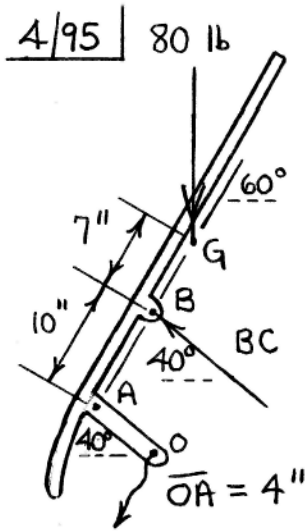
$$\alpha = \beta + 15^\circ = 25.7^\circ$$

$$\sum M_C = 0 : 120(200) - A \cos 25.7^\circ (80) = 0, \quad A = 333 \text{ N}$$

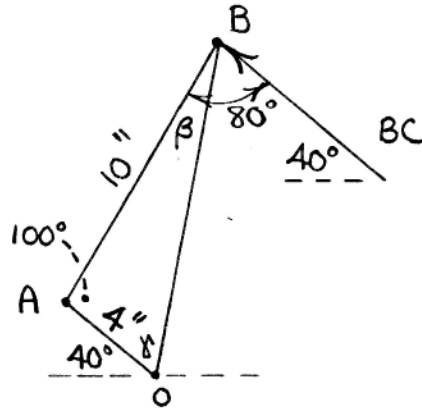
$$\sum F_y = 0 : C_y - 120 \cos 15^\circ - 333 \cos 10.74^\circ = 0, \quad C_y = 443 \text{ N}$$

$$\text{Piston : } p(\text{Area}) = C_y, \quad p = \frac{443}{1.662(10^{-3})} = 267(10^3) \text{ N/m}^2$$

$$\text{or } p = \underline{267 \text{ kPa}}$$



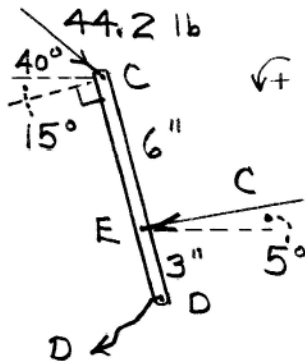
Blowup :



$$\overline{OB}^2 = 4^2 + 10^2 - 2(4)(10) \cos 100^\circ, \quad \overline{OB} = 11.40 \text{ in.}$$

$$\frac{\sin \beta}{4''} = \frac{\sin 100^\circ}{11.40''}, \quad \beta = 20.2^\circ$$

$$\sqrt{+} \sum M_O = 0: -80(17 \cos 60^\circ - 4 \cos 40^\circ) + BC(11.40 \sin(80^\circ - 20.2^\circ)) = 0, \quad BC = 44.2 \text{ lb}$$

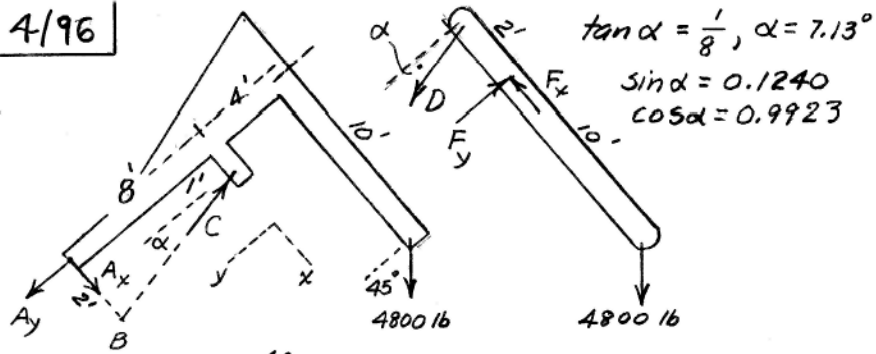


$$\sqrt{+} \sum M_D = 0: -44.2 \cos 55^\circ (9) + C \cos 10^\circ (3) = 0$$

$$\underline{C = 77.2 \text{ lb}}$$



4/96



$$\sum M_B = 0; 2A_y - \frac{4800}{\sqrt{2}}(10-2+12) = 0, A_y = 33,940 \text{ lb}$$

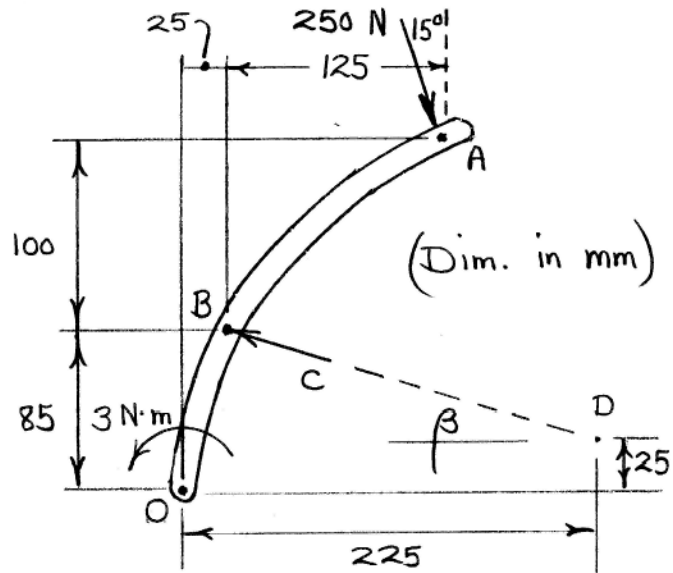
$$\sum M_C = 0; 8A_x + 33900(1) - \frac{4800}{\sqrt{2}}(10-1+4) = 0, A_x = 1273 \text{ lb}$$

$$A = \sqrt{1273^2 + 33,940^2} = 34,000 \text{ lb}$$

$$\sum M_F = 0; \frac{4800}{\sqrt{2}}(10) - 0.9923D(2) = 0, D = 17,100 \text{ lb}$$

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$$\beta = \tan^{-1} \frac{60}{200} \\ = 16.70^\circ$$



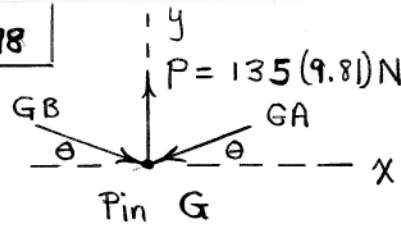
$$\sum M_O = 0 : 3000 - 250 \cos 15^\circ (150) - \\ 250 \sin 15^\circ (185) + C \cos \beta (85) + C \sin \beta (25) = 0$$

$$C = 510 \text{ N}$$

$$C = pA : 510 = p \left( \frac{\pi 45^2}{4} \right)$$

$$p = 0.321 \frac{\text{N}}{\text{mm}^2} \text{ or } \underline{321,000 \text{ Pa}} \\ \text{(gauge pressure)}$$

4/98

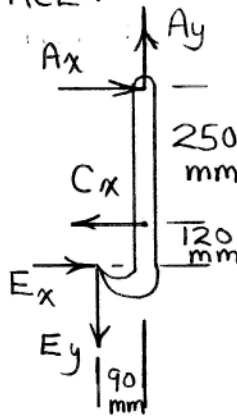


$$\theta = \cos^{-1} \frac{340}{350} = 13.73^\circ$$

$$\sum F_y = 0 : 135(9.81) - 2 G_A \sin 13.73^\circ = 0$$

$$G_A = G_B = 2790 \text{ N}$$

ACE :



$$A_x = 2790 \cos 13.73^\circ = 2710 \text{ N}$$

$$A_y = 2790 \sin 13.73^\circ = 662 \text{ N}$$

$$\sum F_y = 0 \Rightarrow E_y = 662 \text{ N}$$

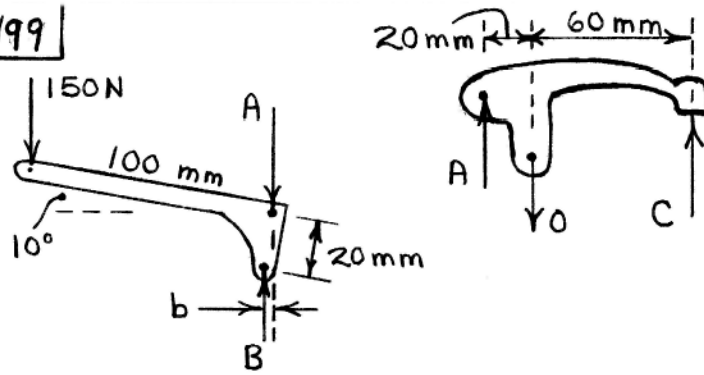
$$\sum M_C = 0 : 2710(250) - 662(90)$$

$$-E_x(120) = 0, \quad E_x = 5150 \text{ N}$$

$$E = \sqrt{E_x^2 + E_y^2} = 5190 \text{ N}$$

$$\text{or } \underline{5.19 \text{ kN}}$$

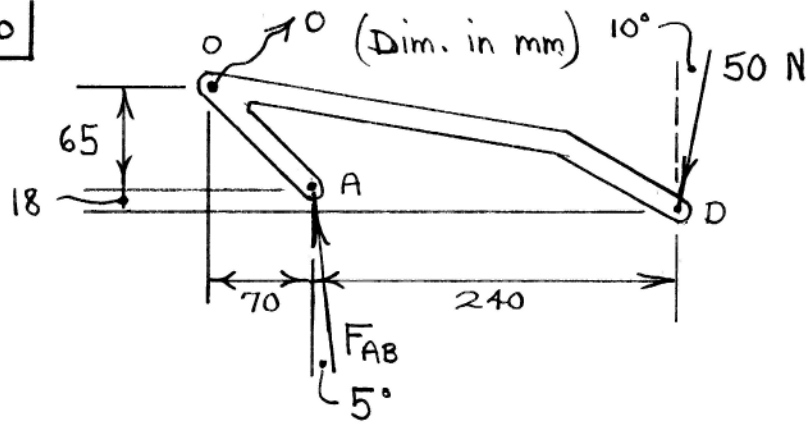
4/99



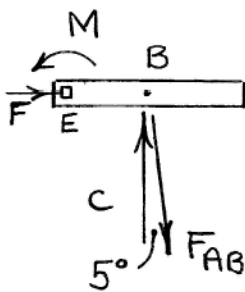
$$\text{(Handle)} \quad \sum M_B = 0: 150 [100 \cos 10^\circ - 20 \sin 10^\circ] - A [20 \sin 10^\circ] = 0, \quad A = 4103 \text{ N}$$

$$\text{(Jaw)} \quad \sum M_O = 0: 60C - 20(4103) = 0, \quad \underline{C = 1368 \text{ N}}$$

4/100

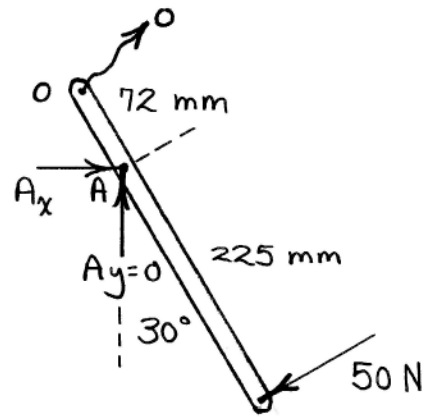
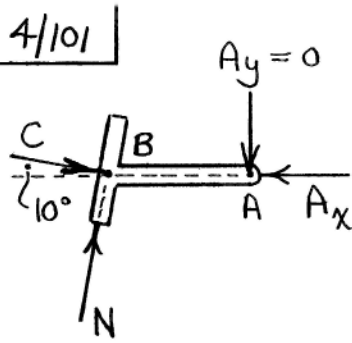


$$\begin{aligned} \sum M_O = 0: & F_{AB} [\cos 5^\circ (70) - \sin 5^\circ (65)] \\ & - 50 [\cos 10^\circ (310) + \sin 10^\circ (83)] = 0 \\ F_{AB} = & 250 \text{ N} \end{aligned}$$



$$\begin{aligned} M &= 0 \quad (\text{from } \sum M_B = 0) \\ +\uparrow \sum F = 0: & C - 250 \cos 5^\circ = 0 \\ C &= 249 \text{ N} \\ & \text{(a factor } \frac{C}{P} = 4.97) \end{aligned}$$

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$A_y = 0$  because AB is a three-force body.

Body OA:

$$\sum M_O = 0 : A_x (72 \cos 30^\circ) - 50(72 + 225) = 0$$

$$A_x = 238 \text{ N}$$

Body AB:

$$\sum F_x = 0 : C - 238 \cos 10^\circ = 0$$

$$C = 235 \text{ N}$$

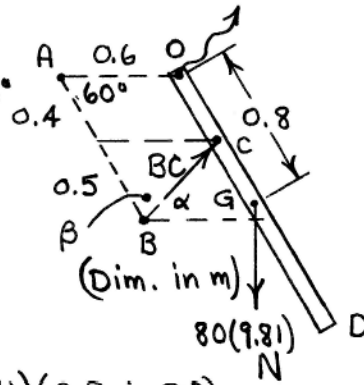
$$\left( \text{a factor } \frac{C}{P} = 4.69 \right)$$

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$$\overline{BC}^2 = 0.5^2 + 0.6^2 - 2(0.5)(0.6) \cos 60^\circ$$

$$BC = 0.557 \text{ m}$$

$$\frac{\sin \beta}{0.6} = \frac{\sin 60^\circ}{0.557} \quad \beta = 68.9^\circ$$
$$\alpha = 51.1^\circ$$

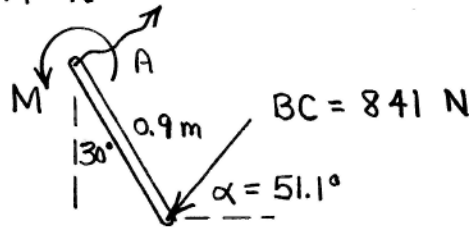


$$\sum M_o = 0 :$$

$$BC(0.4) \cos(51.1^\circ - 30^\circ) - 80(9.81)(0.8 \sin 30^\circ) = 0$$

$$BC = 841 \text{ N}$$

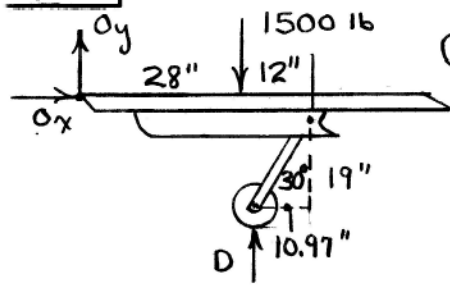
AB:



$$\sum M_A = 0 : M - 841(0.9) \cos(51.1^\circ - 30^\circ) = 0$$

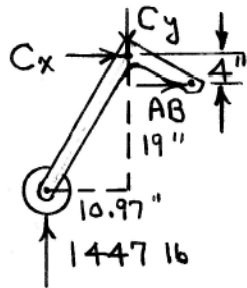
$$\underline{\underline{M = 706 \text{ N}\cdot\text{m}}}$$

4/103 | Entire ramp + mechanism :



$$\begin{aligned} \sum M_O = 0: & D(28+12-10.97) \\ & -1500(28) = 0 \\ & D = 1447 \text{ lb} \end{aligned}$$

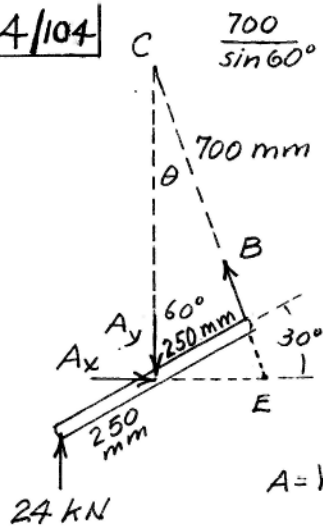
Crank BCD :



$$\begin{aligned} \sum M_C = 0: & -1447(10.97) \\ & + AB(4) = 0, \quad \underline{AB = 3970 \text{ lb}} \\ & \text{(Cylinder is in compression)} \end{aligned}$$



4/104



$$\frac{700}{\sin 60^\circ} = \frac{250}{\sin \theta}, \quad \theta = \sin^{-1} 0.3093 = 18.02^\circ$$

$$\cos \theta = 0.9510, \quad \tan \theta = 0.3252$$

$$\bar{AC} = 700(0.9510) + 250(0.5) = 791 \text{ mm}$$

$$\sum M_C = 0; \quad 24(250)(0.866) - 791 A_x = 0$$

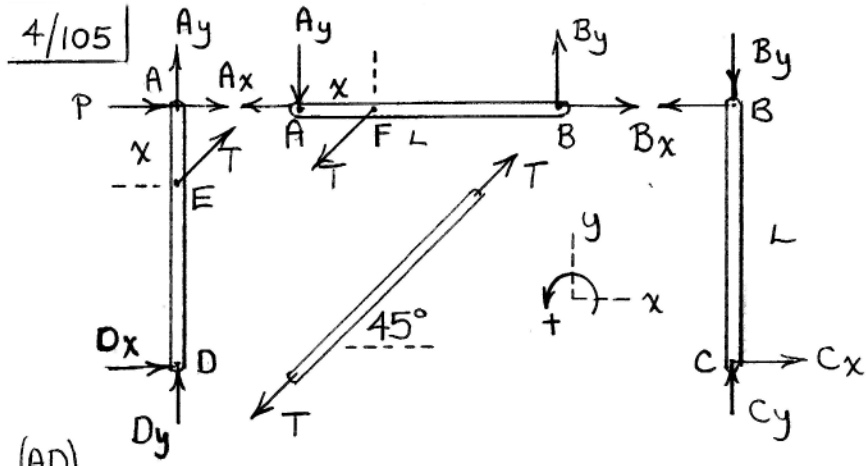
$$A_x = 6.57 \text{ kN}$$

$$\bar{AE} = \bar{AC} \tan \theta = 791(0.3252) = 257 \text{ mm}$$

$$\sum M_E = 0; \quad A_y(257) - 24(250 \times 0.866 + 257) = 0$$

$$A_y = 44.2 \text{ kN}$$

$$A = \sqrt{6.57^2 + 44.2^2} = \underline{44.7 \text{ kN}}$$



(AD)

$$\sum F_x = 0 : D_x + A_x + P + T \frac{\sqrt{2}}{2} = 0 \quad (1)$$

$$\sum F_y = 0 : D_y + A_y + T \frac{\sqrt{2}}{2} = 0 \quad (2)$$

$$\sum M_A = 0 : D_x(L) + T \frac{\sqrt{2}}{2} (x) = 0 \quad (3)$$

(AB)

$$\sum F_x = 0 : -A_x + B_x - T \frac{\sqrt{2}}{2} = 0 \quad (4)$$

$$\sum F_y = 0 : -A_y + B_y - T \frac{\sqrt{2}}{2} = 0 \quad (5)$$

$$\sum M_A = 0 : B_y(L) - T \frac{\sqrt{2}}{2} x = 0 \quad (6)$$

(BC)

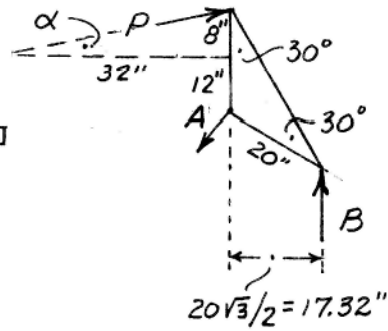
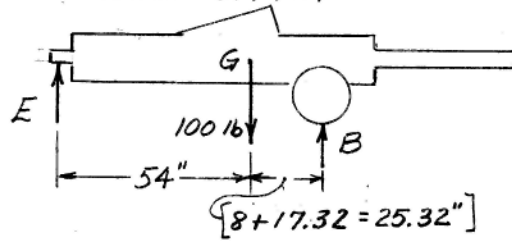
$$\sum F_x = 0 : -B_x + C_x = 0 \quad (7)$$

$$\sum F_y = 0 : -B_y + C_y = 0 \quad (8)$$

$$\sum M_C = 0 : B_x(L) = 0 \quad (9)$$

Solve Eqs. (1)-(9) for  $T = \frac{\sqrt{2} PL}{x} \quad (x \neq 0)$

4/106  $\tan \alpha = \frac{8}{32}, \alpha = 14.04^\circ$   
 $\cos \alpha = 0.9701$



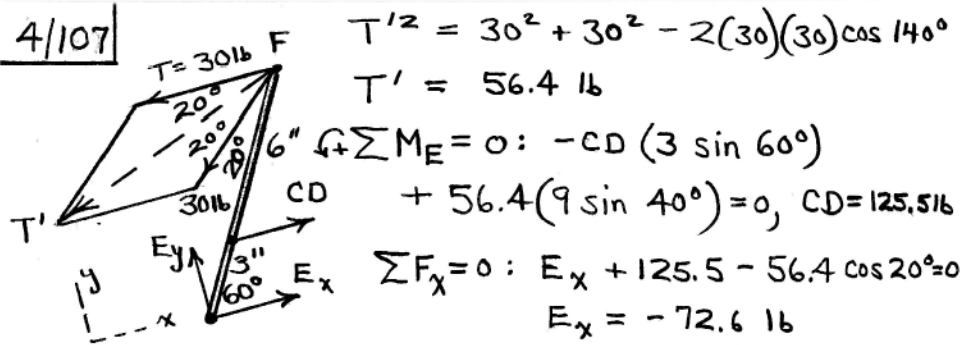
$$\sum M_E = 0; (54 + 25.32) B - 54(100) = 0$$

$$B = 68.1 \text{ lb}$$

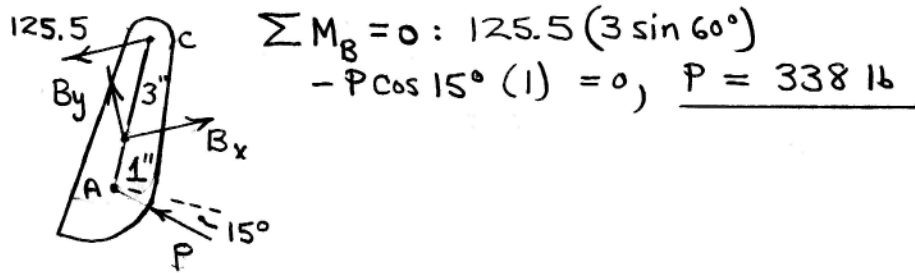
$$\sum M_A = 0; 0.9701 P(20) - 68.1(17.32) = 0$$

$$P = 60.8 \text{ lb}$$

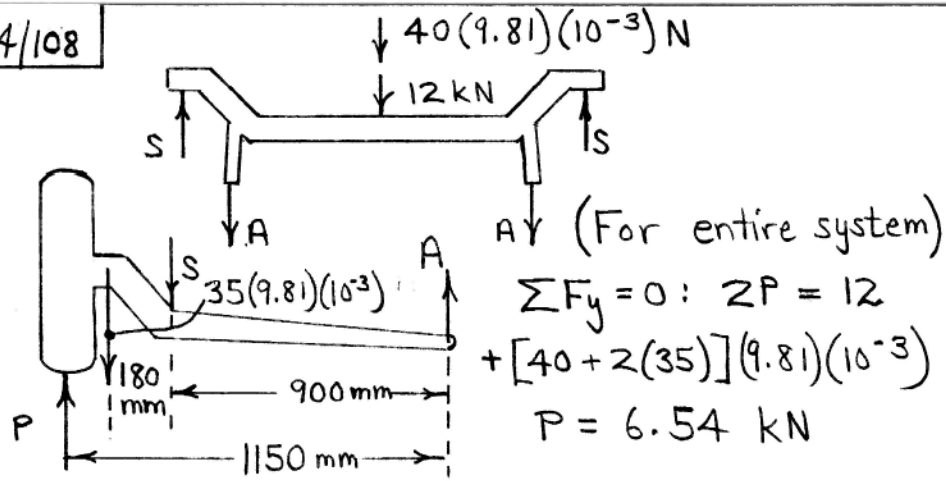
4/107



$\sum F_y = 0: E_y - 56.4 \sin 20^\circ = 0, E_y = 19.28 \text{ lb}$   
 $E = \sqrt{E_x^2 + E_y^2} = 75.1 \text{ lb}$



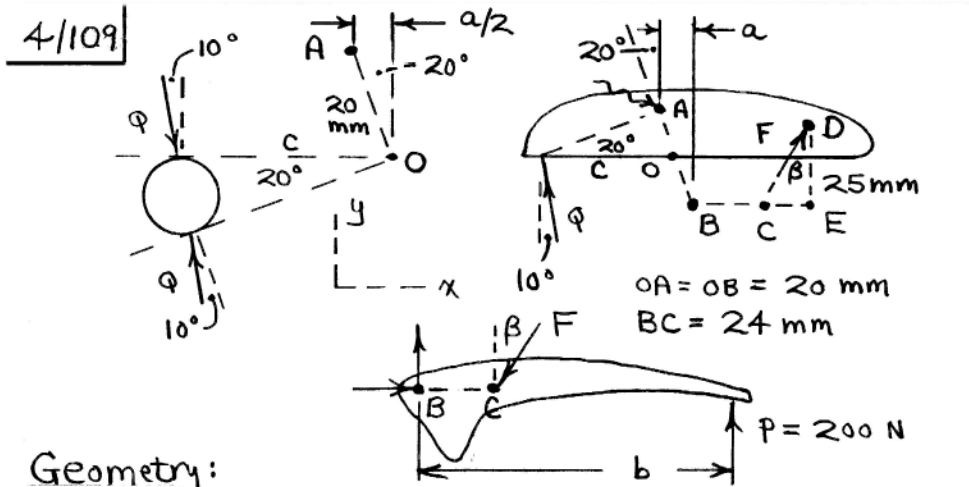
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(Wheel Assembly)  $\sum M_s = 0: 900 A$

$$-6.54(250) + 35(9.81)(10^{-3})(180) = 0$$

$$\underline{A = 1.748 \text{ kN}}$$



Geometry:

$$\left\{ \begin{aligned} c &= \frac{7.5}{\tan 10^\circ} = 42.5 \text{ mm}, & \frac{a}{2} &= 20 \sin 20^\circ = 6.84 \text{ mm} \\ a &= 13.68 \text{ mm}, & b &= 190 - 13.68 = 176.3 \text{ mm} \\ \overline{CE} &= 50 - 13.68 - 24 = 12.32 \text{ mm}, & \beta &= \tan^{-1} \frac{12.32}{25} = 26.2^\circ \end{aligned} \right.$$

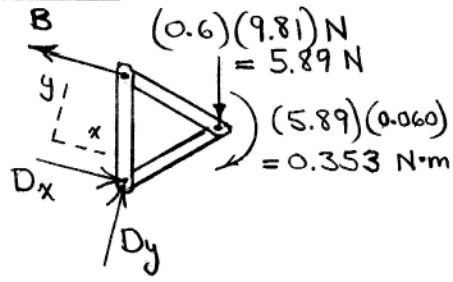
Lower handle:

$$\begin{aligned} \sum M_B = 0: & 200(176.3) - F \cos \beta (24) = 0 \\ & F \cos \beta = 1469 \text{ N}, \quad F \sin \beta = 724 \text{ N} \end{aligned}$$

Upper jaw: (consider F to act at C)

$$\begin{aligned} \sum M_A = 0: & 1469(24 + 13.68) + 724(40 \cos 20^\circ) \\ & - Q \cos 10^\circ (42.5 - 6.84) - Q \sin 10^\circ (20 \cos 20^\circ) = 0 \\ & Q = \underline{2150 \text{ N}} \quad \text{or} \quad \underline{2.15 \text{ kN}} \end{aligned}$$

4/110 | Note that AB is a two-force member.



$$\sum F_y = 0: D_y - 5.89 \cos 15^\circ = 0$$

$$D_y = 5.69 \text{ N}$$

$$\sum M_D = 0: B(0.065) \cos 15^\circ + 5.89(0.060) - 0.353 = 0$$

$$B = 11.25 \text{ N}$$

$$\sum F_x = 0: -11.25 + D_x + 5.89 \sin 15^\circ = 0$$

$$D_x = 9.73 \text{ N}$$



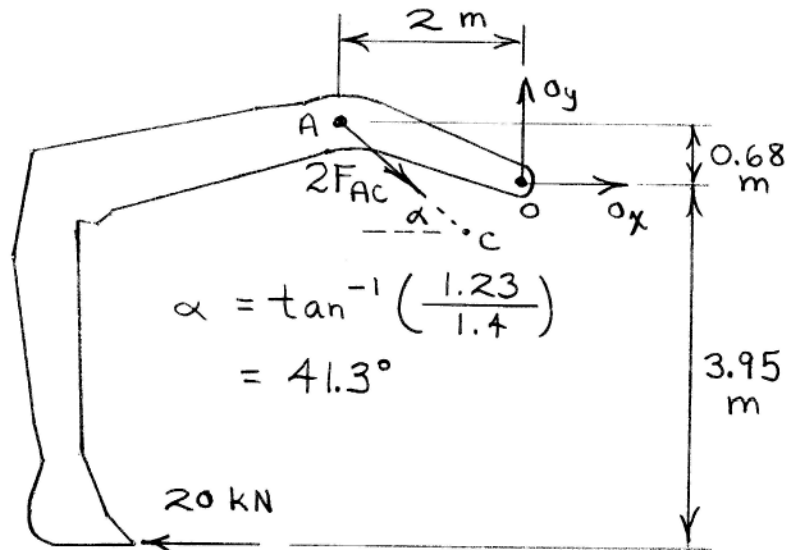
$$\alpha = \tan^{-1} \left( \frac{65 \cos 15^\circ}{225 + 65 \sin 15^\circ} \right)$$

$$= 14.55^\circ$$

$$\sum M_F = 0: (F_s \sin \alpha)(0.225) - 5.69(0.450) = 0$$

$$F_s = \underline{45.2 \text{ N}}$$

4/III



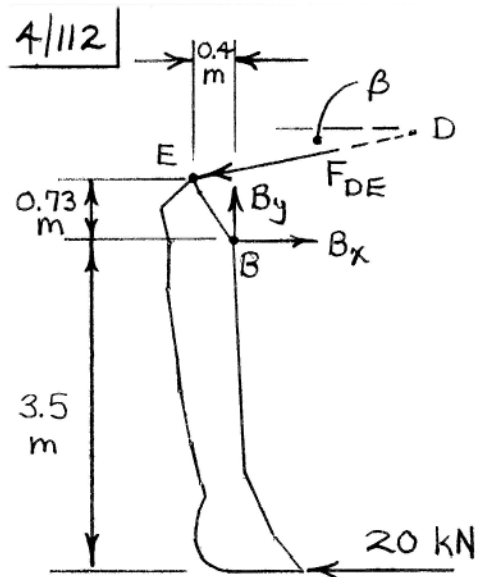
$$\sum M_o = 0 : -20000(3.95) - 2F_{AC} \cos \alpha (0.68) + 2F_{AC} \sin \alpha (2) = 0$$

$$F_{AC} = 48800 \text{ N or } \underline{48.8 \text{ kN}}$$

$$F_{AC} = pA : 48800 = p \left( \pi \frac{0.095^2}{4} \right)$$

$$p = 6.89 (10^6) \text{ Pa or } \underline{6.89 \text{ MPa}}$$





$$\beta = \tan^{-1} \left( \frac{0.5}{2.5} \right)$$

$$= 11.31^\circ$$

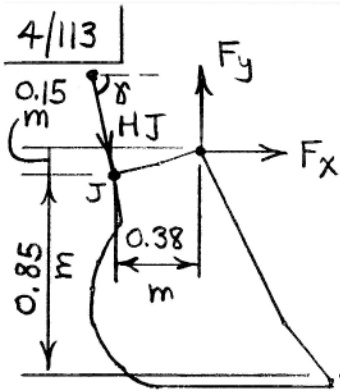
$$\sum M_B = 0: -20000(3.5) + F_{DE} \cos \beta (0.73) + F_{DE} \sin \beta (0.4) = 0$$

$$F_{DE} = 88100 \text{ N or } \underline{88.1 \text{ kN}}$$

$$F_{DE} = pA: 88100 = p \left( \pi \frac{0.105^2}{4} \right)$$

$$p = 10.18(10^6) \text{ Pa or } \underline{10.18 \text{ MPa}}$$

4/113



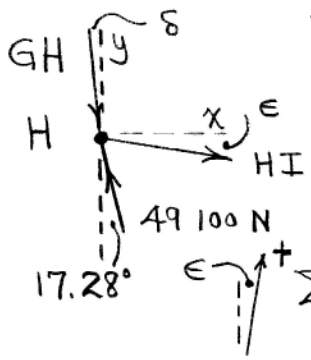
$$\delta = \tan^{-1} \left( \frac{0.45}{0.14} \right) = 72.7^\circ$$

$$\begin{aligned} \sum M_F = 0: & -20000(1) \\ & + HJ \sin \delta (0.38) \\ & + HJ \cos \delta (0.15) = 0 \end{aligned}$$

$$HJ = 49100 \text{ N C}$$

Joint H:

$$\begin{cases} \delta = \tan^{-1} \left( \frac{0.18}{1.75} \right) = 5.87^\circ \\ \epsilon = \tan^{-1} \left( \frac{0.09}{0.47} \right) = 10.84^\circ \end{cases}$$



$$\sum F = 0:$$

$$49100 \cos(17.28^\circ + 10.84^\circ) - GH \cos(5.87^\circ + 10.84^\circ) = 0$$

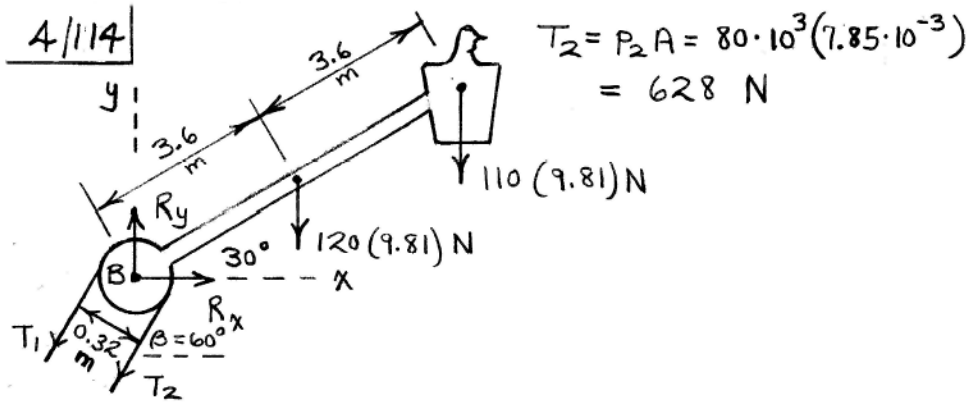
$$GH = 45200 \text{ N or } 45.2 \text{ kN}$$

$$GH = pA : 45200 = p \left( \pi \frac{0.095^2}{4} \right)$$

$$p = 6.38(10^6) \text{ Pa or } \underline{6.38 \text{ MPa}}$$



ارائه دهنده خدمات متنوع مهندسی



$$\sum M_B = 0 : 110 (9.81) (7.2 \cos 30^\circ) + 120 (9.81) (3.6 \cos 30^\circ) + 628 (0.160) - T_1 (0.160) = 0, \quad T_1 = 65.6 (10^3) \text{ N}$$

$$p = T/A = 65.6 (10^3) / (7.85 \cdot 10^{-3}) = 8360 (10^3) \text{ Pa}$$

$$\text{or } p = 8360 \text{ kPa}$$

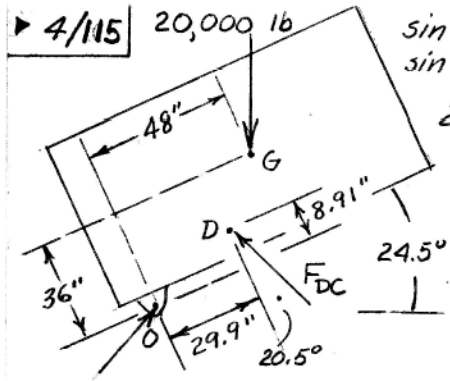
$$\sum F_x = 0 : R_x - [65.6 (10^3) + 628] \cos 60^\circ = 0, \quad R_x = 33.1 \text{ kN}$$

$$\sum F_y = 0 : R_y - [65.6 (10^3) + 628] \sin 60^\circ - 230 (9.81) = 0$$

$$R_y = 59.6 \text{ kN}$$

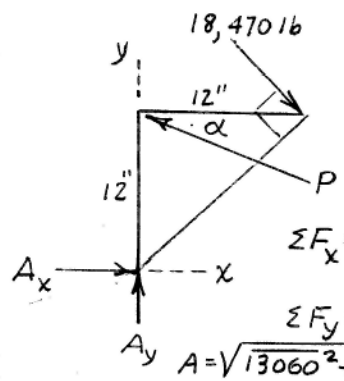
$$R = \sqrt{R_x^2 + R_y^2} = 68.2 \text{ kN}$$

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$\sin 24.5^\circ = 0.4147, \cos 24.5^\circ = 0.9100$   
 $\sin 20.5^\circ = 0.3502, \cos 20.5^\circ = 0.9367$

$\sum M_O = 0; 20,000(0.9100)48$   
 $-20,000(0.4147)(36)$   
 $-F_{DC}(0.9367)(29.9) - F_{DC}(0.3502)(8.91)$   
 $= 0$   
 $F_{DC} = 18,470 \text{ lb}$



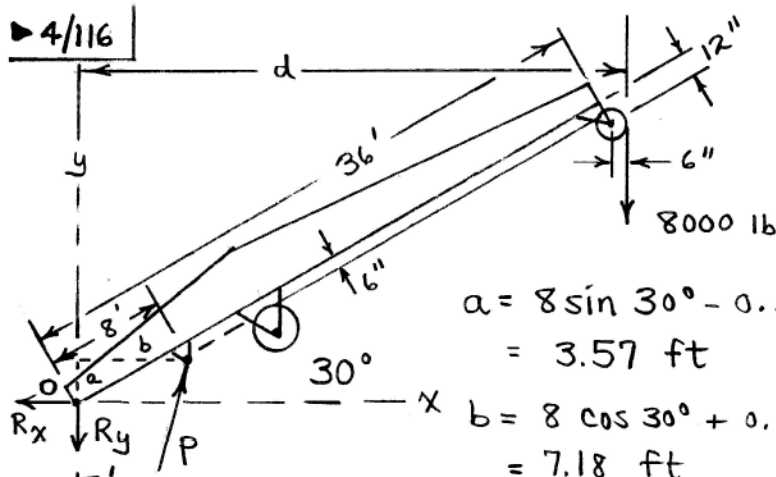
$\alpha = \tan^{-1} \frac{12}{48} = 14.04^\circ$   
 $\sin \alpha = 0.2425, \cos \alpha = 0.9701$   
 $\sum M_A = 0; 0.9701P(12) - 18,470(12\sqrt{2}) = 0$   
 $P = 26,900 \text{ lb}$

$\sum F_x = 0; A_x + 18,470/\sqrt{2} - 26,900(0.9701) = 0$   
 $A_x = 13,060 \text{ lb}$

$\sum F_y = 0; A_y + 26,900(0.2425) - 18,470/\sqrt{2} = 0$   
 $A_y = 6,530 \text{ lb}$   
 $A = \sqrt{13060^2 + 6530^2} = 14,600 \text{ lb}$



ارائه دهنده خدمات متنوع مهندسی



$$a = 8 \sin 30^\circ - 0.5 \cos 30^\circ = 3.57 \text{ ft}$$

$$b = 8 \cos 30^\circ + 0.5 \sin 30^\circ = 7.18 \text{ ft}$$

$$d = 36 \cos 30^\circ + 1 \sin 30^\circ + 0.5 = 32.2 \text{ ft}$$

$$\tan \alpha = \frac{7.18 - 4}{8 + 3.57}, \quad \alpha = 15.36^\circ$$

$$\overline{FE} = 4 / \tan \alpha = 14.56 \text{ ft}$$

$$\sum M_E = 0: 8000 (32.2) - (14.56 + 8) R_x = 0$$

$$R_x = 11,410 \text{ lb}$$

$$\sum M_O = 0: 8000 (32.2) - P \cos (15.36^\circ + 30^\circ) 8 - P \sin (15.36^\circ + 30^\circ) (0.5) = 0, \quad P = 43,100 \text{ lb}$$

$$\sum F_y = 0: 43,100 \cos 15.36^\circ - 8000 - R_y = 0$$

$$R_y = 33,500 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = 35,400 \text{ lb}$$

4/117 | Frame as a whole:

$$\theta = \tan^{-1} \frac{5 \sin 50^\circ}{7 + 5 \cos 50^\circ} = 20.6^\circ$$

$$d = 7 \sin 20.6^\circ = 2.46 \text{ ft}$$

$$\beta + 20.6^\circ = 50^\circ, \beta = 29.4^\circ$$

$$\sum M_x = 0 : (5 \cos 50^\circ)(1200) - 2.46 T = 0, \underline{T = 1569 \text{ lb}}$$

$$\sum F_{(C-D)-B} = 0 : R - 1200 \cos 40^\circ - 1569 \cos 29.4^\circ = 0$$

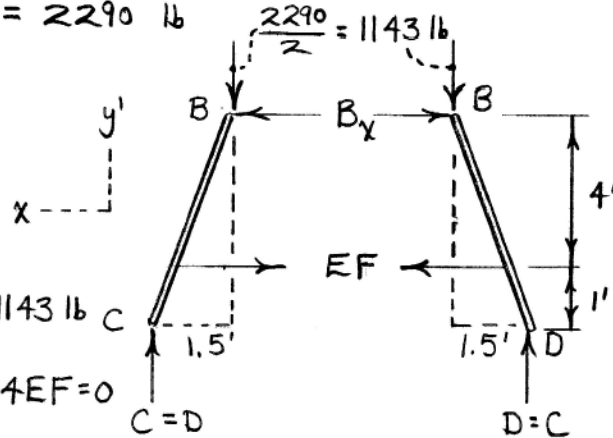
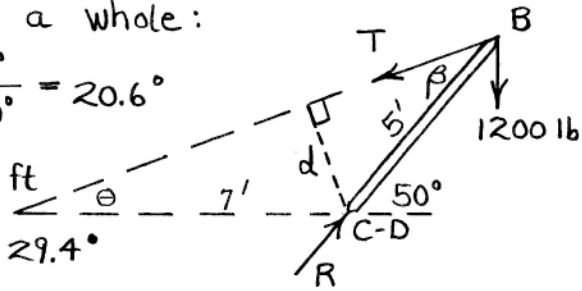
$$R = 2290 \text{ lb}$$

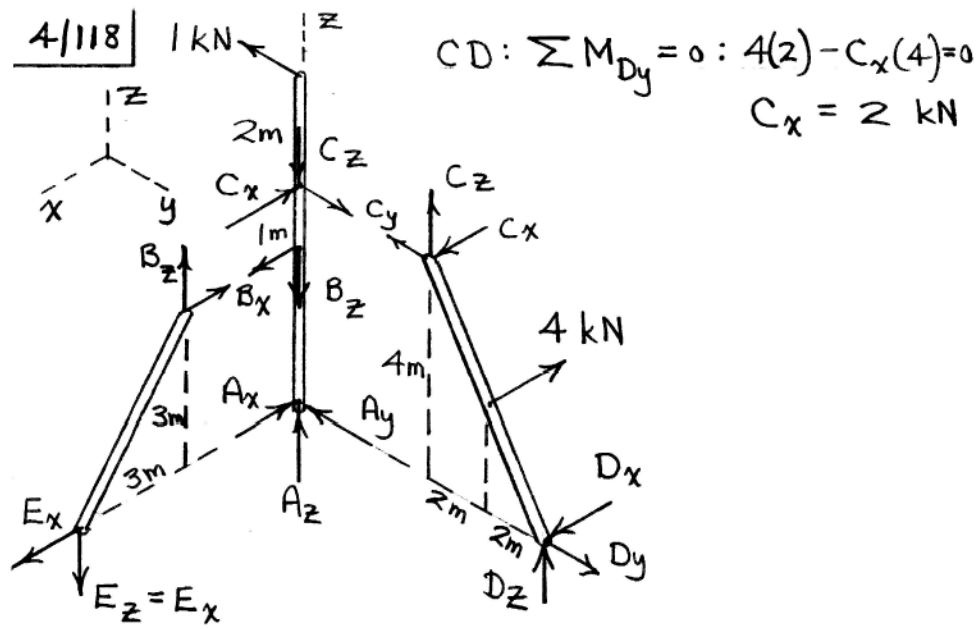
Plane of frame:

$$\sum F_{y'} = 0 : C = D = 1143 \text{ lb}$$

$$\sum M_B = 0 : 1143(1.5) - 4EF = 0$$

$$\underline{EF = 429 \text{ lb}}$$





ABC:  $\sum M_{By} = 0: 2(1) - A_x(3) = 0, \quad A_x = 0.667 \text{ kN}$

$\sum F_x = 0: B_x - 0.667 - 2 = 0, \quad B_x = 2.67 \text{ kN}$

EB:  $B_z = E_z = B_x = E_x = 2.67 \text{ kN}$

ABC:  $\sum M_{Ax} = 0: 1(6) - C_y(4) = 0, \quad C_y = 1.50 \text{ kN}$

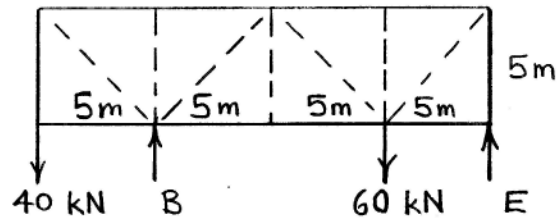
CD:  $\sum M_{Dx} = 0: C_z = C_y = 1.50 \text{ kN}$

ABC:  $\sum F_z = 0: A_z - 2.67 - 1.50 = 0, \quad A_z = 4.17 \text{ kN}$

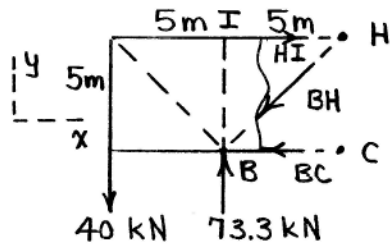
$\sum F_y = 0: A_y + 1 - 1.50 = 0, \quad A_y = 0.50 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \underline{4.25 \text{ kN}}$

4/119



$$\sum M_E = 0 : 40(20) + 60(5) - 15B = 0 \quad , \quad B = 73.3 \text{ kN}$$



$$\sum F_y = 0 : -\frac{\sqrt{2}}{2} BH + 73.3 - 40 = 0 \quad ,$$

$$\underline{BH = 47.1 \text{ kN C}}$$

$$\sum M_B = 0 : HI(5) - 40(5) = 0 \quad ,$$

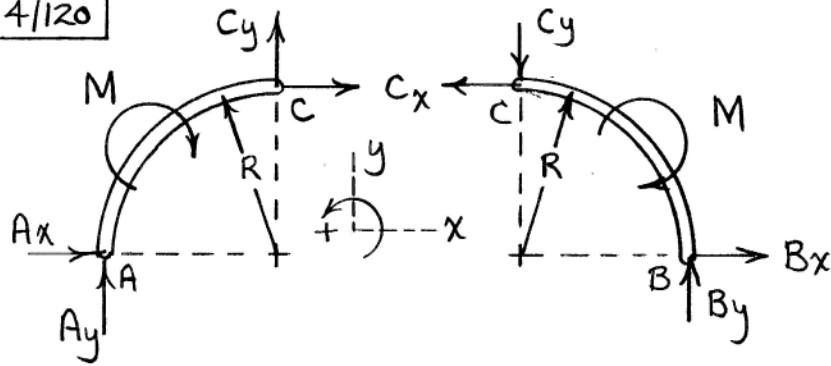
$$\underline{HI = 40 \text{ kN T}}$$

$$\sum F_x = 0 : 40 - 47.1 \frac{\sqrt{2}}{2} - BC = 0 \quad ,$$

$$\underline{BC = 6.67 \text{ kN C}}$$



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Left member :

$$\begin{cases} \sum F_x = 0 : A_x + C_x = 0 & (1) \\ \sum F_y = 0 : A_y + C_y = 0 & (2) \\ \sum M_A = 0 : -M - C_x(R) + C_y(R) = 0 & (3) \end{cases}$$

Right member :

$$\sum F_x = 0 : -C_x + B_x = 0 \quad (4)$$

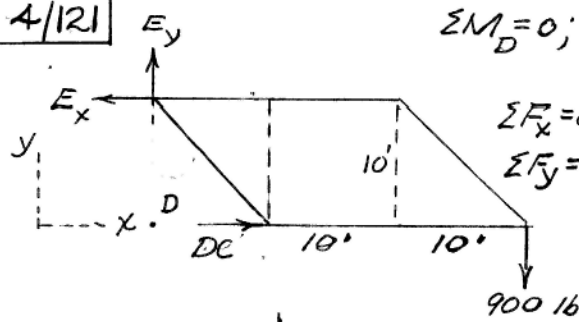
$$\sum F_y = 0 : -C_y + B_y = 0 \quad (5)$$

$$\sum M_B = 0 : -M + C_x(R) + C_y(R) = 0 \quad (6)$$

Solution of Eqs. (1)-(6) :

$$\begin{cases} C_y = B_y = \frac{M}{R}, & A_y = -\frac{M}{R} \\ A_x = B_x = C_x = 0 \end{cases}$$

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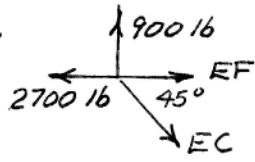
$$\sum M_D = 0; 900(30) - 10E_x = 0$$

$$E_x = 2700 \text{ lb}$$

$$\sum F_x = 0; DC = 2700 \text{ lb C}$$

$$\sum F_y = 0; E_y = 900 \text{ lb}$$

Joint E;



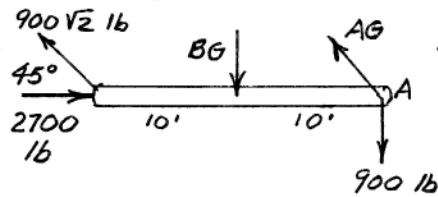
$$\sum F_y = 0; EC/\sqrt{2} - 900 = 0$$

$$EC = 900\sqrt{2} \text{ lb}$$

$$\sum F_x = 0; EF + \frac{900\sqrt{2}}{\sqrt{2}} - 2700 = 0$$

$$EF = 1800 \text{ lb T}$$

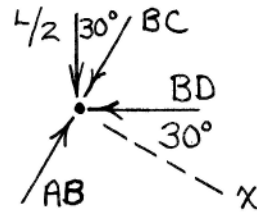
Joint F;  $FG = EF, FC = 0$



$$\sum M_A = 0; 10BG - \frac{900\sqrt{2}}{\sqrt{2}} 20 = 0$$

$$BG = 1800 \text{ lb C}$$

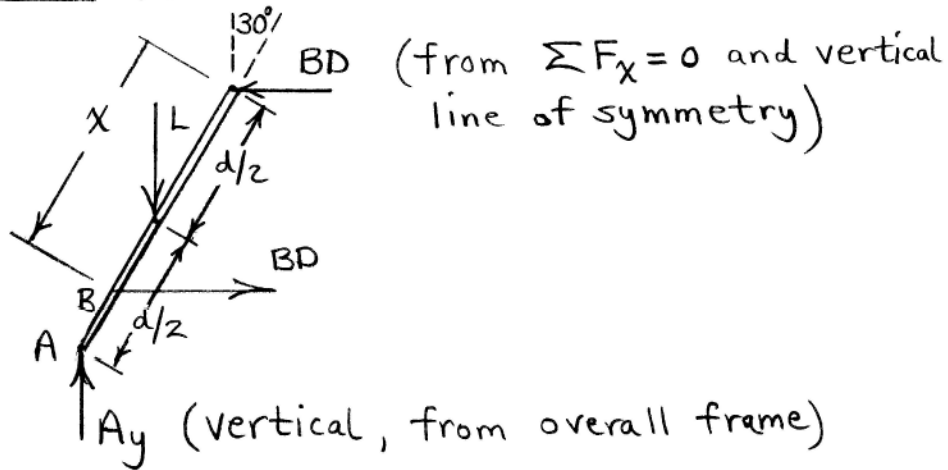
4/122 | Joint B :



$$\sum F_x = 0: \frac{L}{2} \left(\frac{1}{2}\right) - BD \frac{\sqrt{3}}{2} = 0$$

$$BD = \frac{\sqrt{3}}{6} L C, \text{ independent of } x.$$

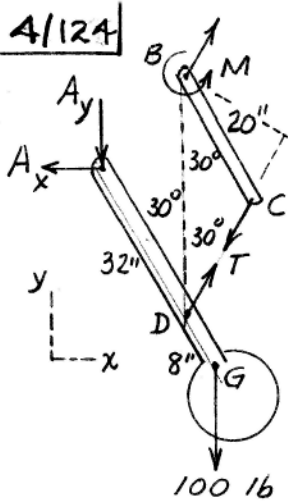
4/123 | Member ABC:



$$\begin{aligned} \curvearrow + \Sigma M_A = 0: & \quad BD(d \cos 30^\circ) - BD(d-x) \cos 30^\circ \\ & \quad - L\left(\frac{d}{2} \sin 30^\circ\right) = 0 \\ & \quad BD = \frac{Ld}{2x} \tan 30^\circ = \frac{0.289Ld}{x} \end{aligned}$$

( $x$  cannot be zero)

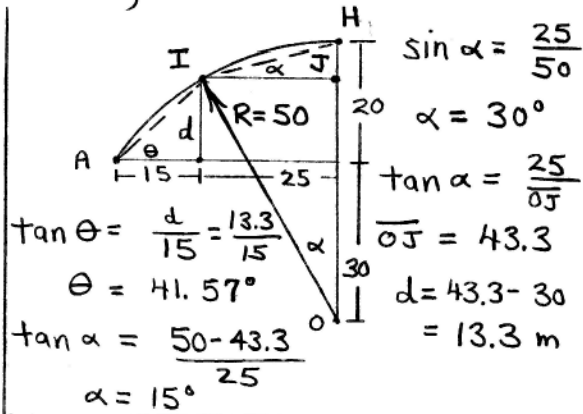
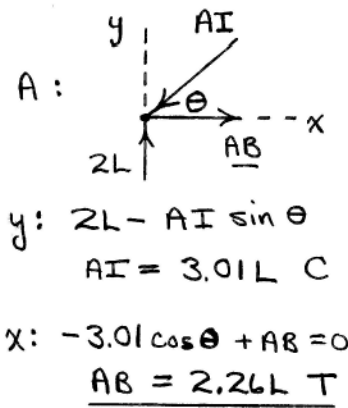
4/124



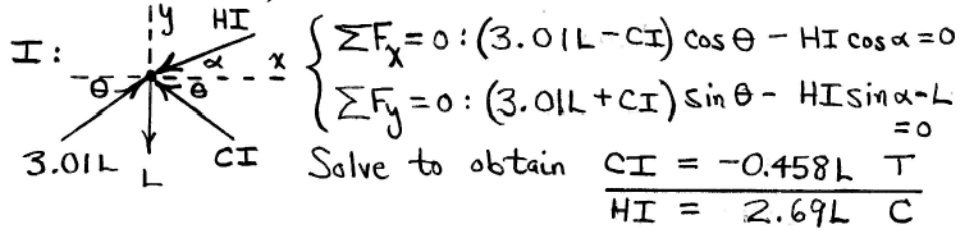
$$\begin{aligned} ADG; \sum M_A = 0; & 100(40 \sin 30^\circ) \\ & - T(32 \cos 30^\circ) = 0 \\ & T = 72.2 \text{ lb} \end{aligned}$$

$$\begin{aligned} BC; \sum M_B = 0; & 72.2(20 \cos 30^\circ) - M = 0 \\ & \underline{M = 1250 \text{ lb-in.}} \end{aligned}$$

4/125 | From whole truss,  $A = F = 2L$

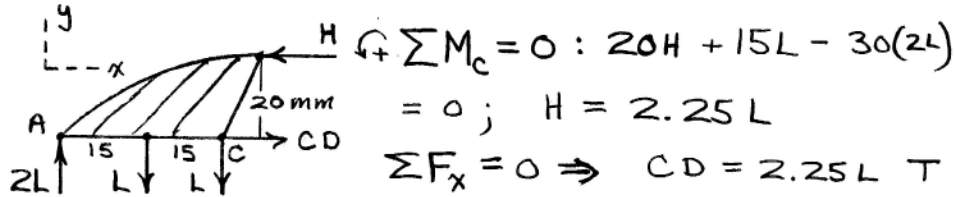


From joint B,  $BI = L \text{ T}$



4/126 | From whole structure,  $A = F = 2L$

Half of structure:



$$\sum M_c = 0 : 20H + 15L - 30(2L)$$

$$= 0 ; H = 2.25L$$

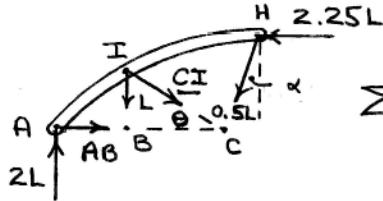
$$\sum F_x = 0 \Rightarrow CD = 2.25L T$$

From joint B,  $BI = LT$

Member AIH:

From previous solution

$$\theta = 41.57, \alpha = \tan^{-1}\left(\frac{10}{20}\right) = 26.57^\circ$$



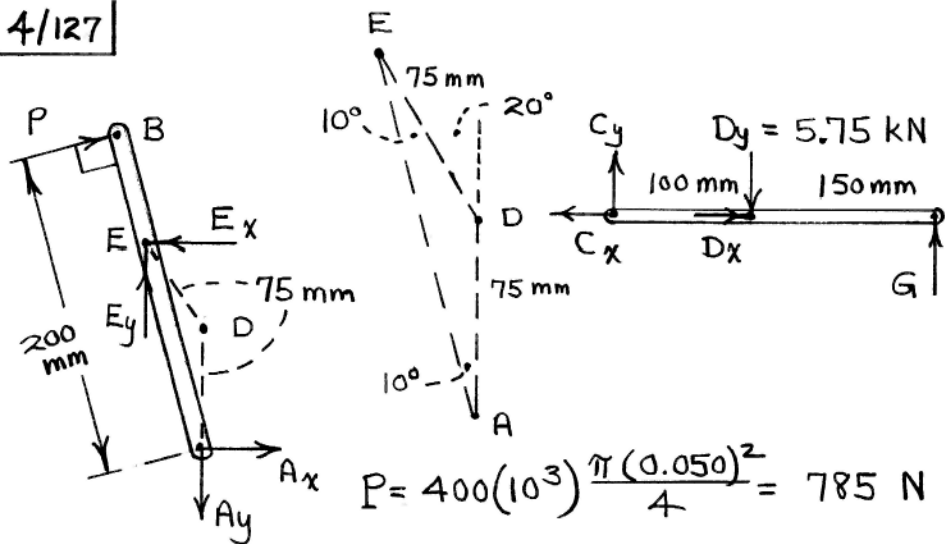
$$\sum F_y = 0 : 2L - L - CI \sin \theta - 0.5L \cos \alpha = 0, \quad \underline{CI = 0.833L T}$$

$$\sum F_x = 0 : AB + 0.833L \cos \theta - 0.5L \sin \alpha - 2.25L = 0$$

$$\underline{AB = 1.850L T}$$

Problem not solvable without CH data.

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$$AB: \sum M_A = 0: 785(200) + E_y(75 \sin 20^\circ) - E_x(75 + 75 \cos 20^\circ) = 0$$

But ED is a two-force member so that

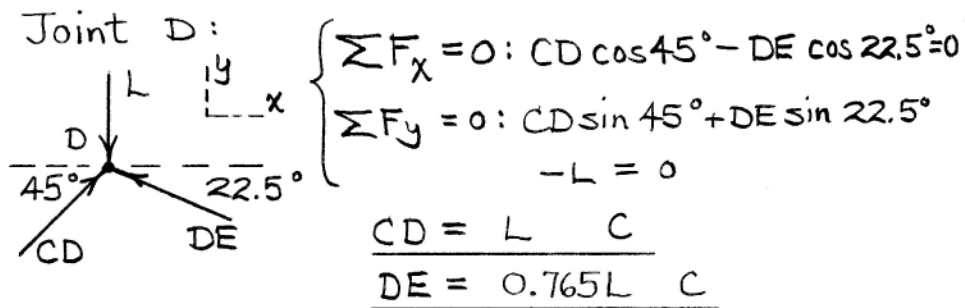
$$E_x = E_y \tan 20^\circ = 0.364 E_y$$

Solve moment equation:  $E_y = 5.75 \text{ kN}$

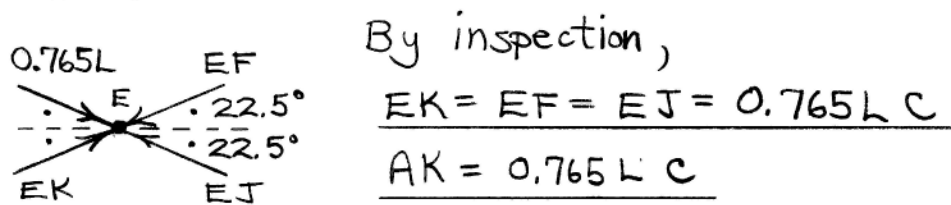
$$CG: \sum M_C = 0: 250 G - 100(5.75) = 0, \quad \underline{G = 2.30 \text{ kN}}$$



4/128 | By inspection of joints B, K, and C,  
 $BK = CK = CE = 0$ .

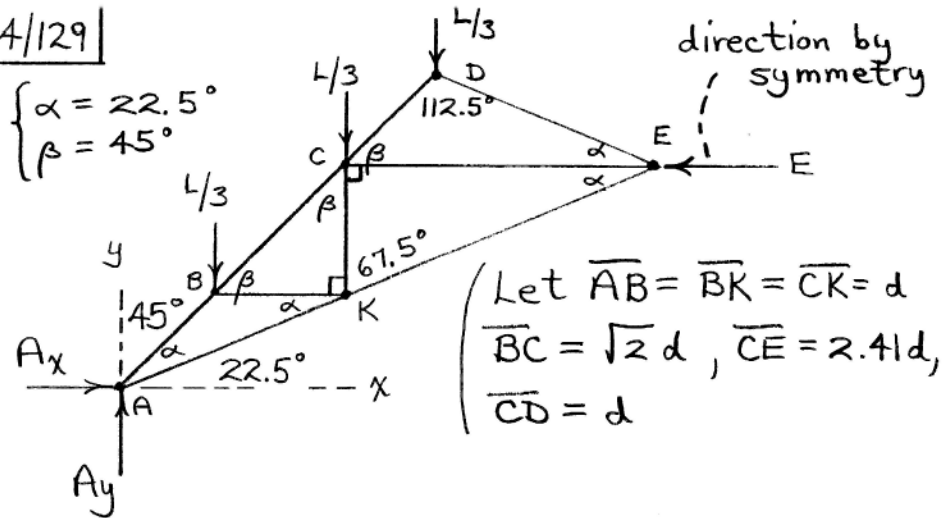


Then  $AB = BC = L \cdot C$



By symmetry,  $EG = GJ = HJ = 0$ ,  
 $FG = GH = HI = L \cdot C$ ,  $IJ = 0.765L \cdot C$ .

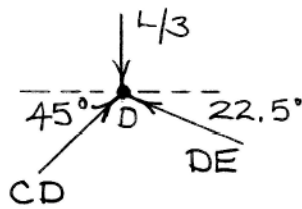
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$$\sum M_A = 0: E(d + \sqrt{2}d) \sin 45^\circ - \frac{L}{3}(d \cos 45^\circ) - \frac{L}{3}(d + \sqrt{2}d) \cos 45^\circ - \frac{L}{3}(d + \sqrt{2}d + d) \cos 45^\circ = 0$$

$$E = 0.943L$$

Joint D:



$$\sum F_x = 0: CD \cos 45^\circ - DE \cos 22.5^\circ = 0$$

$$\sum F_y = 0: CD \sin 45^\circ + DE \sin 22.5^\circ - \frac{L}{3} = 0$$

$$-\frac{L}{3} = 0$$

$$DE = 0.255L \quad C$$

$$CD = 0.333L \quad C$$

$0.943L + CE - 0.943L = 0$   
 $\sum F_y = 0 \Rightarrow \underline{EK = 0.255L C}$   
 So  $\underline{CE = +0.471L C}$

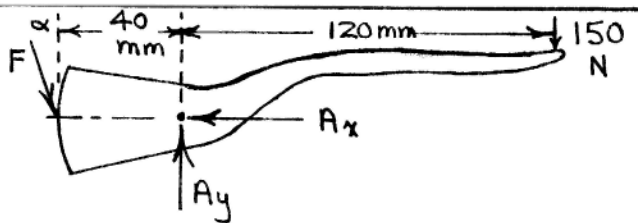
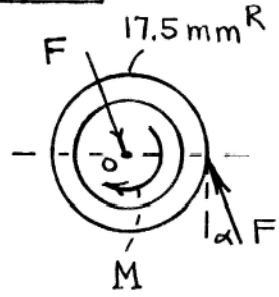
Joint C:  $\sum F_x = 0: BC \frac{\sqrt{2}}{2} - 0.333L \frac{\sqrt{2}}{2} - 0.471L = 0, \underline{BC = L C}$   
 $\sum F_y = 0: -\frac{L}{3} - 0.333L \frac{\sqrt{2}}{2} + L \frac{\sqrt{2}}{2} + CK = 0, \underline{CK = 0.1381L T}$

Joint K:  $\sum F = 0: -BK \sin 22.5^\circ + 0.1381L \cos 22.5^\circ = 0$   
 $BK = \underline{0.333L C}$   
 $\sum F_y = 0: 0.1381L - 0.255L \sin 22.5^\circ + AK \sin 22.5^\circ = 0, \underline{AK = 0.1057L T}$

Joint B:  $\sum F_x = 0: AB \frac{\sqrt{2}}{2} - L \frac{\sqrt{2}}{2} - 0.333L = 0, \underline{AB = 1.471L C}$

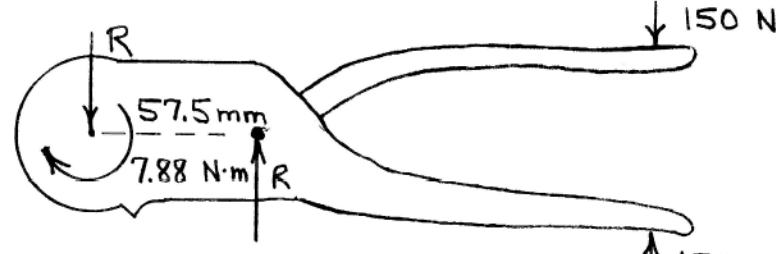
(Right-half members by symmetry)

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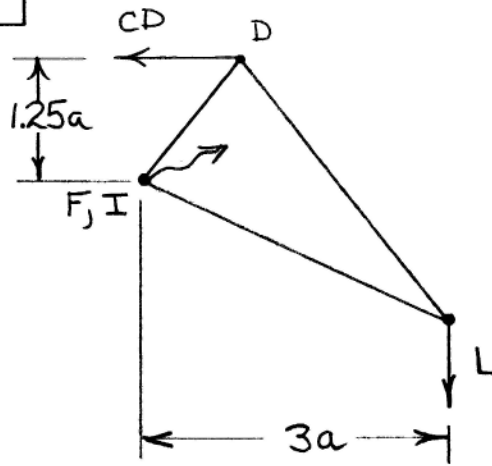
$$\sum M_A = 0: 40 F \cos \alpha = 120 (150)$$
$$F \cos \alpha = 450 \text{ N}$$

$$\sum M_o = 0: 450 (0.0175) = M = 7.88 \text{ N}\cdot\text{m}$$



$$\sum M = 0: 57.5 (10^{-3}) R = 7.88$$
$$R = 137.0 \text{ N}$$

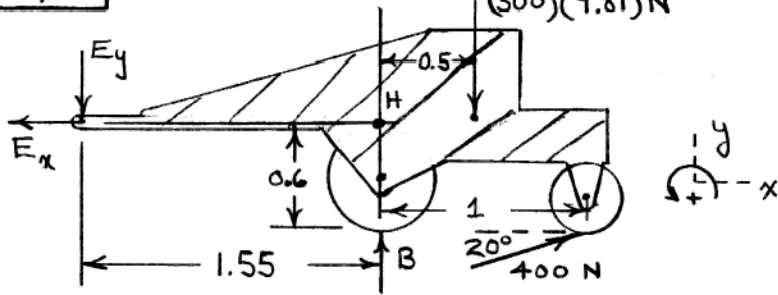
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$$\curvearrow + \sum M_F = 0: CD(1.25a) - L(3a) = 0$$

$$\underline{CD = 2.4L \text{ T}}$$

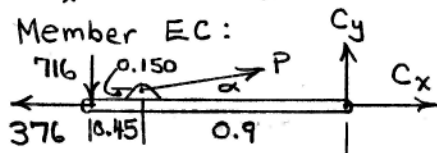
4/132 Entire machine :  $(300)(9.81)N$



$$\sum M_H = 0 : E_y(1.55) - (300)(9.81)(0.5) + (400 \sin 20^\circ)(1) + (400 \cos 20^\circ)(0.6) = 0, \quad E_y = 716 \text{ N}$$

$$\sum F_x = 0 : -E_x + 400 \cos 20^\circ = 0, \quad E_x = 376 \text{ N}$$

Member EC:

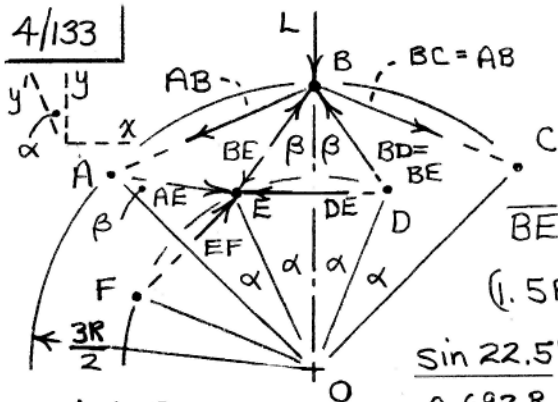


$$\alpha = \tan^{-1} \left( \frac{230}{1300} \right) = 10.03^\circ$$

$$\left\{ \begin{array}{l} \sum M_C = 0 \text{ yields } P = 3170 \text{ N} \\ \sum F_x = 0 \text{ yields } C_x = 2750 \text{ N} \\ \sum F_y = 0 \text{ yields } C_y = 162.9 \text{ N} \end{array} \right.$$

$$C = \sqrt{C_x^2 + C_y^2} = 2750 \text{ N}$$

4/133



$$\begin{cases} \alpha = 22.5^\circ \\ \angle ABO = 67.5^\circ \end{cases}$$

$$\overline{BE}^2 = R^2 + (1.5R)^2 - 2R \times (1.5R) \cos 22.5^\circ, \overline{BE} = 0.692R$$

$$\frac{\sin 22.5^\circ}{0.692R} = \frac{\sin \beta}{R}, \beta = 33.6^\circ$$

Joint B:

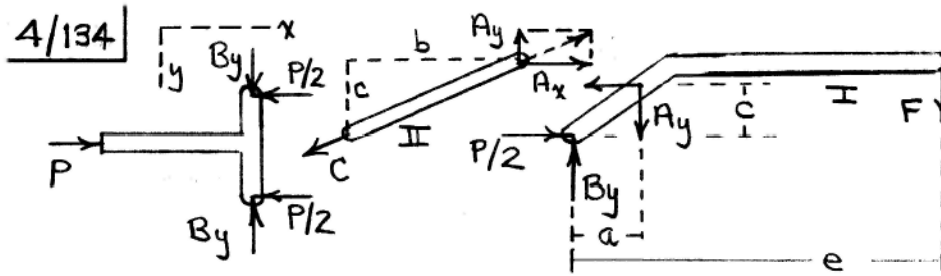
$$\sum F_y = 0 : 2(0.8L) \cos 33.6^\circ - 2AB \cos 67.5^\circ - L = 0$$

$$\underline{AB = 0.434L \text{ T}}$$

For joint E, note that  $EF = DE$  &  $AE = BE$ .

$$\sum F_{y'} = 0 : -2(0.8L) \cos 56.1^\circ + 2DE \cos 67.5^\circ = 0$$

$$\underline{DE = 1.166L \text{ C}}$$



$$\text{II. } A_x = \frac{b}{c} A_y, \quad A_y = \frac{c}{b} A_x$$

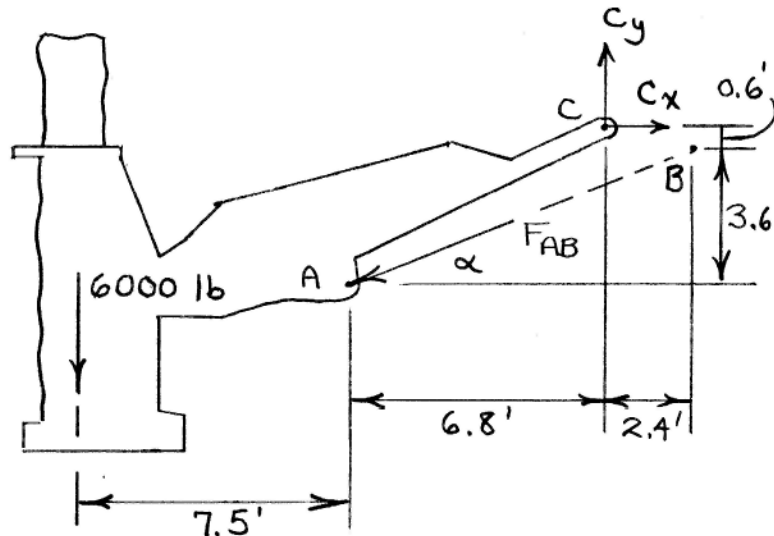
$$\text{I. } \sum M_B = 0: Fe + \frac{c}{b} A_x a - A_x c = 0$$

$$A_x = \frac{Fe}{c(1-\frac{a}{b})}$$

$$\sum F_x = 0: \frac{Fe}{c(1-\frac{a}{b})} = \frac{P}{2} \quad \underline{P = \frac{2Fe}{c(1-\frac{a}{b})}}$$



4/135



$$\alpha = \tan^{-1} \left( \frac{3.6}{9.2} \right) = 21.4^\circ$$

$$\sum M_C = 0: -F_{AB} \cos \alpha (0.6) - F_{AB} \sin \alpha (2.4) + 6000 (14.3) = 0$$

$$F_{AB} = \underline{59,900 \text{ lb}}$$

$$F_{AB} = pA: 59,900 = p \left( \pi \frac{4.72^2}{4} \right)$$

$$p = \underline{3420 \text{ lb/in.}^2}$$

► 4/136 | Vector expressions for forces at A

(treated as tensions) with  $F_{AE} = F_{AF} = F_1$ ,

$F_{BE} = F_{BF} = P$ ,  $F_{BD} = F_{BC} = C$ , are

$$\underline{F}_{AE} = \frac{F_1}{1.552} (-1.2\underline{i} - 0.4\underline{j} + 0.9\underline{k}), \underline{F}_{AF} = \frac{F_1}{1.552} (-1.2\underline{i} + 0.4\underline{j} + 0.9\underline{k})$$

$$\underline{F}_{AB} = \frac{F_{AB}}{1.432} (-0.3\underline{i} + 1.4\underline{k}), \underline{F} = 2.2\underline{k}. \text{ For joint}$$

$$A, \Sigma \underline{F} = 0 \text{ gives } \left[ \frac{F_{AB}}{1.432} (-0.3) + \frac{2F_1}{1.552} (-1.2) \right] \underline{i}$$

$$+ \left[ 2.2 + \frac{F_{AB}}{1.432} (1.4) + \frac{2F_1}{1.552} (0.9) \right] \underline{k} = \underline{0}$$

Solve to get  $F_{AB} = -2.681 \text{ kN}$ ,  $F_1 = 0.363 \text{ kN}$

$$\text{On B: } \underline{F}_{BE} = \frac{P}{1.105} (-0.9\underline{i} - 0.4\underline{j} - 0.5\underline{k})$$

$$\underline{F}_{BF} = \frac{P}{1.105} (-0.9\underline{i} + 0.4\underline{j} - 0.5\underline{k}), \underline{F}_{BD} = \frac{C}{1.105} (-0.9\underline{i} - 0.4\underline{j} + 0.5\underline{k})$$

$$\underline{F}_{BC} = \frac{C}{1.105} (-0.9\underline{i} + 0.4\underline{j} + 0.5\underline{k})$$

For joint B,  $\Sigma \underline{F} = \underline{0}$  gives

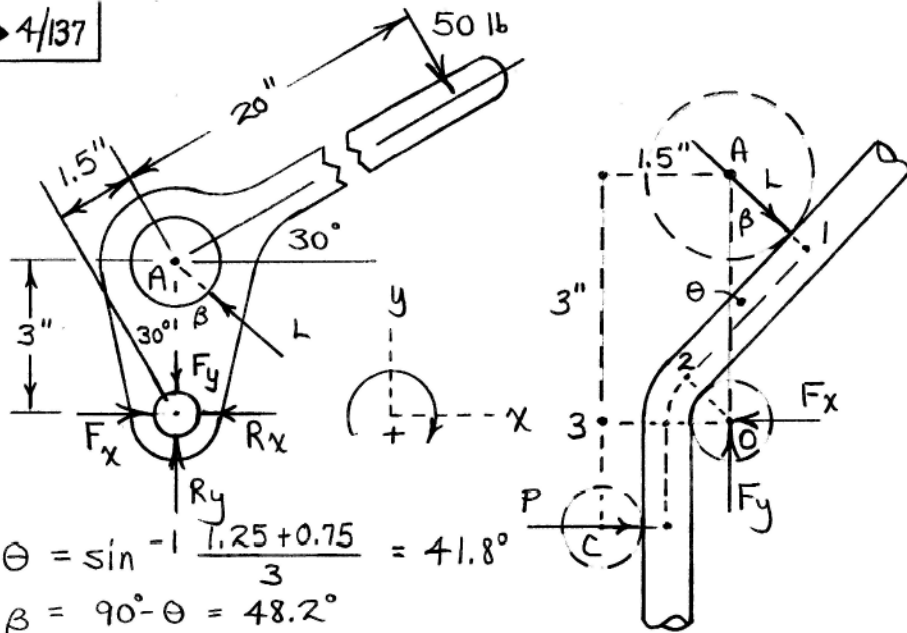
$$\left( \frac{-1.8P}{1.105} - \frac{1.8C}{1.105} + 0.3 \frac{-2.681}{1.432} \right) \underline{i} + \left( \frac{-P}{1.105} + \frac{C}{1.105} - 1.4 \frac{-2.681}{1.432} \right) \underline{k}$$

$$+ 0\underline{j} = \underline{0}. \text{ Solve to get } P = 1.620 \text{ kN},$$

$$C = -1.275 \text{ kN}, \underline{F}_{BE} = P = 1.620 \text{ kN}$$



► 4/137



$$\theta = \sin^{-1} \frac{1.25 + 0.75}{3} = 41.8^\circ$$

$$\beta = 90^\circ - \theta = 48.2^\circ$$

$$1-2 = 3 \sin \beta = 2.24''; \quad C-3 = \sqrt{2^2 - 1.5^2} = 1.323''$$

$$\text{Handle } \sum M_o = 0: 50(21.5) - L \sin 48.2^\circ (3) = 0, \quad L = 481 \text{ lb}$$

$$\text{Bar } \left\{ \begin{array}{l} \sum M_o = 0: 481(2.24) - P(1.323) = 0, \quad P = 813 \text{ lb} \\ \sum F_x = 0: 813 + 481 \sin 48.2^\circ - F_x = 0, \quad F_x = 1171 \text{ lb} \\ \sum F_y = 0: F_y - 481 \cos 48.2^\circ = 0, \quad F_y = 321 \text{ lb} \end{array} \right.$$

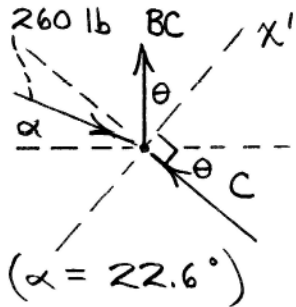
$$\text{Handle } \left\{ \begin{array}{l} \sum F_x = 0: 1171 - R_x - 481 \sin 48.2^\circ + 50 \sin 30^\circ = 0 \\ \sum F_y = 0: R_y - 321 + 481 \cos 48.2^\circ - 50 \cos 30^\circ = 0 \end{array} \right.$$

$$\underline{R_x = 838 \text{ lb}}$$

$$\underline{R_y = 43.3 \text{ lb}}$$

\*4/38 From the solution to Prob. 4/2,  $AC = 260$  lb C and  $AB = 240$  lb T, both constant.

Joint C (BC assumed to be in tension)



$$\sum F_{x'} = 0:$$

$$BC \cos \theta + 260 \sin(\theta - \alpha) = 0$$

$$BC = -260 \frac{\sin(\theta - \alpha)}{\cos \theta}$$

( $\alpha = 22.6^\circ$ )

$BC \rightarrow \infty$  as  $\theta \rightarrow 90^\circ$ , but truss is partially constrained at  $\theta = 90^\circ$ . (b) BC is zero

when  $\sin(\theta - \alpha) = 0$ ,  $\theta = \alpha = 22.6^\circ$ . (Note

that AC & C are collinear if  $\theta = \alpha$ , leaving  $BC = 0$  by inspection.)

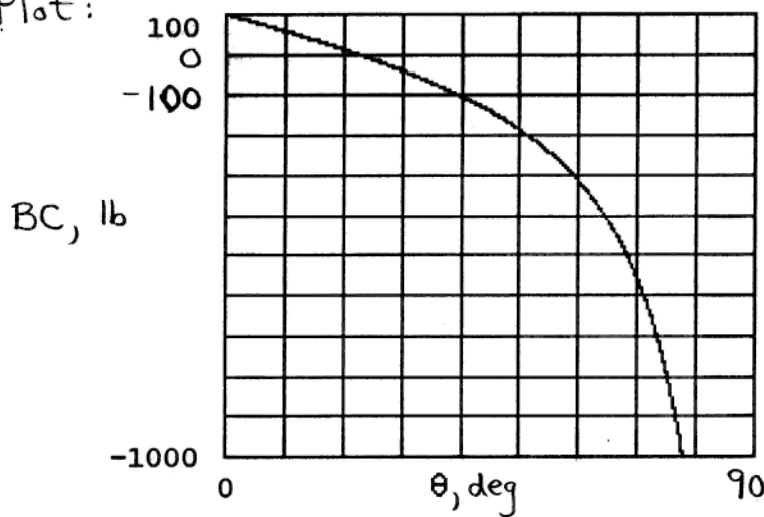
(c) Set  $BC = -1000 = -260 \frac{\sin(\theta - \alpha)}{\cos \theta}$

$$3.85 \cos \theta = \sin \theta \cos \alpha - \sin \alpha \cos \theta$$

$$\Rightarrow \tan \theta = \frac{3.85 + \sin \alpha}{\cos \alpha}, \quad \theta = 77.7^\circ$$

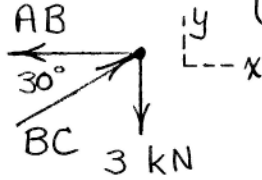
(Note: BC is never +1000 lb over  $0 \leq \theta \leq 90^\circ$ .)

Plot:

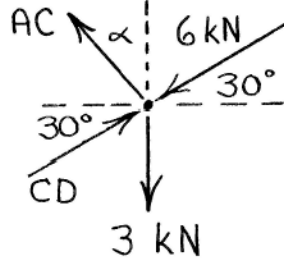


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Joint B :  $\begin{cases} \sum F_y = 0: BC(\frac{1}{2}) - 3 = 0, BC = 6 \text{ kN C} \\ \sum F_x = 0: -AB + 6\frac{\sqrt{3}}{2} = 0, AB = 3\sqrt{3} \text{ kN T} \end{cases}$



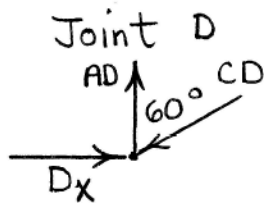
Joint C :  $\begin{cases} \sum F_x = 0: -AC \sin \alpha - 6(\frac{\sqrt{3}}{2}) + CD(\frac{\sqrt{3}}{2}) = 0 \\ \sum F_y = 0: AC \cos \alpha - 6(\frac{1}{2}) + CD(\frac{1}{2}) - 3 = 0 \end{cases}$



Simultaneous solution :

$$AC = \frac{3\sqrt{3} \text{ kN}}{\sqrt{3} \cos \alpha + \sin \alpha} \quad \text{T}$$

$$CD = \frac{15}{2} + \frac{-3\sqrt{3} \cos \alpha + 9 \sin \alpha}{2(\sqrt{3} \cos \alpha + \sin \alpha)} \text{ kN C}$$



Joint D  $\sum F_y = 0: AD - CD(\frac{1}{2}) = 0$

$$AD = \frac{15}{4} + \frac{-3\sqrt{3} \cos \alpha + 9 \sin \alpha}{4(\sqrt{3} \cos \alpha + \sin \alpha)} \text{ kN T}$$

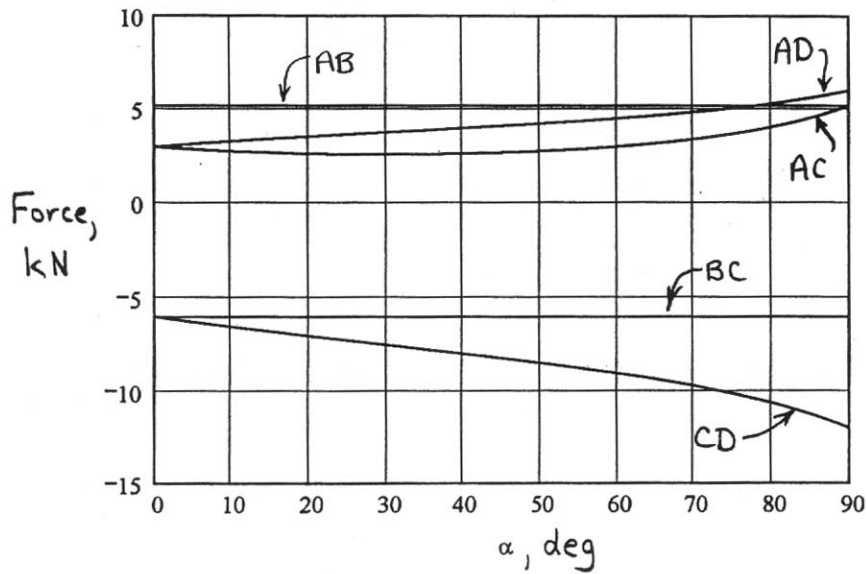
The plots below use (+) for tension and (-) for compression.

$$\frac{d(AC)}{d\alpha} = 0 \Rightarrow -3\sqrt{3}\sqrt{3}(-\sin\alpha) - 3\sqrt{3}\cos\alpha = 0$$

$$\tan\alpha = \frac{1}{\sqrt{3}}, \quad \alpha = 30^\circ$$

$$(AC)_{\min} = \frac{3\sqrt{3}}{\sqrt{3}\frac{\sqrt{3}}{2} + \frac{1}{2}} = \frac{3\sqrt{3}}{2} = \underline{2.60 \text{ kN T}}$$

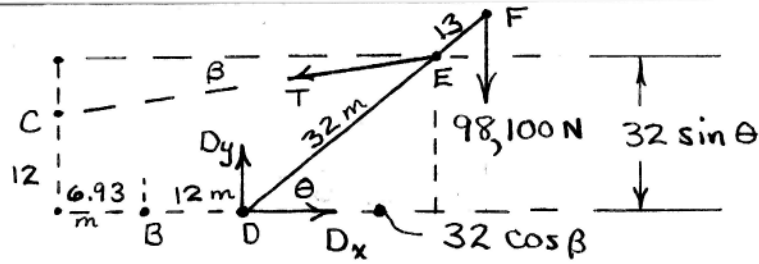
$$\underline{(CD)_{\min} = 6 \text{ kN C @ } \alpha = 0}$$



(Plot uses minus for compression.)

\*4/140

Boom:

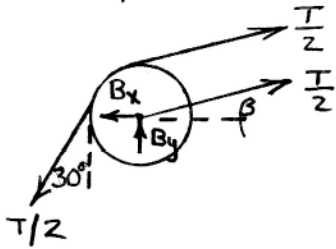


$$\beta = \tan^{-1} \left( \frac{32 \sin \theta - 12}{6.93 + 12 + 32 \cos \theta} \right) = \tan^{-1} \left( \frac{32 \sin \theta - 12}{32 \cos \theta + 18.93} \right) \quad (1)$$

$$\sum M_D = 0: (T \cos \beta)(32 \sin \theta) - (T \sin \beta)(32 \cos \theta) - (98,100)(45 \cos \theta) = 0$$

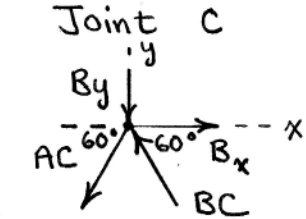
$$\text{So } T = \frac{137,953 \cos \theta}{\cos \beta \sin \theta - \sin \beta \cos \theta} \quad (\text{in N}) \quad (2)$$

Pulley at C:



Equilibrium yields

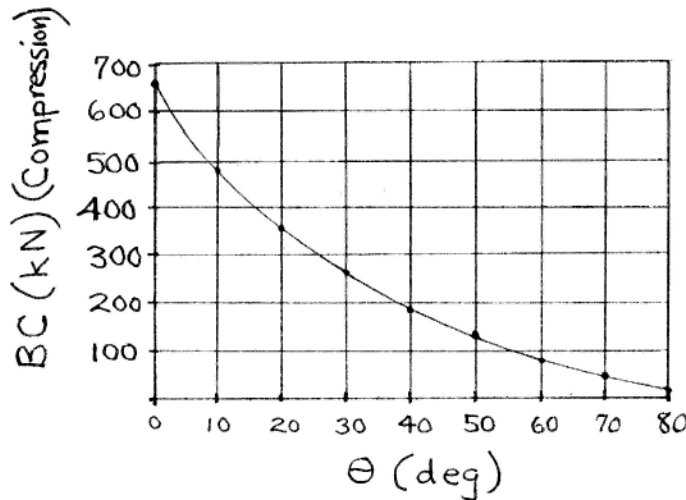
$$\begin{cases} B_x = T \cos \beta - \frac{T}{2} \sin 30^\circ \\ B_y = -T \sin \beta + \frac{T}{2} \cos 30^\circ \end{cases} \quad (3)$$



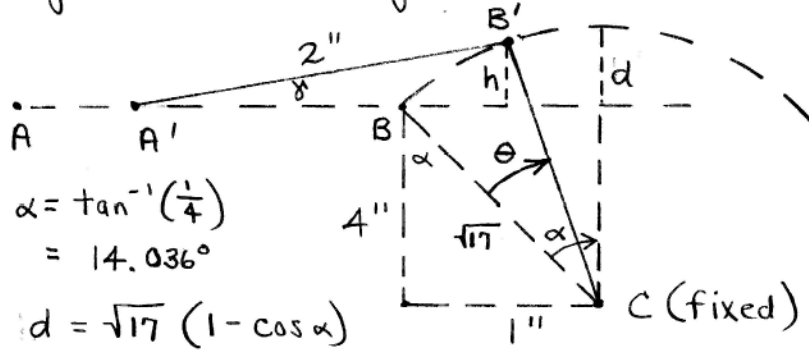
Equilibrium yields

$$\begin{aligned} BC &= B_x + 0.5774 B_y \quad (4) \\ (AC &= B_x - 0.5774 B_y) \end{aligned}$$

Solve Eqs. 1-4 in that order for  $0 \leq \theta \leq 180^\circ$ .



\*4/141 | Geometry considerations. Note that unprimed refers to  $\theta=0$ , primed to  $\theta \neq 0$ . Figure is reduced by a factor of 4 vertically.



$$\alpha = \tan^{-1}\left(\frac{1}{4}\right) = 14.036^\circ$$

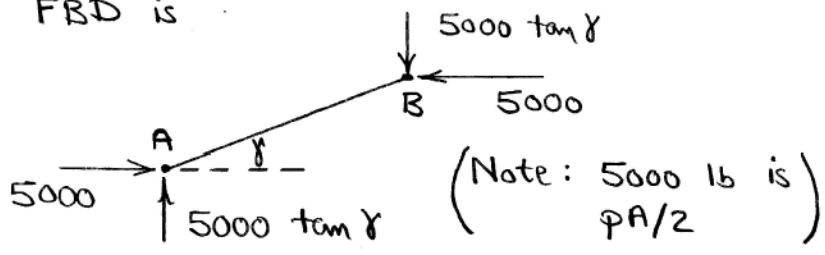
$$d = \sqrt{17} (1 - \cos \alpha)$$

$$d - h = \sqrt{17} (1 - \cos(\alpha - \theta)) \Rightarrow h = \sqrt{17} [\cos(\alpha - \theta) - \cos \alpha]$$

$$\sin \gamma = \frac{h}{2} = \frac{\sqrt{17}}{2} [\cos(\alpha - \theta) - \cos \alpha]^*$$

Note that AB is a two-force member.

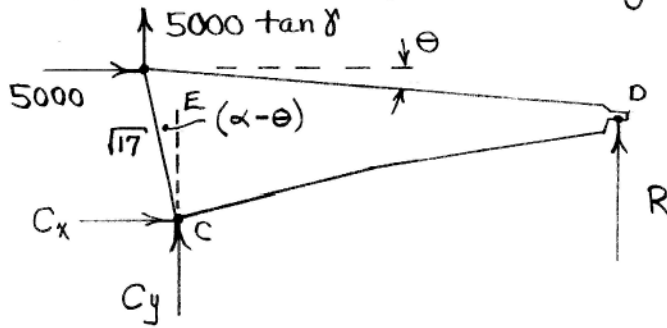
Its FBD is



(Note: 5000 lb is  $\frac{pA}{2}$ )



FBD of lower jaw, for arbitrary  $\theta$ :



$$\overline{CD} = \sqrt{(18)^2 + (3.75)^2} = 18.39''$$

$$\text{When } \theta = 0, \angle ECD = \beta = \tan^{-1}\left(\frac{18}{3.75}\right) = 78.23^\circ$$

$$\text{When } \theta \neq 0, \angle ECD = \beta + \theta$$

$\therefore$  Moment arm for force R about C is

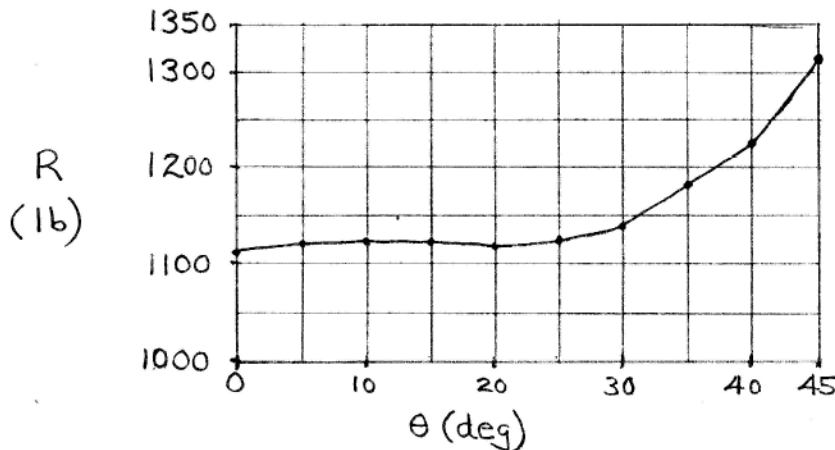
$$\overline{CD} \sin(\beta + \theta)$$

$$\sum M_C = 0: \overline{CD} \sin(\beta + \theta) R - 5000 \sqrt{17} \cos(\alpha - \theta) - 5000 \sqrt{17} \tan \delta \sin(\alpha - \theta) = 0$$

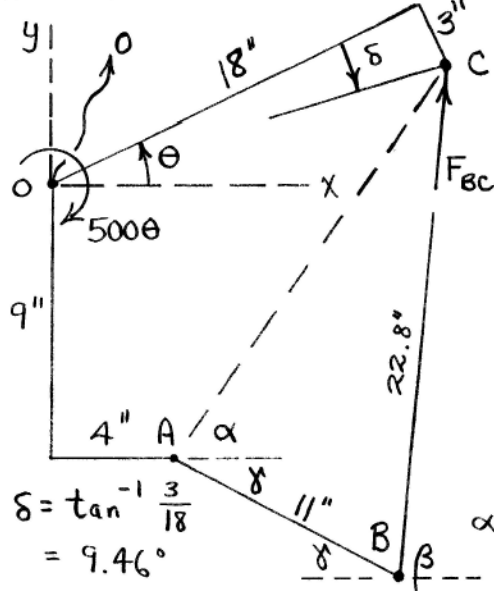
$$R = \frac{1121.233}{\sin(78.23^\circ + \theta)} \left[ \cos(\theta - 14.036^\circ) - \tan \delta \sin(\theta - 14.036^\circ) \right] \text{ lb}$$

( $\delta$  given by \*)

$$R_{\max} = 1314 \text{ lb @ } \theta = 45^\circ$$



$$*4/142 \quad \overline{OC} = \sqrt{18^2 + 3^2}$$



$$\overline{AB} = 11''; \quad \overline{BC} = \sqrt{14^2 + 17^2}$$

$$\overline{AC} = \left\{ [18 \cos \theta + 3 \sin \theta - 4]^2 + [18 \sin \theta - 3 \cos \theta + 9]^2 \right\}^{\frac{1}{2}}$$

$$\angle BAC = \cos^{-1} \left[ \frac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{2 \overline{AB} \overline{AC}} \right]$$

$$\angle ABC = \cos^{-1} \left[ \frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2 \overline{AB} \overline{BC}} \right]$$

$$\alpha = \tan^{-1} \left[ \frac{18 \sin \theta - 3 \cos \theta + 9}{18 \cos \theta + 3 \sin \theta - 4} \right]$$

$$\delta = \tan^{-1} \frac{3}{18}$$

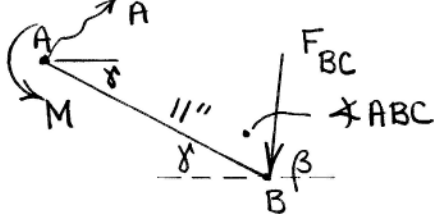
$$= 9.46^\circ$$

$$\gamma = \angle BAC - \alpha; \quad \beta = 180^\circ - \gamma - \angle ABC$$

From above FBD of door:

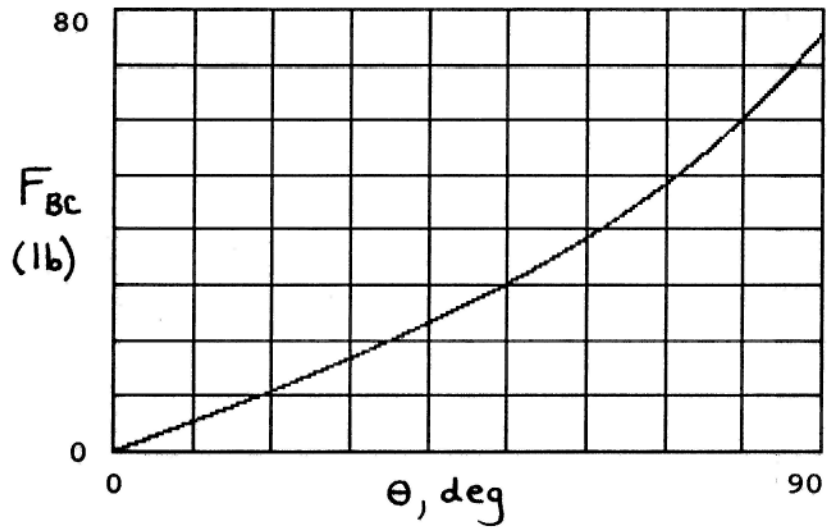
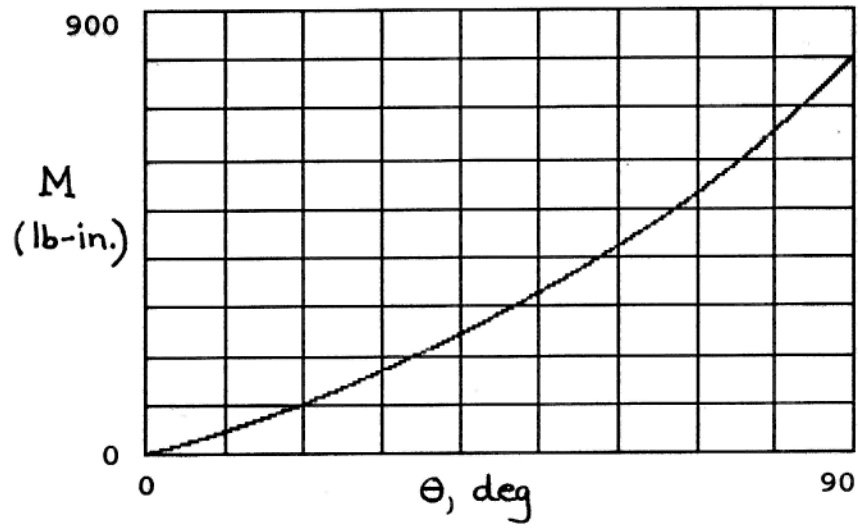
$$\uparrow \sum M_O = 0: -500\theta + F_{BC} (\overline{OC} \sin(\beta - \theta + \delta)) = 0$$

Then from FBD of AB:

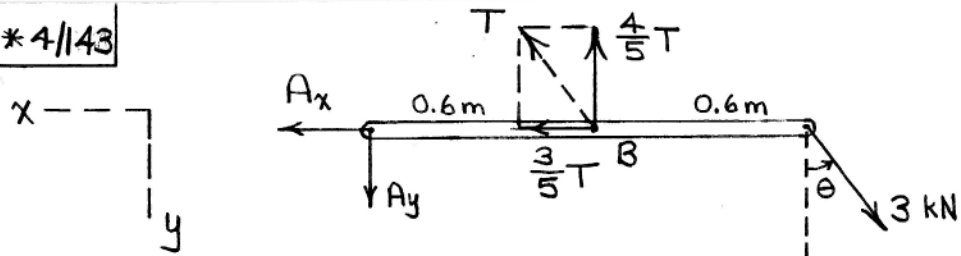


$$\uparrow \sum M_A = 0: M - F_{BC} (11 \sin \angle ABC) = 0$$

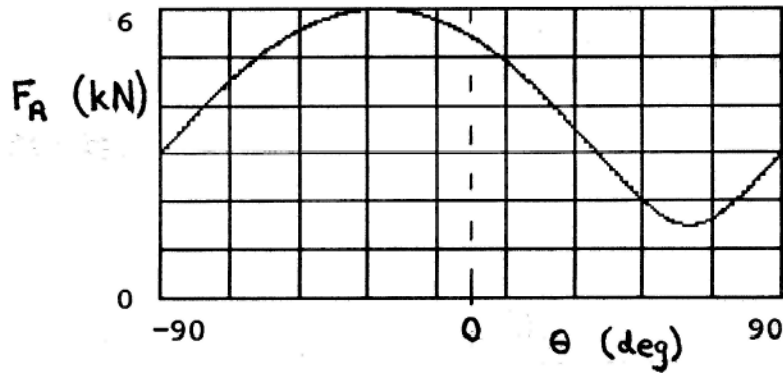
Solve the above equations (in order) with  $\theta$  varied from  $0^\circ$  to  $90^\circ$  to obtain the following plots for  $M$  and  $F_{BC}$ .



\*4/143



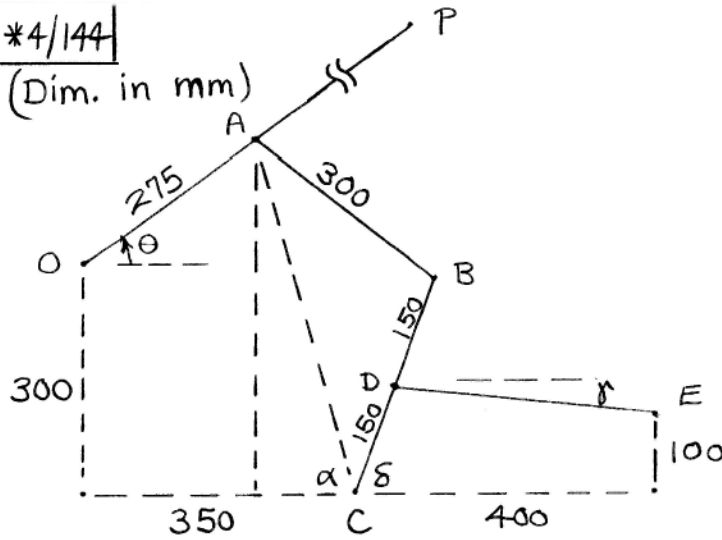
$$\begin{aligned}\sum M_A = 0: & 3 \cos \theta (1.2) - \frac{4}{5} T (0.6) = 0, T = \frac{15}{2} \cos \theta \\ \sum F_x = 0: & A_x + \frac{3}{5} \left( \frac{15}{2} \cos \theta \right) - 3 \sin \theta = 0, A_x = 3 \sin \theta - \frac{9}{2} \cos \theta \\ \sum F_y = 0: & A_y + 3 \cos \theta - \frac{4}{5} \left( \frac{15}{2} \cos \theta \right) = 0, A_y = 3 \cos \theta \\ F_R = & \sqrt{A_x^2 + A_y^2} = 3 \sqrt{1 - 3 \sin \theta \cos \theta + \frac{9}{4} \cos^2 \theta}\end{aligned}$$



$$F_{A \max} = 6 \text{ kN @ } \theta = -26.6^\circ$$

\*4/144

(Dim. in mm)



$$\overline{AC} = \sqrt{(350 - 275 \cos \theta)^2 + (300 + 275 \sin \theta)^2}$$

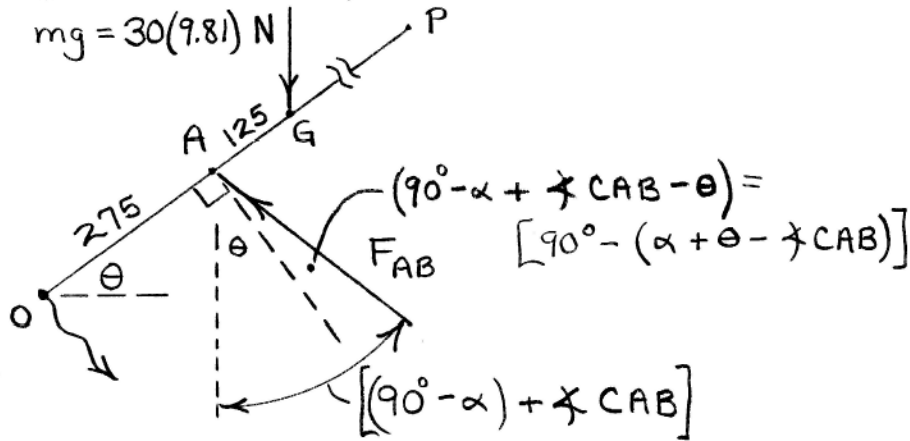
$$\angle ABC = \cos^{-1} \left[ \frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2 \overline{AB} \overline{BC}} \right]$$

$$\alpha = \tan^{-1} \left[ \frac{300 + 275 \sin \theta}{350 - 275 \cos \theta} \right]$$

$$\angle CAB = \angle ACB = \frac{180^\circ - \angle ABC}{2}$$

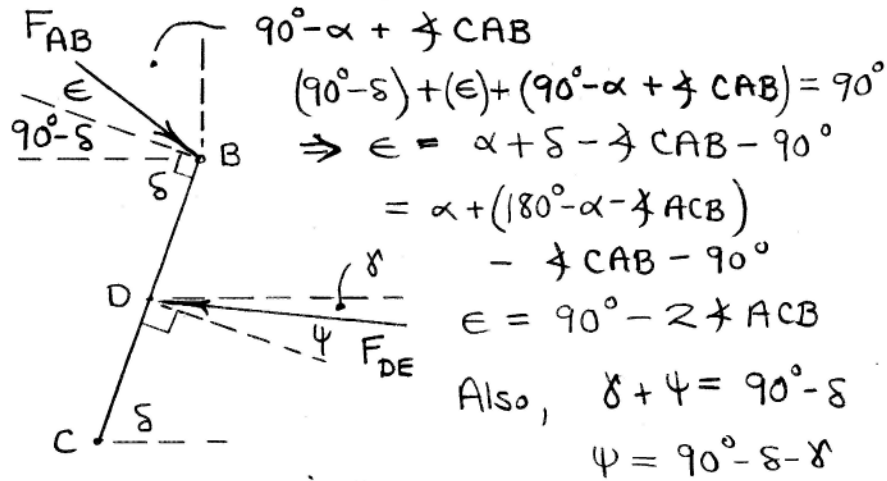
$$\delta = 180^\circ - \alpha - \angle ACB$$

$$mg = 30(9.81) \text{ N}$$



$$\sqrt{\uparrow} \sum M_O = 0: F_{AB} \cos [90^\circ - (\alpha + \theta - \angle CAB)] \overline{OA} - mg \overline{OG} \cos \theta = 0$$

$$F_{AB} = \frac{mg \overline{OG} \cos \theta}{\overline{OA} \sin (\alpha + \theta - \angle CAB)}$$

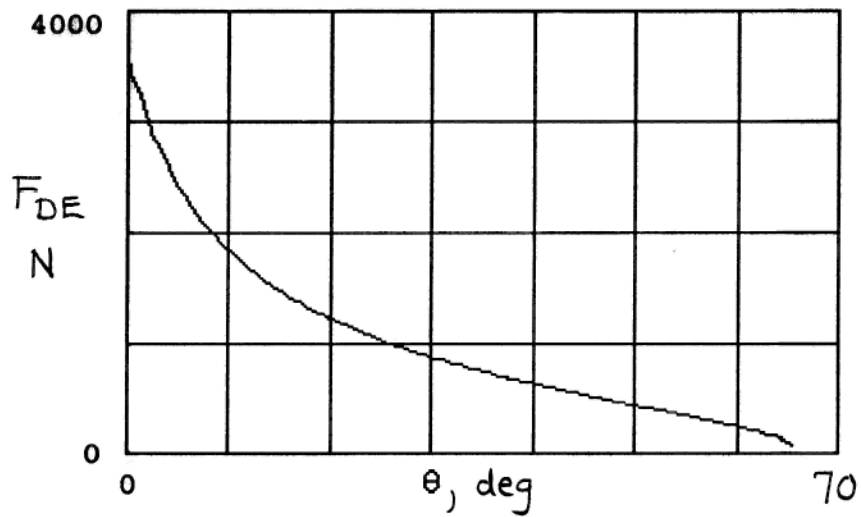


$$\delta = \tan^{-1} \left[ \frac{150 \sin \delta - 100}{400 - 150 \cos \delta} \right]$$

$$\sqrt{\uparrow} \sum M_C = 0: -F_{AB} \cos \epsilon (\overline{CB}) + F_{DE} \cos \psi (\overline{CD}) = 0$$

$$F_{DE} = \frac{F_{AB} \cos \epsilon (\overline{CB})}{\overline{CD} \cos \psi}$$

To find  $\theta_{max}$ , set the above expression for  $\overline{AC}$  to  $\overline{AC} = 600$  mm and solve for  $\theta_{max} = 65.9^\circ$ . Numerically evaluate the above expressions for  $0 \leq \theta \leq \theta_{max}$  to obtain the following plot:



$$(F_{DE})_{\max} = 3580 \text{ N @ } \theta = 0$$

$(F_{DE})_{\min} = 0 \text{ @ } \theta_{\max} = 65.9^\circ$  (links AB and BC are collinear and serve as (an unstable!) prop for the door)

5/1 | From Table D/3, the horizontal coordinate of the centroid is

$$1 + \frac{(9-1) + (7-1)}{3} = \underline{5.67}$$

The vertical coordinate is

$$1 + \frac{9-1}{3} = \underline{3.67}$$



5/2 | From Sample Problem 5/3,

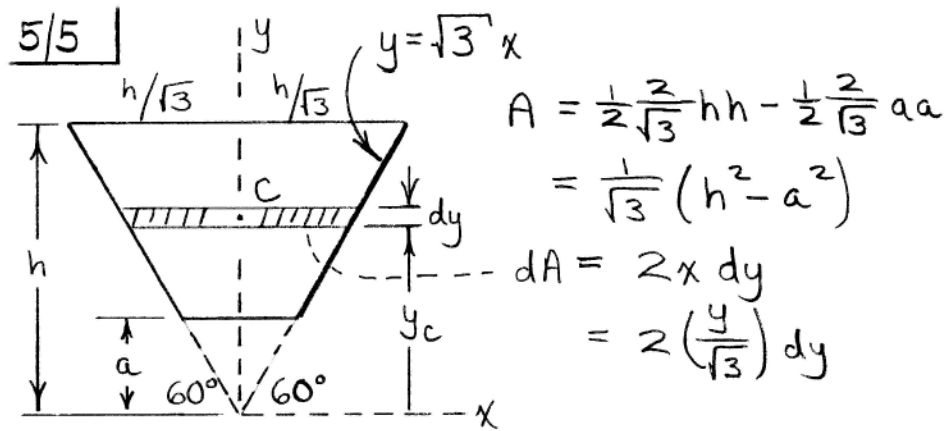
$$\begin{aligned}\bar{x} &= 0 \\ \bar{y} &= \frac{2}{3} \frac{r \sin \alpha}{\alpha} = \frac{2}{3} \frac{200 \sin (120^\circ/2)}{\frac{120^\circ}{2} (\pi/180^\circ)} \\ &= \underline{110.3 \text{ mm}}\end{aligned}$$

$$\frac{5}{3} \quad | \quad \bar{x} = 0$$

$$\bar{y} = -\frac{4r}{3\pi} = -\frac{4(120)}{3\pi} = \underline{-50.9 \text{ mm}}$$

$$\bar{z} = -\frac{360}{2} = \underline{-180 \text{ mm}}$$

$$\frac{5}{4} \quad \bar{x} = \bar{y} = -\frac{2r}{\pi} = -\frac{2(120)}{\pi} = \underline{-76.4 \text{ mm}}$$
$$\underline{\bar{z} = -180 \text{ mm}}$$

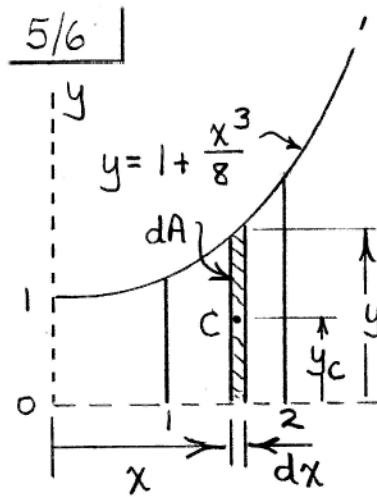


$$\int y_c dA = \int_a^h y \frac{2}{\sqrt{3}} y dy = \frac{2}{\sqrt{3}} \frac{y^3}{3} \Big|_a^h$$

$$= \frac{2}{3\sqrt{3}} (h^3 - a^3)$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{2}{3\sqrt{3}} (h^3 - a^3)}{\frac{1}{\sqrt{3}} (h^2 - a^2)} = \frac{2(h^3 - a^3)}{3(h^2 - a^2)}$$

(For  $a=0$ ,  $\bar{y} = \frac{2}{3}h$ , the correct value.)



$$dA = y dx = \left(1 + \frac{x^3}{8}\right) dx$$

$$A = \int dA = \int_1^2 \left(1 + \frac{x^3}{8}\right) dx$$

$$= \left(x + \frac{x^4}{32}\right) \Big|_1^2 = \frac{47}{32}$$

$$\int x_c dA = \int_1^2 x \left(1 + \frac{x^3}{8}\right) dx$$

$$= \left(\frac{x^2}{2} + \frac{x^5}{40}\right) \Big|_1^2 = \frac{91}{40}$$

$$\int y_c dA = \int \frac{y}{2} y dx = \frac{1}{2} \int y^2 dx$$

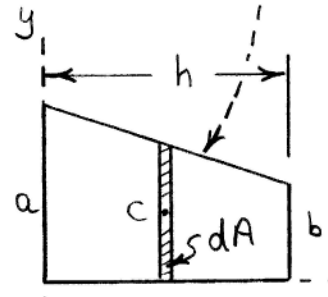
$$= \frac{1}{2} \int_1^2 \left(1 + \frac{x^3}{8}\right)^2 dx = \frac{1}{2} \int_1^2 \left(1 + \frac{x^3}{4} + \frac{x^6}{64}\right) dx$$

$$= \frac{1}{2} \left(x + \frac{x^4}{16} + \frac{x^7}{448}\right) \Big|_1^2 = \frac{995}{896}$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{91/40}{47/32} = \underline{1.549}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{995/896}{47/32} = \underline{0.756}$$

5/7



$$y = \left(\frac{b-a}{h}\right)x + a$$

$$dA = y dx$$

$$A = \int dA = \int_0^h \left[\left(\frac{b-a}{h}\right)x + a\right] dx$$

$$= \left[\frac{b-a}{h} \frac{x^2}{2} + ax\right]_0^h = \frac{h}{2}(a+b)$$

$$\int x_c dA = \int_0^h \left[\left(\frac{b-a}{h}\right)x^2 + ax\right] dx$$

$$= \left[\frac{b-a}{h} \frac{x^3}{3} + \frac{ax^2}{2}\right]_0^h = h^2 \left(\frac{b}{3} + \frac{a}{6}\right)$$

$$\int y_c dA = \int \frac{y}{2} y dx = \frac{1}{2} \int_0^h y^2 dx$$

$$= \frac{1}{2} \int_0^h \left[\left(\frac{b-a}{h}\right)^2 x^2 + 2\left(\frac{b-a}{h}\right)ax + a^2\right] dx$$

$$= \frac{1}{2} \left[\left(\frac{b-a}{h}\right)^2 \frac{x^3}{3} + 2\left(\frac{b-a}{h}\right)a \frac{x^2}{2} + a^2 x\right]_0^h$$

$$= \frac{h}{6} [a^2 + ab + b^2]$$

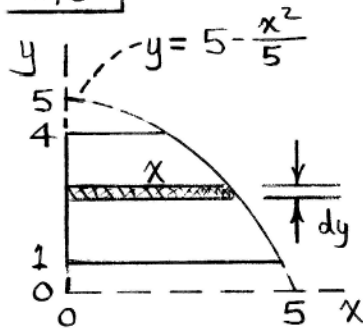
$$\bar{x} = \frac{\int x_c dA}{A} = \frac{h^2 \left(\frac{b}{3} + \frac{a}{6}\right)}{\frac{h}{2}(a+b)} = \frac{h(a+2b)}{3(a+b)}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{h}{6} (a^2 + ab + b^2)}{\frac{h}{2}(a+b)} = \frac{(a^2 + ab + b^2)}{3(a+b)}$$



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5/8



$$A = \int x dy = \int_1^4 \sqrt{5(5-y)} dy$$

$$= \sqrt{5} \left( -\frac{2}{3} [5-y]^{3/2} \right) \Big|_1^4 = \frac{14\sqrt{5}}{3}$$

$$\int x_c dA = \int_1^4 \frac{x}{2} x dy = \int_1^4 \frac{5}{2} (5-y) dy$$

$$= \frac{5}{2} \left( 5y - \frac{y^2}{2} \right) \Big|_1^4 = \frac{75}{4}$$

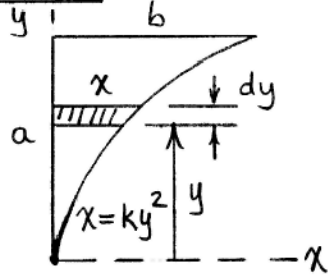
$$\bar{x} = \frac{1}{A} \int x_c dA = \frac{75/4}{14\sqrt{5}/3} = \underline{1.797}$$

$$\int y_c dA = \int_1^4 y x dy = \int_1^4 y \sqrt{5(5-y)} dy$$

$$= \sqrt{5} \left( \frac{-2}{15} \right) (3y+10)(5-y)^{3/2} \Big|_1^4 = \frac{164\sqrt{5}}{15}$$

$$\bar{y} = \frac{1}{A} \int y_c dA = \frac{164\sqrt{5}/15}{14\sqrt{5}/3} = \underline{2.34}$$

5/9



$$b = ka^2, \quad k = \frac{b}{a^2}, \quad x = \frac{b}{a^2} y^2$$

$$A = \int x dy = \int_0^a \frac{b}{a^2} y^2 dy \\ = \frac{b}{a^2} \frac{a^3}{3} = \frac{1}{3} ab$$

$$\int x_c dA = \int \frac{x}{2} x dy = \int \frac{b^2 y^4}{2a^4} dy = \frac{ab^2}{10}$$

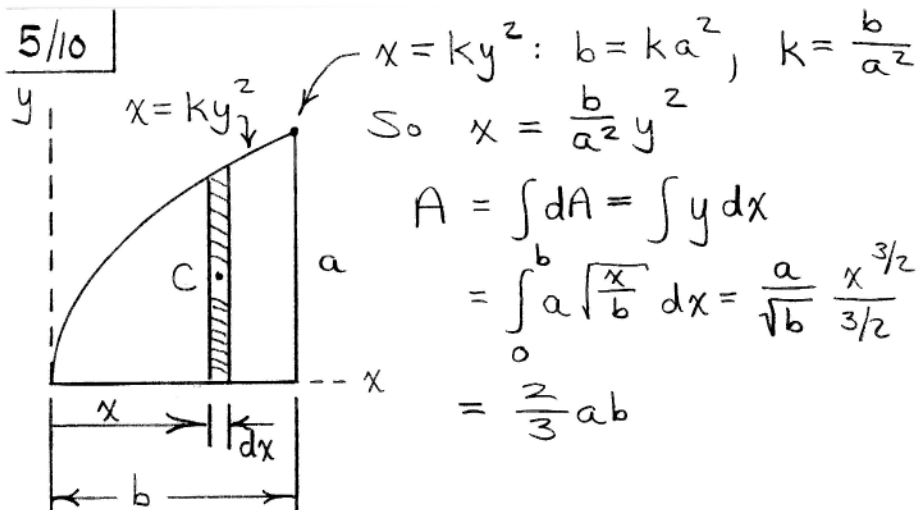
$$\bar{x} = \frac{\int x_c dA}{A} = \frac{ab^2/10}{ab/3} = \underline{\underline{\frac{3}{10} b}}$$

$$\int y_c dA = \int y x dy = \int_0^a y \frac{b}{a^2} y^2 dy = \frac{ba^2}{4}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{ba^2/4}{ab/3} = \underline{\underline{\frac{3}{4} a}}$$



5/10



$$\begin{aligned}
 A &= \int dA = \int y dx \\
 &= \int_0^b a \sqrt{\frac{x}{b}} dx = \frac{a}{\sqrt{b}} \frac{x^{3/2}}{3/2} \Big|_0^b \\
 &= \frac{2}{3} ab
 \end{aligned}$$

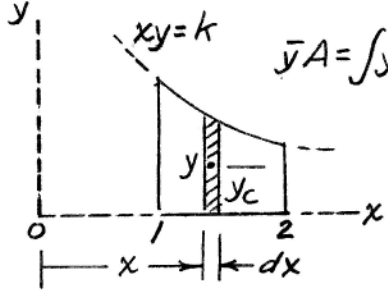
$$\int x_c dA = \int_0^b \frac{a}{\sqrt{b}} x^{3/2} dx = \frac{a}{\sqrt{b}} \frac{x^{5/2}}{5/2} \Big|_0^b = \frac{2}{5} ab^2$$

$$\begin{aligned}
 \int y_c dA &= \int \frac{y}{2} y dx = \frac{1}{2} \int y^2 dx \\
 &= \frac{1}{2} \int_0^b \frac{a^2}{b} x dx = \frac{1}{2} \frac{a^2}{b} \frac{x^2}{2} \Big|_0^b = \frac{1}{4} a^2 b
 \end{aligned}$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{\frac{2}{5} ab^2}{\frac{2}{3} ab} = \frac{3}{5} b$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{1}{4} a^2 b}{\frac{2}{3} ab} = \frac{3}{8} a$$

5/11

$$dA = y dx, \quad A = \int_1^2 \frac{k}{x} dx = k \ln x \Big|_1^2 = k \ln 2$$


$$\bar{y}A = \int y_c dA = \int_1^2 \frac{y}{2} y dx = \frac{1}{2} \int_1^2 \frac{k^2}{x^2} dx$$

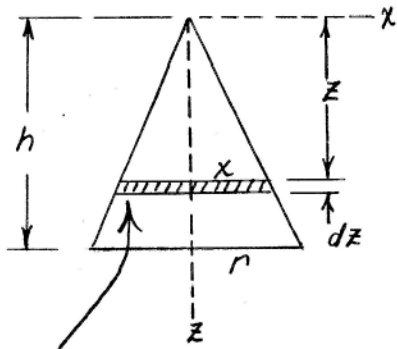
$$= \frac{k^2}{2} \left[ -\frac{1}{x} \right]_1^2 = \frac{k^2}{2} \left[ -\frac{1}{2} + 1 \right] = \frac{k^2}{4}$$

$$\bar{y} = \frac{k^2/4}{k \ln 2} = \frac{k}{4 \ln 2} = 0.361 k$$

$$\bar{x}A = \int x dA = \int_1^2 x y dx = \int_1^2 k dx = k(2-1) = k$$

$$\bar{x} = \frac{k}{k \ln 2} = \frac{1}{\ln 2} = 1.443$$

$$5/12 \quad x = \frac{r}{h} z, \quad dV = \pi x^2 dz = \pi \frac{r^2}{h^2} z^2 dz$$



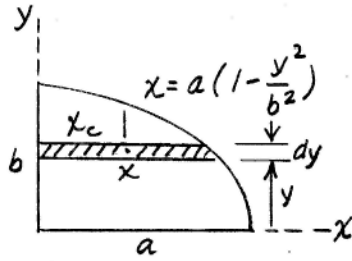
$$V = \pi \frac{r^2}{h^2} \int_0^h z^2 dz = \frac{\pi r^2 h}{3}$$

$$\int z dV = \pi \frac{r^2}{h^2} \int_0^h z^3 dz = \frac{\pi r^2 h^2}{4}$$

$$\bar{z} = \int z dV / V = \underline{3h/4}$$

(Disk-shaped element viewed edge-on.)

$$\frac{5}{13} \quad dA = x dy, \quad A = \int_0^b a \left(1 - \frac{y^2}{b^2}\right) dy = a \left[ y - \frac{y^3}{3b^2} \right]_0^b = \frac{2}{3} ab$$



$$\begin{aligned} \bar{x}A &= \int x_c dA = \int \frac{x}{2} x dy \\ &= \frac{1}{2} \int_0^b a^2 \left(1 - \frac{2y^2}{b^2} + \frac{y^4}{b^4}\right) dy \\ &= \frac{a^2}{2} \left[ y - \frac{2y^3}{3b^2} + \frac{y^5}{5b^4} \right]_0^b = \frac{4}{15} a^2 b \end{aligned}$$

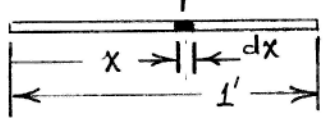
$$\bar{x} = \frac{4a^2b/15}{2ab/3} = \frac{2}{5}a$$

$$\bar{y}A = \int y dA = \int_0^b a \left(y - \frac{y^3}{b^2}\right) dy = a \left[ \frac{y^2}{2} - \frac{y^4}{4b^2} \right]_0^b = \frac{1}{4} ab^2$$

$$\bar{y} = \frac{ab^2/4}{2ab/3} = \frac{3}{8}b$$

5/14

$dm = \rho dx$  ( $\rho =$  mass per unit length)



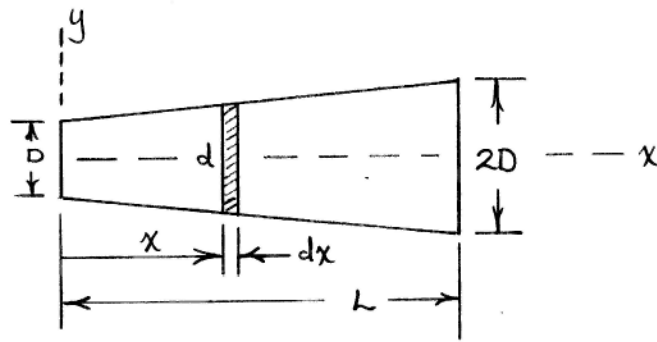
$m = \int dm = \int \rho dx = \int_0^{l'} \rho_0 \left(1 - \frac{x}{2}\right) dx$

$= \rho_0 \left[ x - \frac{x^2}{4} \right]_0^{l'} = \frac{3}{4} \rho_0 l'$

$\int x dm = \int_0^{l'} x \rho_0 \left(1 - \frac{x}{2}\right) dx = \rho_0 \left[ \frac{x^2}{2} - \frac{x^3}{6} \right]_0^{l'} = \frac{\rho_0 l'^3}{3}$

$\bar{x} = \frac{\int x dm}{\int dm} = \frac{\rho_0 l'^3 / 3}{3 \rho_0 l' / 4} = \frac{4}{9} l'$

5/15



For constant density,  $\bar{x}V = \int x dV$

Diameter  $d = D \left(1 + \frac{x}{L}\right)$

So  $dV = \frac{\pi d^2}{4} dx = \frac{\pi D^2}{4} \left(1 + \frac{x}{L}\right)^2 dx$

$$V = \frac{\pi D^2}{4} \int_0^L \left(1 + \frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4} \left[ x + \frac{x^2}{L} + \frac{x^3}{3L^2} \right]_0^L$$

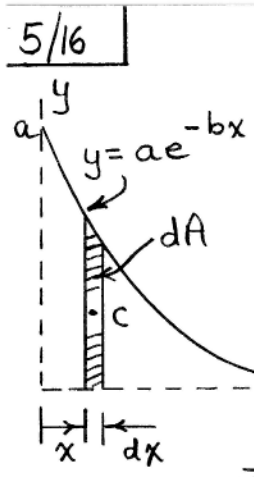
$$= \frac{7}{12} \pi D^2 L$$

$$\int x dV = \frac{\pi D^2}{4} \int_0^L x \left(1 + \frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4} \left[ x^2 + \frac{2x^3}{3L} + \frac{x^4}{4L^2} \right]_0^L$$

$$= \frac{17}{48} \pi D^2 L^2$$

$$\bar{x} = \frac{\frac{17}{48} \pi D^2 L^2}{\frac{7}{12} \pi D^2 L} = \frac{17}{28} L$$

5/16



$$dA = y dx = a e^{-bx} dx$$

$$A = \int dA = \int_0^{\infty} a e^{-bx} dx$$

$$= -\frac{a}{b} e^{-bx} \Big|_0^{\infty} = -\frac{a}{b} [0 - 1]$$

$$= a/b$$

$$\int x_c dA = a \int_0^{\infty} x e^{-bx} dx$$

$$= a \frac{e^{-bx}}{b^2} [-bx - 1] \Big|_0^{\infty}$$

$$= -\frac{a}{b^2} [bx e^{-bx} + e^{-bx}] \Big|_0^{\infty} = -\frac{a}{b^2} [0 + 0 - (0 + 1)]$$

$$= a/b^2$$

$$\int y_c dA = \int \frac{y}{2} y dx = \int \frac{y^2}{2} dx$$

$$= \frac{1}{2} \int_0^{\infty} a^2 e^{-2bx} dx = \frac{a^2}{2} \frac{e^{-2bx}}{-2b} \Big|_0^{\infty}$$

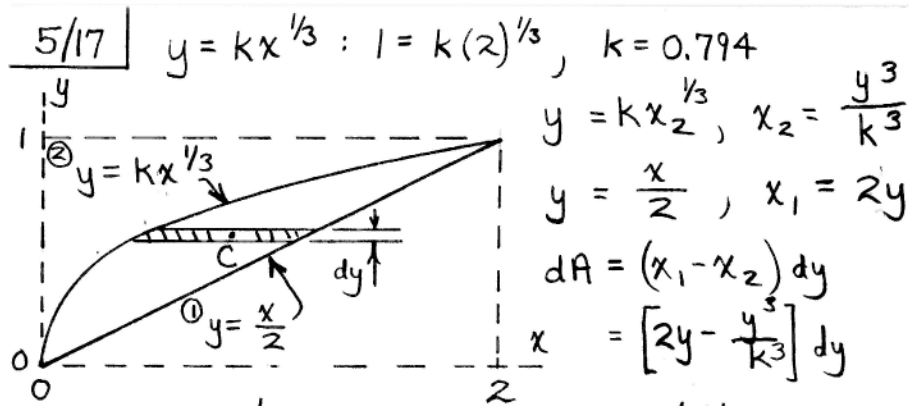
$$= -\frac{a^2}{4b} [0 - 1] = \frac{a^2}{4b}$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{a/b^2}{a/b} = \frac{1}{b}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{a^2/4b}{a/b} = \frac{a}{4}$$



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$$A = \int dA = \int_0^1 \left( 2y - \frac{y^3}{k^3} \right) dy = \left( y^2 - \frac{y^4}{4k^3} \right) \Big|_0^1 = 0.5$$

$$\int x_c dA = \int \left( \frac{x_1 + x_2}{2} \right) (x_1 - x_2) dy = \frac{1}{2} \int (x_1^2 - x_2^2) dy$$

$$= \frac{1}{2} \int_0^1 \left( 4y^2 - \frac{y^6}{k^6} \right) dy = \frac{1}{2} \left[ \frac{4}{3} y^3 - \frac{y^7}{7k^6} \right] \Big|_0^1 = 0.381$$

$$\int y_c dA = \int y \left( 2y - \frac{y^3}{k^3} \right) dy = \int \left( 2y^2 - \frac{y^4}{k^3} \right) dy$$

$$= \left( \frac{2y^3}{3} - \frac{y^5}{5k^3} \right) \Big|_0^1 = 0.267$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{0.381}{0.5} = \underline{0.762}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{0.267}{0.5} = \underline{0.533}$$



5/18

$$A = \int_0^a (y_2 - y_1) dx$$

$$= \int_0^a \left[ b \left( 1 + \frac{x}{a} \right) - b \left( \frac{x}{a} \right)^{\frac{1}{2}} \right] dx$$

$$= b \left( x + \frac{x^2}{2a} \right) - \frac{b}{\sqrt{a}} \frac{2}{3} x^{\frac{3}{2}} \Big|_0^a$$

$$= \frac{5}{6} ab$$

$$\int y_c dA = \int_0^a \left( \frac{y_1 + y_2}{2} \right) (y_2 - y_1) dx = \frac{1}{2} \int_0^a (y_2^2 - y_1^2) dx$$

$$= \frac{1}{2} \int_0^a \left[ b^2 \left( 1 + \frac{x}{a} \right)^2 - \frac{b^2}{a} x \right] dx$$

$$= \frac{1}{2} \left[ b^2 \left( x + \frac{x^2}{a} + \frac{x^3}{3a^2} \right) - \frac{b^2 x^2}{2a} \right] \Big|_0^a = \frac{11}{12} ab^2$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{11ab^2/12}{5ab/6} = \frac{11}{10} b$$

$$\frac{5/19}{y} \quad x_2^2 + y^2 = a^2 \quad A = \frac{1}{4}\pi a^2 - \frac{1}{2} \frac{a}{2} a = \frac{a^2}{4}(\pi - 1)$$

$$dA = (x_2 - x_1) dy = \left[ \sqrt{a^2 - y^2} - \frac{a-y}{2} \right] dy$$

$$\int x_c dA = \int_0^a \left( \frac{x_1 + x_2}{2} \right) (x_2 - x_1) dy = \frac{1}{2} \int_0^a (x_2^2 - x_1^2) dy$$

$$= \frac{1}{2} \int_0^a \left[ (a^2 - y^2) - \left( \frac{a-y}{2} \right)^2 \right] dy$$

$$= \frac{1}{2} \int_0^a \left[ a^2 - y^2 - \frac{1}{4}(a^2 - 2ay + y^2) \right] dy$$

$$= \frac{1}{2} \int_0^a \left[ \frac{3}{4}a^2 - \frac{5}{4}y^2 + \frac{1}{2}ay \right] dy$$

$$= \frac{1}{2} \left[ \frac{3}{4}a^2 y - \frac{5}{12}y^3 + \frac{1}{4}ay^2 \right]_0^a = \frac{7}{24}a^3$$

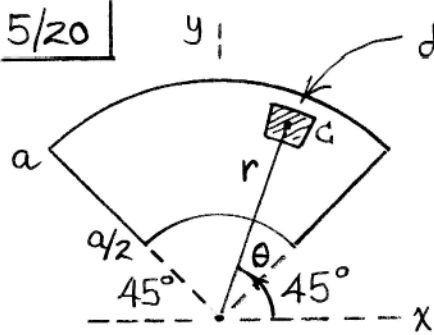
$$\int y_c dA = \int_0^a y \left[ \sqrt{a^2 - y^2} - \frac{a-y}{2} \right] dy$$

$$= \left[ -\frac{1}{3} \sqrt{(a^2 - y^2)^3} - \frac{a}{2} \frac{y^2}{2} + \frac{y^3}{6} \right]_0^a = \frac{a^3}{4}$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{\frac{7}{24}a^3}{\frac{a^2}{4}(\pi - 1)} = \frac{7a}{6(\pi - 1)}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{a^3/4}{\frac{a^2}{4}(\pi - 1)} = \frac{a}{\pi - 1}$$

5/20



$$dA = r dr d\theta$$

$$A = \frac{\pi a^2}{4} - \frac{\pi}{4} \left(\frac{a}{2}\right)^2 = \frac{3}{16} \pi a^2$$

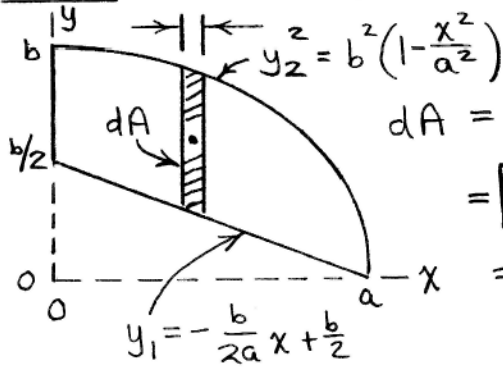
$$\int y_c dA = \iint (r \sin \theta) r dr d\theta$$

$$= \int_{\pi/4}^{3\pi/4} \int_{a/2}^a \sin \theta r^2 dr d\theta = \int_{\pi/4}^{3\pi/4} \sin \theta \frac{7}{24} a^3 d\theta$$

$$= \frac{7}{24} a^3 (-\cos \theta) \Big|_{\pi/4}^{3\pi/4} = \frac{7\sqrt{2}}{24} a^3$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{7\sqrt{2}}{24} a^3}{\frac{3}{16} \pi a^2} = \frac{14\sqrt{2}}{9\pi} a$$

$$5/21 \quad dx \quad A = \pi \frac{ab}{4} - \frac{1}{2} a \frac{b}{2} = \frac{ab}{4} (\pi - 1)$$



$$dA = (y_2 - y_1) dx$$

$$= \left[ b \sqrt{1 - \frac{x^2}{a^2}} - \left( -\frac{b}{2a}x + \frac{b}{2} \right) \right] dx$$

$$= \frac{b}{a} \left[ \sqrt{a^2 - x^2} + \frac{x}{2} - \frac{a}{2} \right] dx$$

$$\int x_c dA = \int_0^a x \frac{b}{a} \left[ \sqrt{a^2 - x^2} + \frac{x}{2} - \frac{a}{2} \right] dx$$

$$= \frac{b}{a} \left[ -\frac{1}{3} \sqrt{(a^2 - x^2)^3} + \frac{x^3}{6} - \frac{ax^2}{4} \right]_0^a = \frac{1}{4} ba^2$$

$$\int y_c dA = \int_0^a \left( \frac{y_2 + y_1}{2} \right) (y_2 - y_1) dx = \frac{1}{2} \int_0^a (y_2^2 - y_1^2) dx$$

$$= \frac{1}{2} \int_0^a \left[ b^2 \left( 1 - \frac{x^2}{a^2} \right) - \left( -\frac{b}{2a}x + \frac{b}{2} \right)^2 \right] dx$$

$$= \frac{1}{2} \int_0^a \left[ b^2 \left( 1 - \frac{x^2}{a^2} \right) - \frac{b^2}{4a^2}x^2 + \frac{b^2}{2a}x - \frac{b^2}{4} \right] dx$$

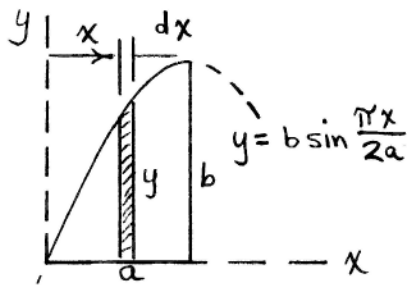
$$= \frac{1}{2} \left[ b^2 \left( x - \frac{x^3}{3a^2} \right) - \frac{b^2}{4a^2} \frac{x^3}{3} + \frac{b^2}{2a} \frac{x^2}{2} - \frac{b^2}{4}x \right]_0^a$$

$$= \frac{7}{24} ab^2$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{\frac{1}{4} ba^2}{\frac{ab}{4} (\pi - 1)} = \frac{a}{\pi - 1}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{7}{24} ab^2}{\frac{ab}{4} (\pi - 1)} = \frac{7b}{6(\pi - 1)}$$

5/22



$$A = \int y dx = \int_0^a b \sin \frac{\pi x}{2a} dx$$

$$= -\frac{2ab}{\pi} \cos \frac{\pi x}{2a} \Big|_0^a = \frac{2ab}{\pi}$$

$$\int x_c dA = \int_0^a x y dx = \int_0^a b x \sin \frac{\pi x}{2a} dx$$

$$= b \left(\frac{2a}{\pi}\right)^2 \left[ \sin \frac{\pi x}{2a} - \frac{\pi x}{2a} \cos \frac{\pi x}{2a} \right]_0^a$$

$$= 4a^2 b / \pi^2$$

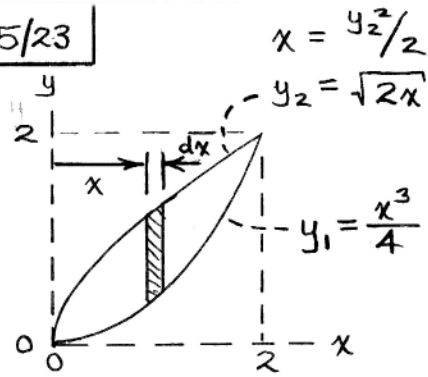
$$\bar{x} = \frac{\int x_c dA}{A} = \frac{4a^2 b / \pi^2}{2ab / \pi} = \frac{2a}{\pi}$$

$$\int y_c dA = \int_0^a \frac{y}{2} y dx = \frac{b^2}{2} \int_0^a \sin^2 \frac{\pi x}{2a} dx$$

$$= \frac{ab^2}{\pi} \left[ \frac{\pi x}{4a} - \frac{1}{4} \sin \frac{\pi x}{a} \right]_0^a = \frac{ab^2}{4}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{ab^2/4}{2ab/\pi} = \frac{\pi b}{8}$$

5/23



$$A = \int dA = \int_0^{\frac{1}{2}} (y_2 - y_1) dx = \int_0^{\frac{1}{2}} (\sqrt{2x} - \frac{x^3}{4}) dx$$

$$= \left( \frac{2\sqrt{2}}{3} x^{3/2} - \frac{x^4}{16} \right) \Big|_0^{\frac{1}{2}} = \frac{5}{3}$$

$$\int x_c dA = \int_0^{\frac{1}{2}} x (\sqrt{2x} - \frac{x^3}{4}) dx$$

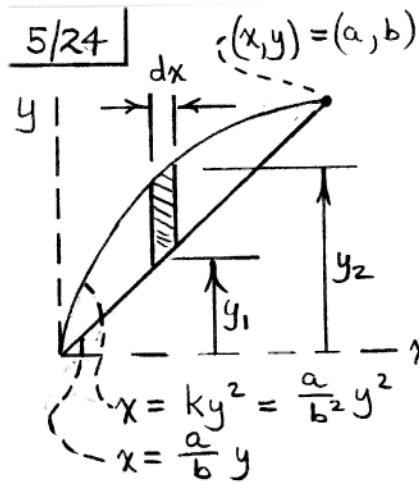
$$= \left( \frac{2\sqrt{2}}{5} x^{5/2} - \frac{x^5}{20} \right) \Big|_0^{\frac{1}{2}} = \frac{8}{5}$$

$$\int y_c dA = \int_0^{\frac{1}{2}} \left( \frac{y_1 + y_2}{2} \right) (y_2 - y_1) dx = \int_0^{\frac{1}{2}} \frac{1}{2} (y_2^2 - y_1^2) dx$$

$$= \frac{1}{2} \int_0^{\frac{1}{2}} (2x - \frac{x^6}{16}) dx = \frac{1}{2} \left[ x^2 - \frac{x^7}{7(16)} \right] \Big|_0^{\frac{1}{2}} = \frac{10}{7}$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{8/5}{5/3} = \frac{24}{25}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{10/7}{5/3} = \frac{6}{7}$$



$$A = \int_0^a (y_2 - y_1) dx$$

$$= \int_0^a \left( b\sqrt{\frac{x}{a}} - x\frac{b}{a} \right) dx$$

$$= b \left[ \frac{1}{\sqrt{a}} \frac{2x^{3/2}}{3} - \frac{1}{2a} x^2 \right]_0^a$$

$$= \frac{ab}{6}$$

$$\int x_c dA = \int_0^a x(y_2 - y_1) dx = \int_0^a \left[ \frac{b}{\sqrt{a}} x^{3/2} - \frac{b}{a} x^2 \right] dx$$

$$= b \left[ \frac{2x^{5/2}}{5\sqrt{a}} - \frac{x^3}{3a} \right]_0^a = \frac{a^2 b}{15}$$

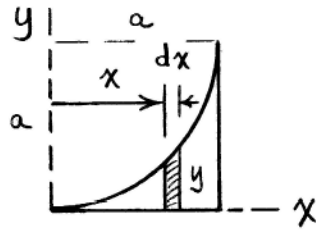
$$\bar{x} = \frac{\int x_c dA}{A} = \frac{a^2 b / 15}{ab/6} = \frac{2}{5} a$$

$$\int y_c dA = \int_0^a \left( \frac{y_1 + y_2}{2} \right) (y_2 - y_1) dx = \frac{1}{2} \int_0^a (y_2^2 - y_1^2) dx$$

$$= \frac{1}{2} \int_0^a \left( \frac{xb^2}{a} - \frac{x^2 b^2}{a^2} \right) dx = \frac{1}{12} ab^2$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{ab^2/12}{ab/6} = \frac{b}{2}$$

5/25



$$x^2 + (y-a)^2 = a^2$$

$$y = a - \sqrt{a^2 - x^2}$$

(use minus sign)

$$A = \int y dx = \int_0^a [a - \sqrt{a^2 - x^2}] dx$$
$$= \left[ ax - \frac{1}{2} (x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a}) \right]_0^a = a^2 \left( 1 - \frac{\pi}{4} \right)$$

$$\int x_c dA = \int_0^a x y dx = \int_0^a [ax - x\sqrt{a^2 - x^2}] dx$$
$$= \left[ \frac{ax^2}{2} + \frac{1}{3} (a^2 - x^2)^{3/2} \right]_0^a = \frac{a^3}{6}$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{a^3/6}{a^2(1 - \frac{\pi}{4})} = \frac{2a}{3(4 - \pi)} = \underline{0.777a}$$

From symmetry,  $\bar{y} = a - \bar{x} = a - \frac{2a}{3(4 - \pi)} = \underline{\frac{10 - 3\pi}{3(4 - \pi)}a}$

or  $\bar{y} = \underline{0.223a}$



$$\frac{5}{26} \quad V\bar{z} = \int z dV$$

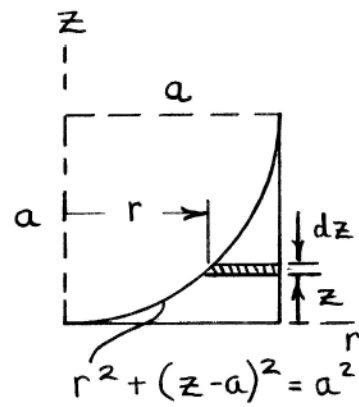
$$dV = A dz = \frac{\pi}{2} (a^2 - r^2) dz$$

$$= \frac{\pi}{2} (z-a)^2 dz$$

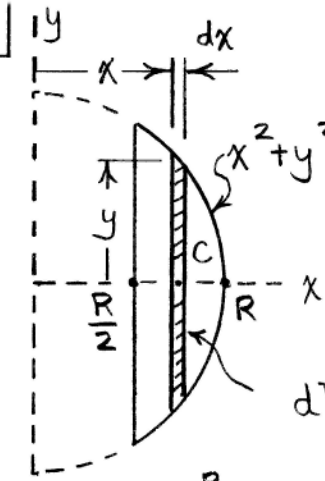
$$V = \frac{\pi}{2} \int_0^a (z-a)^2 dz = \frac{\pi a^3}{6}$$

$$\int z dV = \int_0^a z \frac{\pi}{2} (z-a)^2 dz = \frac{\pi a^4}{24}$$

$$\bar{z} = \frac{\frac{\pi a^4}{24}}{\frac{\pi a^3}{6}} = \underline{\underline{a/4}}$$



5/27



(Note: Shaded element is a circular slice viewed edge-on.)

$$dV = \pi y^2 dx = \pi (R^2 - x^2) dx$$

$$V = \int dV = \int_{R/2}^R \pi (R^2 - x^2) dx = \pi \left[ R^2 x - \frac{x^3}{3} \right]_{R/2}^R$$

$$= \pi \left[ R^3 - \frac{R^3}{3} - \left( \frac{R^3}{2} - \frac{R^3}{24} \right) \right] = \frac{5}{24} \pi R^3$$

$$\int x_c dV = \int_{R/2}^R x \pi (R^2 - x^2) dx = \pi \left[ \frac{R^2 x^2}{2} - \frac{x^4}{4} \right]_{R/2}^R$$

$$= \pi \left[ \frac{R^4}{2} - \frac{R^4}{4} - \left( \frac{R^4}{8} - \frac{R^4}{64} \right) \right] = \frac{9}{64} \pi R^4$$

$$\bar{x} = \frac{\int x_c dV}{\int dV} = \frac{\frac{9}{64} \pi R^4}{\frac{5}{24} \pi R^3} = \frac{27}{40} R$$

5/28 | Choose elemental cylindrical shell of radius  $x$ .

$$\bar{x} = \frac{\int x_c dV}{\int dV}$$

From Sample Problem 5/1 or Table D/3,  $x_c = \frac{2x}{\pi}$

$$dV = \frac{\pi x}{2} (a-z) dz = \frac{\pi x}{2} (2a-x) dx$$

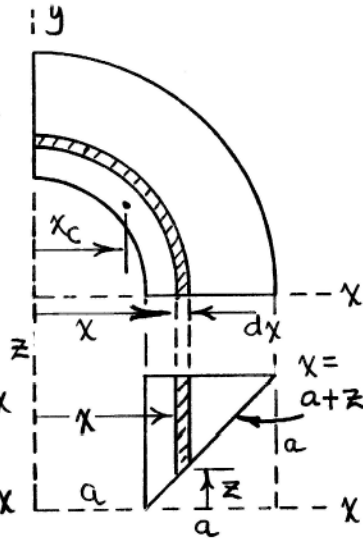
$$\int x_c dV = \int_a^{2a} \frac{2x}{\pi} \frac{\pi x}{2} (2a-x) dx$$

$$= \int_a^{2a} (2ax^2 - x^3) dx = \frac{11}{12} a^4$$

$$\int dV = \int_a^{2a} \frac{\pi x}{2} (2a-x) dx = \pi \int_a^{2a} \left( xa - \frac{x^2}{2} \right) dx$$

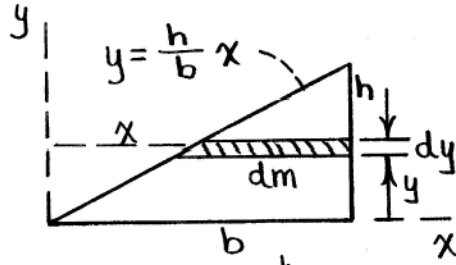
$$= \frac{\pi a^3}{3}$$

$$\text{So } \bar{x} = \frac{11a^4/12}{\pi a^3/3} = \frac{11a}{4\pi} = \bar{y} \text{ by symmetry}$$



5/29

$$\begin{aligned}
 dm &= \rho dV = \rho dA t = t \rho (b-x) dy \\
 &= \left[ t_0 \left( \frac{y}{h} + 1 \right) \right] \rho (b-x) dy \\
 &= t_0 \rho \left( \frac{y}{h} + 1 \right) \left( b - \frac{b}{h} y \right) dy \\
 &= t_0 \rho b \left( 1 - \frac{y^2}{h^2} \right) dy
 \end{aligned}$$

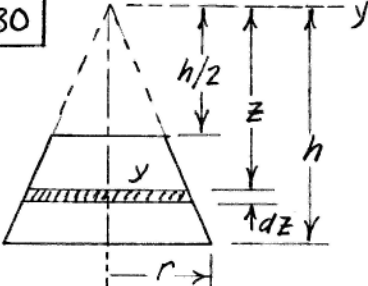


$$\begin{aligned}
 m &= \int dm = \int_0^h t_0 \rho b \left( 1 - \frac{y^2}{h^2} \right) dy = t_0 \rho b \left[ y - \frac{y^3}{3h^2} \right]_0^h \\
 &= \frac{2}{3} \rho t_0 b h
 \end{aligned}$$

$$\begin{aligned}
 \int y_c dm &= \int t_0 \rho b \left( 1 - \frac{y^2}{h^2} \right) y dy = t_0 \rho b \left[ \frac{y^2}{2} - \frac{y^4}{4h^2} \right]_0^h \\
 &= \frac{1}{4} \rho t_0 b h^2
 \end{aligned}$$

$$\bar{y} = \frac{\int y_c dm}{m} = \frac{\frac{1}{4} \rho t_0 b h^2}{\frac{2}{3} \rho t_0 b h} = \frac{3}{8} h$$

5/30



$dV = \pi y^2 dz$  where  $y = \frac{r}{h} z$

$= \pi \frac{r^2}{h^2} z^2 dz$

$V = \pi \frac{r^2}{h^2} \int_{h/2}^h z^2 dz = \frac{7\pi r^2 h}{24}$

$$\int z_c dV = \int_{h/2}^h z \pi \frac{r^2}{h^2} z^2 dz = \frac{15}{64} \pi r^2 h^2$$

$$\bar{z} = \int z_c dV / V = \frac{15}{64} \pi r^2 h^2 / \frac{7}{24} \pi r^2 h = \frac{45}{56} h$$

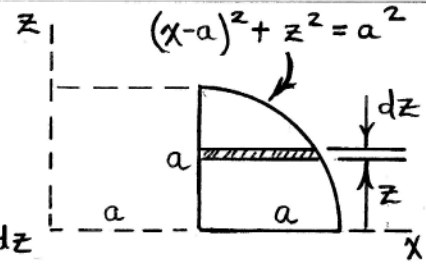
$$\bar{h} = h - \bar{z} = \frac{11}{56} h$$

$$\frac{5/31}{\bar{z}} = \frac{\int z dV}{V}$$

$$dV = \frac{\pi}{4} (x^2 - a^2) dz$$

$$= \frac{\pi}{4} [(a + \sqrt{a^2 - z^2})^2 - a^2] dz$$

$$= \frac{\pi}{4} [a^2 - z^2 + 2a\sqrt{a^2 - z^2}] dz$$



$$V = \frac{\pi}{4} \int_0^a [a^2 - z^2 + 2a\sqrt{a^2 - z^2}] dz$$

$$= \frac{\pi}{4} \left[ a^2 z - \frac{z^3}{3} + 2a \left( \frac{1}{2} \right) (z\sqrt{a^2 - z^2} + a^2 \sin^{-1} \frac{z}{a}) \right]_0^a$$

$$= \frac{\pi a^3}{2} \left( \frac{1}{3} + \frac{\pi}{4} \right)$$

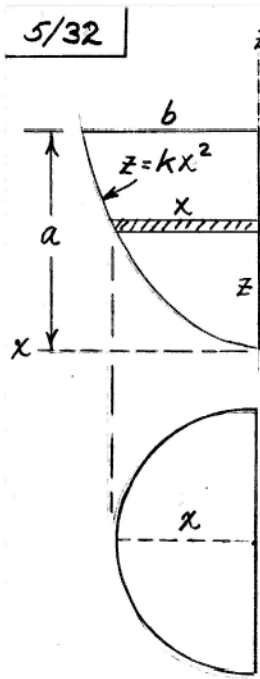
$$\int z dV = \frac{\pi}{4} \int_0^a [a^2 z - z^3 + 2a z \sqrt{a^2 - z^2}] dz$$

$$= \frac{\pi}{4} \left[ \frac{a^2 z^2}{2} - \frac{z^4}{4} - \frac{2a}{3} \sqrt{(a^2 - z^2)^3} \right]_0^a$$

$$= \frac{11}{48} \pi a^4$$

$$\bar{z} = \frac{\frac{11}{48} \pi a^4}{\frac{\pi a^3}{2} \left( \frac{1}{3} + \frac{\pi}{4} \right)} = \frac{11a}{2(4+3\pi)}$$

5/32



$$z = kb^2 \text{ so } k = a/b^2 \text{ \& } z = \frac{a}{b^2} x^2$$

$$dV = \frac{\pi x^2}{2} dz = \frac{\pi b^2}{2a} z dz$$

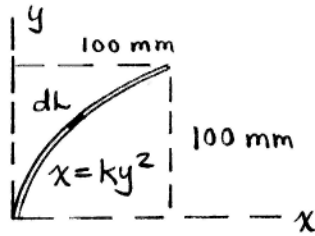
$$V = \frac{\pi b^2}{2a} \int_0^a z dz = \frac{\pi}{4} ab^2$$

$$\int \bar{z}_c dV = \int z dV = \frac{\pi b^2}{2a} \int_0^a z^2 dz = \frac{\pi}{6} a^2 b^2$$

$$\bar{z} = \frac{\int \bar{z}_c dV}{V} = \frac{\frac{\pi}{6} a^2 b^2}{\frac{\pi}{4} ab^2} = \underline{\underline{2a/3}}$$

5/33

$$x = ky^2 = \frac{y^2}{100}, \quad \frac{dx}{dy} = \frac{y}{50}$$



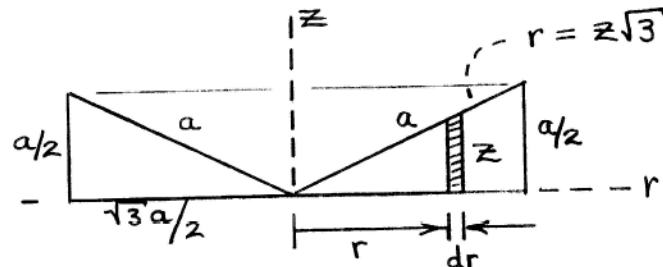
$$\begin{aligned} L &= \int dL = \int_0^{100} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \int_0^{100} \sqrt{1 + \frac{y^2}{50^2}} dy \\ &= \frac{1}{50} \int_0^{100} \sqrt{50^2 + y^2} dy = \frac{1}{50 \cdot 2} \left[ y \sqrt{50^2 + y^2} + 50^2 \ln(y + \sqrt{50^2 + y^2}) \right]_0^{100} \\ &= 147.9 \text{ mm} \end{aligned}$$

$$\begin{aligned} \int y_c dL &= \frac{1}{50} \int_0^{100} y \sqrt{50^2 + y^2} dy = \frac{1}{50} \frac{1}{3} (50^2 + y^2)^{3/2} \Big|_0^{100} \\ &= 8480 \text{ mm}^2 \end{aligned}$$

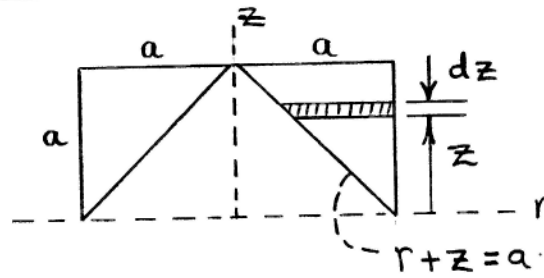
$$\bar{y} = \frac{\int y_c dL}{L} = \frac{8480}{147.9} = \underline{57.4 \text{ mm}}$$



5/34



$$\begin{aligned}
 V &= \int dV = \int_0^{\sqrt{3}a/2} 2\pi r z dr = \frac{2\pi}{\sqrt{3}} \int_0^{\sqrt{3}a/2} r^2 dr \\
 &= \frac{2\pi}{\sqrt{3}} \left( \frac{\sqrt{3}a/2}{3} \right)^3 = \frac{\pi a^3}{4} \\
 \int z_c dV &= \int_0^{\sqrt{3}a/2} \frac{r}{2\sqrt{3}} \frac{2\pi}{\sqrt{3}} r^2 dr = \frac{\pi}{3} \left[ \frac{r^4}{4} \right]_0^{\sqrt{3}a/2} = \frac{3\pi a^4}{64} \\
 \bar{z} &= \frac{\int z_c dV}{V} = \frac{3\pi a^4/64}{\pi a^3/4} = \frac{3a}{16}
 \end{aligned}$$



$$V = \int_0^a \pi (a^2 - r^2) dz = \pi \int_0^a [a^2 - (a-z)^2] dz$$

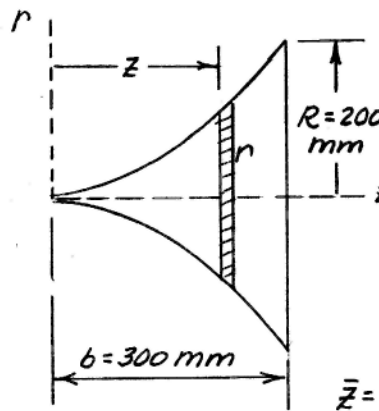
$$= \pi \int_0^a (2az - z^2) dz = \pi \left[ az^2 - \frac{z^3}{3} \right]_0^a = \frac{2}{3} \pi a^3$$

$$\int z_c dV = \int_0^a \pi (2az^2 - z^3) dz = \pi \left[ \frac{2az^3}{3} - \frac{z^4}{4} \right]_0^a$$

$$= \frac{5}{12} \pi a^4$$

$$\bar{z} = \frac{\int z_c dV}{V} = \frac{\frac{5}{12} \pi a^4}{\frac{2}{3} \pi a^3} = \frac{5a}{8}$$

$$5/36 \quad r = kz^3, R = kb^3, \text{ so } r = \frac{R}{b^3} z^3$$



$$dV = \pi r^2 dz = \pi \frac{R^2}{b^6} z^6 dz$$

$$V = \frac{\pi R^2}{b^6} \int_0^b z^6 dz = \frac{\pi}{7} R^2 b$$

$$\int \bar{z} dV = \int z dV = \frac{\pi R^2}{b^6} \int_0^b z^7 dz$$

$$= \frac{\pi R^2}{8} b$$

$$\bar{z} = \frac{\int \bar{z} dV}{V} = \frac{\frac{\pi R^2 b^2}{8}}{\frac{\pi R^2 b}{7}}$$

$$= \frac{7b}{8} = \frac{7(300)}{8} = \underline{263 \text{ mm}}$$

► 5/37

$x^2 + y^2 = a^2, \quad x = +\sqrt{a^2 - y^2}$   
 $dA = 2x dy = 2\sqrt{a^2 - y^2} dy$   
 $A = \int dA = \int_h^a 2\sqrt{a^2 - y^2} dy$   
 $= 2\left(\frac{1}{2}\right) \left[ y\sqrt{a^2 - y^2} + a^2 \sin^{-1} \frac{y}{a} \right]_h^a$   
 $= a^2 \left( \frac{\pi}{2} - \sin^{-1} \frac{h}{a} \right) - h\sqrt{a^2 - h^2}$

$$\int y dA = \int_h^a y 2\sqrt{a^2 - y^2} dy = 2 \left( -\frac{1}{3} \right) (a^2 - y^2)^{3/2} \Big|_h^a$$

$$= \frac{2}{3} (a^2 - h^2)^{3/2}$$

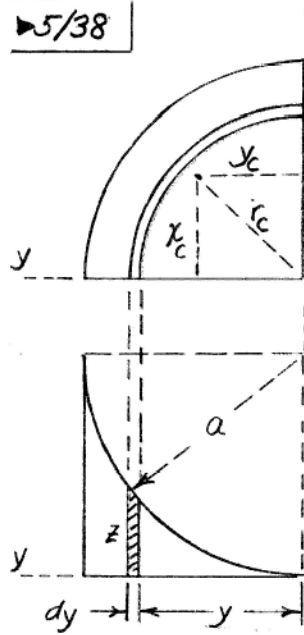
$$\bar{y} = \frac{\int y dA}{\int dA} = \frac{\frac{2}{3} (a^2 - h^2)^{3/2}}{a^2 \left( \frac{\pi}{2} - \sin^{-1} \frac{h}{a} \right) - h\sqrt{a^2 - h^2}}$$

Special cases

$$h = 0 : \quad \bar{y} = \frac{\frac{2}{3} a^3}{a^2 \frac{\pi}{2}} = \frac{4a}{3\pi} \quad (\text{the correct result})$$

$$h = \frac{a}{4} : \quad \bar{y} = \frac{\frac{2}{3} \left( a^2 - \left( \frac{a}{4} \right)^2 \right)^{3/2}}{a^2 \left( \frac{\pi}{2} - \sin^{-1} \frac{1}{4} \right) - \frac{a}{4} \sqrt{a^2 - \left( \frac{a}{4} \right)^2}} = \underline{0.562a}$$

$$h = \frac{a}{2} : \quad \bar{y} = \frac{\frac{2}{3} \left( a^2 - \left( \frac{a}{2} \right)^2 \right)^{3/2}}{a^2 \left( \frac{\pi}{2} - \sin^{-1} \frac{1}{2} \right) - \frac{a}{2} \sqrt{a^2 - \left( \frac{a}{2} \right)^2}} = \underline{0.705a}$$



From Sample Problem 5/1, for elemental shell,  $x_c = y_c = r_c / \sqrt{2} = \frac{2y}{\pi}$

$$dV = \frac{\pi y}{2} (z dy) = \frac{\pi}{2} (ay - y\sqrt{a^2 - y^2}) dy$$

$$V = \frac{\pi}{2} \int_0^a (ay - y\sqrt{a^2 - y^2}) dy$$

$$= \frac{\pi}{2} \left[ \frac{ay^2}{2} + \frac{1}{3} \sqrt{a^2 - y^2}^3 \right]_0^a = \frac{\pi a^3}{2} \cdot \frac{1}{6} = \frac{\pi a^3}{12}$$

$$\int y_c dV = \frac{\pi}{2} \int_0^a \frac{2y}{\pi} (ay - y\sqrt{a^2 - y^2}) dy$$

$$= \left[ \frac{ay^3}{3} + \frac{y}{4} \sqrt{a^2 - y^2}^3 - \frac{a^2}{8} (y\sqrt{a^2 - y^2} + a^2 \sin^{-1} \frac{y}{a}) \right]_0^a$$

$$= \left[ \frac{a^4}{3} + 0 - \frac{a^2}{8} (0 + a^2 \frac{\pi}{2}) \right] = a^4 \left[ \frac{1}{3} - \frac{\pi}{16} \right]$$

$$\bar{y} = \int y_c dV / V = a^4 \left( \frac{1}{3} - \frac{\pi}{16} \right) / \frac{\pi a^3}{12} = \left( \frac{4}{\pi} - \frac{3}{4} \right) a = \bar{y}$$

$$y^2 + (z-a)^2 = a^2 \quad \int z_c dV = \int \frac{z}{2} dV = \int_0^a \frac{a - \sqrt{a^2 - y^2}}{2} \pi (ay - y\sqrt{a^2 - y^2}) dy$$

$$z = a - \sqrt{a^2 - y^2} \quad = \frac{\pi a^4}{48}; \quad \bar{z} = \int z_c dV / V = \frac{\pi a^4}{48} / \frac{\pi a^3}{12} = \frac{a}{4}$$

(Note sign)

► 5/39 | Let  $\rho$  = mass per unit area of shell

$$z = k\theta = \frac{h}{\pi}\theta$$

$$dm = \rho z r d\theta = \frac{\rho h r}{\pi} \theta d\theta$$

$$m = \frac{\rho h r}{\pi} \int_0^{\pi} \theta d\theta = \frac{1}{2} \rho h r \pi$$

$$m\bar{x} = \int x dm = \int_0^{\pi} r \sin\theta \frac{\rho h r}{\pi} \theta d\theta$$

$$= \frac{\rho r^2 h}{\pi} \int_0^{\pi} \theta \sin\theta d\theta$$

$$= \frac{\rho r^2 h}{\pi} (\sin\theta - \theta \cos\theta)_0^{\pi} = \rho r^2 h$$

$$\text{So } \bar{x} = \frac{\rho r^2 h}{\rho h r \pi / 2} = \frac{2r}{\pi}$$

$$m\bar{y} = \int y dm = \int (-r \cos\theta) \frac{\rho h r}{\pi} \theta d\theta$$

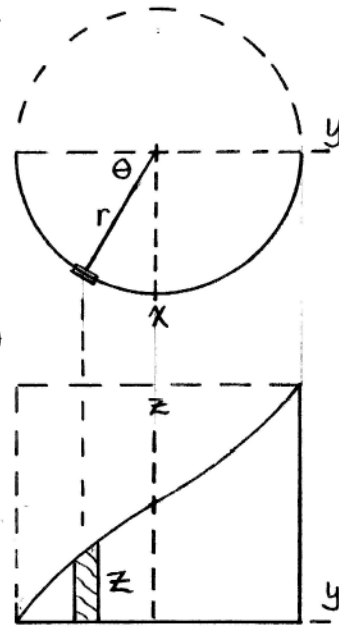
$$= -\frac{\rho h r^2}{\pi} \int_0^{\pi} \theta \cos\theta d\theta = -\frac{\rho h r^2}{\pi} (\cos\theta + \theta \sin\theta)_0^{\pi} = \frac{2\rho h r^2}{\pi}$$

$$\text{So } \bar{y} = \frac{2\rho h r^2 / \pi}{\rho h r \pi / 2} = \frac{4r}{\pi^2}$$

$$m\bar{z} = \int z dm = \int \frac{1}{2} \frac{h}{\pi} \theta \frac{\rho h r}{\pi} \theta d\theta = \frac{\rho h^2 r^2}{2\pi^2} \int_0^{\pi} \theta^2 d\theta$$

$$= \frac{\rho h r^2}{2\pi^2} \left(\frac{\theta^3}{3}\right)_0^{\pi} = \frac{1}{6} \rho h^2 r \pi$$

$$\text{So } \bar{z} = \frac{\rho h^2 r \pi / 6}{\rho h r \pi / 2} = \frac{1}{3} h$$

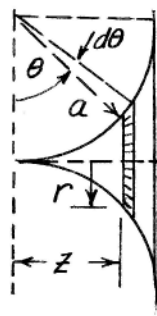


► 5/40  $dA = 2\pi r a d\theta = 2\pi a^2(1 - \cos\theta) d\theta$

$$\int z dA = \int_0^{\pi/2} (a \sin\theta)(2\pi a^2)(1 - \cos\theta) d\theta$$

$$= 2\pi a^3 \int_0^{\pi/2} (\sin\theta - \sin\theta \cos\theta) d\theta$$

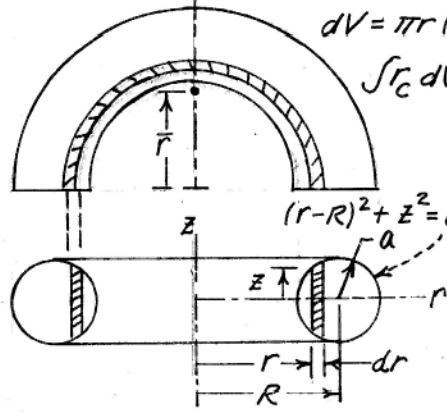
$$= 2\pi a^3 \left(1 - \frac{1}{2}\right) = \pi a^3$$



$$\int dA = 2\pi a^2 \int_0^{\pi/2} (1 - \cos\theta) d\theta = 2\pi a^2 \left(\frac{\pi}{2} - 1\right)$$

$$\bar{z} = \frac{\int z dA}{A} = \frac{\pi a^3}{2\pi a^2 \left(\frac{\pi}{2} - 1\right)} = \frac{a}{\pi - 2}$$

► 5/41 | From Sample Problem 5/1, centroidal coordinate for elemental ring is  $r_c = 2r/\pi$



$$dV = \pi r (2z) dr = 2\pi r \sqrt{a^2 - (r-R)^2} dr$$

$$\int r_c dV = 4 \int_{R-a}^{R+a} r^2 \sqrt{a^2 - (r-R)^2} dr$$

$$= \textcircled{1} + \textcircled{2} + \textcircled{3}$$

$$(r-R)^2 + z^2 = a^2 \text{ where } u = r-R \text{ \& } \textcircled{1} = 4 \int_{-a}^a u^2 \sqrt{a^2 - u^2} du = \frac{\pi a^4}{2}$$

$$\textcircled{2} = 4 \int_{-a}^a 2Ru \sqrt{a^2 - u^2} du = 0$$

$$\textcircled{3} = 4 \int_{-a}^a R^2 \sqrt{a^2 - u^2} du = 2\pi a^2 R^2$$

$$\int dV = 2\pi \int_{-a}^a (u+R) \sqrt{a^2 - u^2} du = 0 + 2\pi R \frac{\pi a^2}{2} = \pi^2 a^2 R$$

$$\bar{r} = \frac{\int r_c dV}{V} = \frac{\frac{\pi a^4}{2} + 2\pi a^2 R^2}{\pi^2 a^2 R} = \frac{a^2 + 4R^2}{2\pi R}$$



► 5/42

$$dV = dr (r d\phi) (r \sin \phi d\theta)$$

$$= r^2 \sin \phi d\phi d\theta dr$$

$$V = \int dV = \int_{R/2}^R \int_0^{\pi} \int_0^{\pi} r^2 \sin \phi d\phi d\theta dr$$

$$= \frac{7}{12} \pi R^3$$

(Check V by

$$V = \frac{\frac{4}{3} \pi R^3}{2} - \frac{\frac{4}{3} \pi (\frac{R}{2})^3}{2} = \frac{7}{12} \pi R^3)$$

$$\int x_c dV = \int (r \sin \phi \sin \theta) (r^2 \sin \phi d\phi d\theta dr)$$

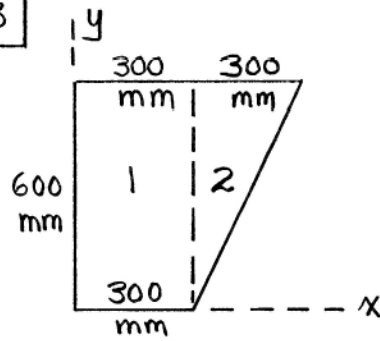
$$= \int_{R/2}^R \int_0^{\pi} \int_0^{\pi} r^3 \sin^2 \phi \sin \theta d\phi d\theta dr = \frac{15}{64} \pi R^4$$

$$\bar{x} = \frac{\int x_c dV}{V} = \frac{\frac{15}{64} \pi R^4}{\frac{7}{12} \pi R^3} = \frac{45}{112} R$$

(Compare to  $\bar{x} = \frac{3}{8} R$  for no hole.)

Note: A hemispherical shell of radius  $r$  and thickness  $dr$  would be a better element.

5/43



$$A_1 = 18(10^4) \text{ mm}^2, \quad \bar{x}_1 = 150 \text{ mm}, \quad \bar{y}_1 = 300 \text{ mm}$$

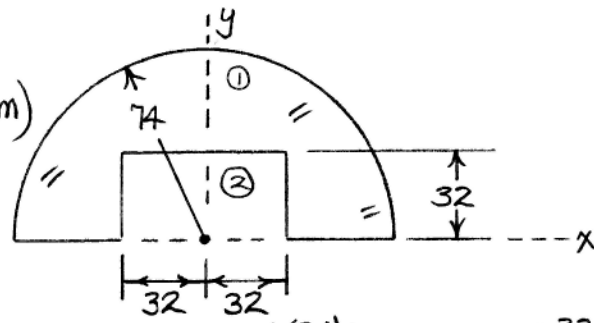
$$A_2 = 9(10^4) \text{ mm}^2, \quad \bar{x}_2 = 300 + \frac{1}{3}(300) = 400 \text{ mm}$$

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{18(10^4)(150) + 9(10^4)(400)}{18(10^4) + 9(10^4)} = \underline{233 \text{ mm}}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{18(10^4)(300) + 9(10^4)(400)}{18(10^4) + 9(10^4)} = \underline{333 \text{ mm}}$$

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(Dim. in mm)

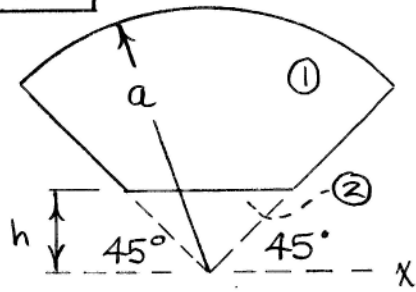


$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{\pi \frac{74^2}{2} \left( \frac{4(74)}{3\pi} \right) - 64(32) \left( \frac{32}{2} \right)}{\pi \frac{74^2}{2} - 64(32)}$$

$$= \underline{36.2 \text{ mm}}$$

5/45

1y



Circular sector (full) ①:

$$A_1 = \frac{\pi}{4} a^2$$

$$\bar{y}_1 = \frac{2}{3} a \frac{\sin 45^\circ}{\pi/4}$$

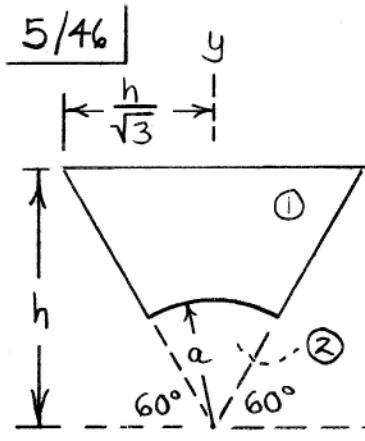
$$= \frac{4\sqrt{2}}{3\pi} a$$

Triangular "hole" ②:

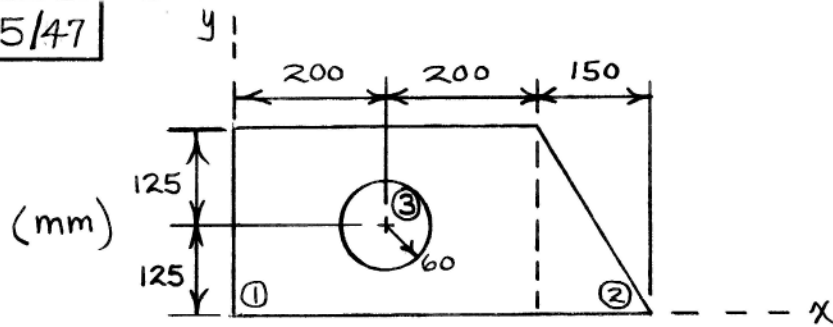
$$A_2 = \frac{1}{2} h(2h) = h^2, \quad \bar{y}_2 = \frac{2}{3} h$$

$$\bar{Y} = \frac{\sum A \bar{y}}{\sum A} = \frac{\frac{\pi}{4} a^2 \left( \frac{4\sqrt{2}}{3\pi} a \right) - h^2 \left( \frac{2}{3} h \right)}{\frac{\pi}{4} a^2 - h^2}$$

$$= \frac{4(\sqrt{2} a^3 - 2h^3)}{3(\pi a^2 - 4h^2)}$$



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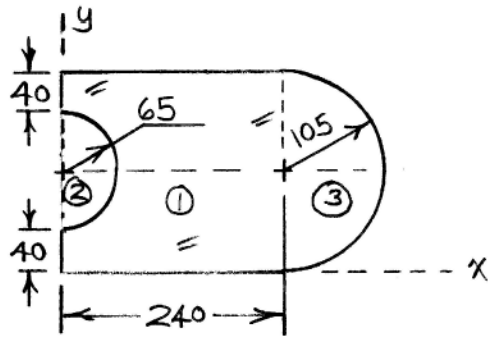


Comp.	A mm <sup>2</sup>	$\bar{x}$ mm	$\bar{y}$ mm	A $\bar{x}$ mm <sup>3</sup>	A $\bar{y}$ mm <sup>3</sup>
Rect. 1	100 (10 <sup>3</sup> )	200	125	20 (10 <sup>6</sup> )	12.50 (10 <sup>6</sup> )
Triangle 2	18.75 (10 <sup>3</sup> )	450	250/3	8.44 (10 <sup>6</sup> )	1.563 (10 <sup>6</sup> )
Circle 3	-11.31 (10 <sup>3</sup> )	200	125	-2.26 (10 <sup>6</sup> )	-1.414 (10 <sup>6</sup> )
Totals	107.4 (10 <sup>3</sup> )			26.2 (10 <sup>6</sup> )	12.65 (10 <sup>6</sup> )

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{26.2 (10^6)}{107.4 (10^3)} = \underline{244 \text{ mm}}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{12.65 (10^6)}{107.4 (10^3)} = \underline{117.7 \text{ mm}}$$

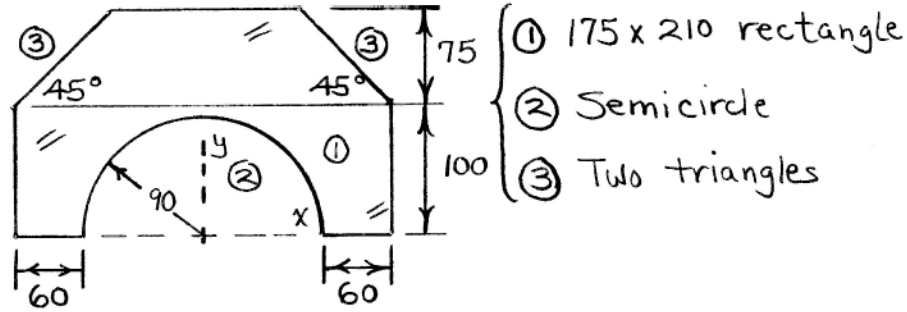
5/48



$$\bar{Y} = \frac{1}{2} (40 + 2(65) + 40) = 105 \text{ mm}$$
$$\bar{X} = \frac{210(240)\left(\frac{240}{2}\right) - \frac{\pi 65^2}{2} \frac{4(65)}{3\pi} + \frac{\pi 105^2}{2} \left(240 + \frac{4(105)}{3\pi}\right)}{(210)(240) - \frac{\pi 65^2}{2} + \frac{\pi 105^2}{2}}$$
$$= \underline{176.7 \text{ mm}}$$

5/49 | (Dim. in mm)

Components :

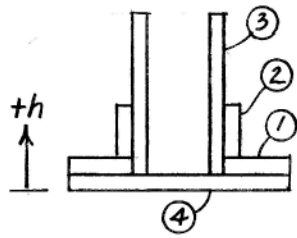


Comp.	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$
①	$175(210)$	$\frac{175}{2}$	3 215 625
②	$-\frac{\pi(90^2)}{2}$	$\frac{4(90)}{3\pi}$	- 486 000
③	$-2 \frac{1}{2}(75)(75)$	$(100 + \frac{2}{3}75)$	- 843 750
$\Sigma A = 18 400$		$\Sigma A\bar{y} = 1 886 000$	

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{102.5 \text{ mm}}$$



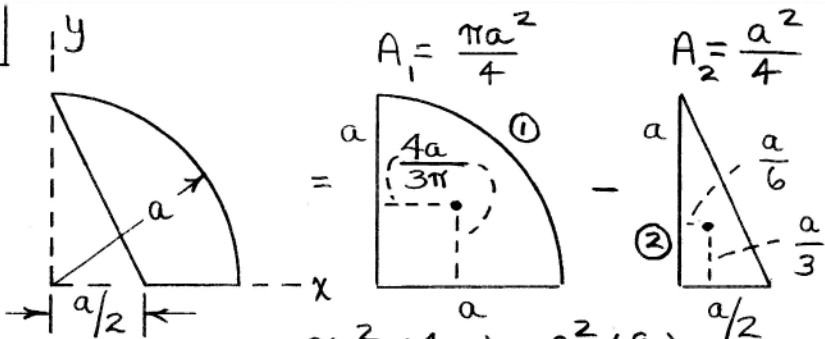
5/50



Part	Size mm	A mm <sup>2</sup>	$\bar{h}$ mm	$A\bar{h}$ mm <sup>3</sup>
①	10x40	800	15	12000
②	10x40	800	40	32000
③	10x120	2400	70	168000
④	10x160	1600	5	8000
$\Sigma$ 's		5600		220000

$$\bar{H} = \frac{\Sigma A\bar{h}}{\Sigma A} = \frac{220000}{5600} = \underline{39.3 \text{ mm}}$$

5/51



$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{\frac{\pi a^2}{4} \left(\frac{4a}{3\pi}\right) - \frac{a^2}{4} \left(\frac{a}{6}\right)}{\frac{\pi a^2}{4} - \frac{a^2}{4}}$$

$$= \frac{7a}{6(\pi-1)}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{\frac{\pi a^2}{4} \left(\frac{4a}{3\pi}\right) - \frac{a^2}{4} \left(\frac{a}{3}\right)}{\frac{\pi a^2}{4} - \frac{a^2}{4}}$$

$$= \frac{a}{\pi-1}$$

$$\frac{5/52}{\bar{X}} = \frac{\sum \bar{x}L}{\sum L}, \quad \bar{Y} = \frac{\sum \bar{y}L}{\sum L}$$

$$L_1 = \pi r = 150\pi \text{ mm}$$

$$\bar{x}_1 = \frac{2r}{\pi} = \frac{300}{\pi} \text{ mm}$$

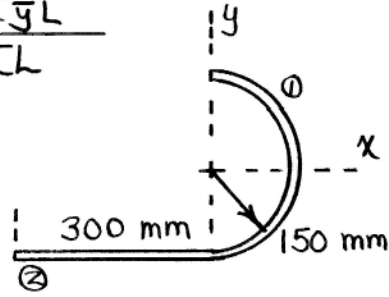
(from Sample Prob. 5/1)

$$\bar{y}_1 = 0$$

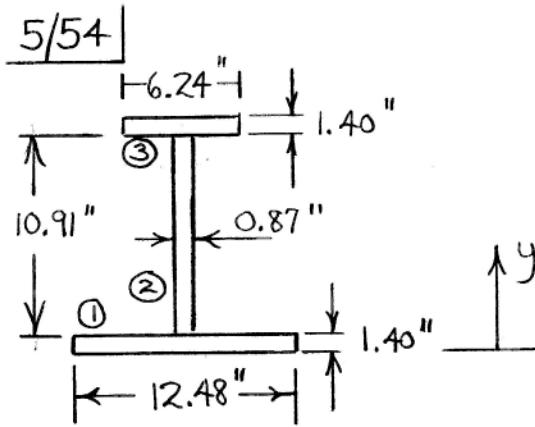
$$L_2 = 300 \text{ mm}, \quad \bar{x}_2 = -150 \text{ mm}, \quad \bar{y}_2 = -150 \text{ mm}$$

$$\text{So } \left\{ \begin{array}{l} \bar{X} = \frac{\frac{300}{\pi}(150\pi) + 300(-150)}{150\pi + 300} = \underline{0} \end{array} \right.$$

$$\left\{ \begin{array}{l} \bar{Y} = \frac{\frac{300}{\pi}(0) + 300(-150)}{150\pi + 300} = \underline{-58.3 \text{ mm}} \end{array} \right.$$



$$\frac{5/53}{\bar{z}} = \frac{\sum m \bar{z}}{\sum m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70 \text{ mm}}$$



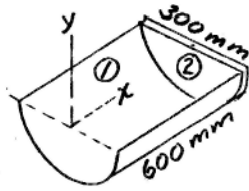
Comp.	$A$ (in. <sup>2</sup> )	$\bar{y}$ (in.)	$A\bar{y}$ (in. <sup>3</sup> )
①	$(12.48)(1.40)$	$\frac{1.40}{2}$	12.23
②	$(10.91)(0.87)$	$1.4 + \frac{10.91}{2}$	65.1
③	$(6.24)(1.40)$	$1.4 + 10.91 + \frac{1.4}{2}$	113.7
	$\Sigma A = 35.7$		$\Sigma A\bar{y} = 191.0$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{5.35 \text{ in.}}$$

5/55

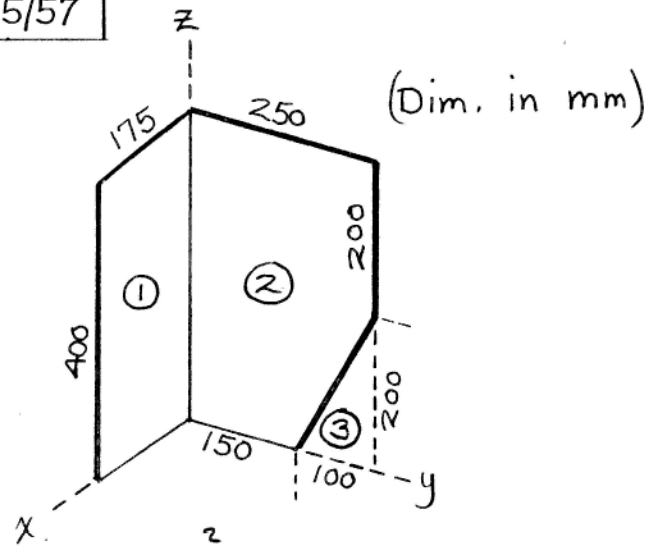
$$\bar{X} = \frac{\sum V\bar{x}}{\sum V} = \frac{\frac{2}{3}\pi R^3 \left(\frac{3R}{8}\right) - \frac{2}{3}\pi \left(\frac{R}{2}\right)^3 \frac{3\left(\frac{R}{2}\right)}{8}}{\frac{2}{3}\pi R^3 - \frac{2}{3}\pi \left(\frac{R}{2}\right)^3}$$
$$= \frac{45}{112} R$$

5/56



Part	$A$ $m^2$	$m$ $kg$	$\bar{x}$ $mm$	$\bar{y}$ $mm$	$m\bar{x}$ $kg \cdot mm$	$m\bar{y}$ $kg \cdot mm$
①	0.283	6.79	300	-95.5	2036	-648
②	0.0353	1.27	600	-63.7	763	-81
		8.06			2799	-729

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} = \frac{2799}{8.06} = 347.4 \text{ mm}, \quad \bar{Y} = \frac{\sum m\bar{y}}{\sum m} = \frac{-729}{8.06} = -90.5 \text{ mm}$$



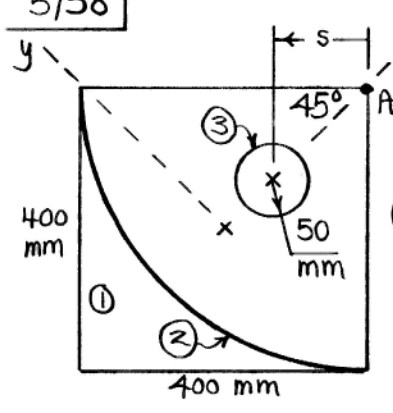
Comp.	mm				in $10^6 \text{ mm}^3$		
	$A$	$\bar{x}$	$\bar{y}$	$\bar{z}$	$A\bar{x}$	$A\bar{y}$	$A\bar{z}$
①	$400(175)$	$\frac{175}{2}$	0	$\frac{400}{2}$	6.13	0	14
②	$400(250)$	0	$\frac{250}{2}$	$\frac{400}{2}$	0	12.5	20
③	$\frac{1}{2}(100)(200)$	0	$\left(\frac{150+}{2}100\right)$	$\frac{200}{3}$	0	-2.17	-0.667
	160.000				6.13	10.33	33.3

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \underline{38.3 \text{ mm}}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \underline{64.6 \text{ mm}}$$

$$\bar{Z} = \frac{\sum A\bar{z}}{\sum A} = \underline{208 \text{ mm}}$$





$$\begin{cases} \rho_{Al} = 2690 (10^{-9}) \text{ kg/mm}^3 \\ \rho_{St} = 7830 (10^{-9}) \text{ kg/mm}^3 \end{cases}$$

(Note  $s\sqrt{2}$  must not be between 350 and 450 mm, or else the hole would be beneath the wire.)

$$m_1 = (400)^2 (6) (2690) (10^{-9}) = 2.58 \text{ kg}$$

$$\bar{x}_1 = 0$$

$$m_2 = \frac{\pi (400)^2}{4} (6) (10^{-9}) = 0.314 \text{ kg}$$

$$\bar{x}_2 = - \left[ \frac{400 \sin \frac{\pi}{4}}{\frac{\pi}{4}} - \frac{400}{\sqrt{2}} \right] = -77.3 \text{ mm}$$

$$m_3 = -\pi (50)^2 (6) (2690) (10^{-9}) = -0.1268 \text{ kg}$$

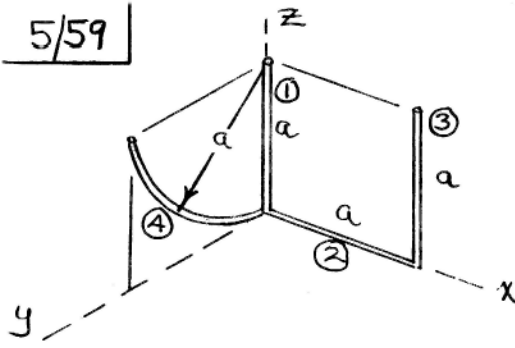
$$\bar{x}_3 = \frac{400}{\sqrt{2}} - s\sqrt{2}$$

$$\bar{X} = \frac{\sum m \bar{x}}{\sum m} = 0 \Rightarrow \sum m \bar{x} = 0$$

$$\text{So } 2.58(0) + 0.314(-77.3) - 0.1268(200\sqrt{2} - s\sqrt{2}) = 0$$

Solve to obtain  $s = +335 \text{ mm}$  ( $335\sqrt{2} = 474 \text{ mm}$  from point A; so hole clears slender wire by 24 mm!)

5/59


 $(\rho = \text{mass/length})$ 

Comp.	$m$	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}$	$m\bar{y}$	$m\bar{z}$
1	$\rho a$	0	0	$a/2$	0	0	$\rho a^2/2$
2	$\rho a$	$a/2$	0	0	$\rho a^2/2$	0	0
3	$\rho a$	$a$	0	$a/2$	$\rho a^2$	0	$\rho a^2/2$
4	$\rho \pi a/2$	0	$-2a/\pi$	$a(1-\frac{2}{\pi})$	0	$-\rho a^2$	$\rho a^2(\frac{\pi}{2}-1)$
Totals	$\rho a(3+\frac{\pi}{2})$				$\frac{3}{2}\rho a^2$	$-\rho a^2$	$\frac{\pi}{2}\rho a^2$

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} = \frac{\frac{3}{2}\rho a^2}{\rho a(3+\frac{\pi}{2})} = \frac{3a}{6+\pi}$$

$$\bar{Y} = \frac{\sum m\bar{y}}{\sum m} = \frac{-\rho a^2}{\rho a(3+\frac{\pi}{2})} = -\frac{2a}{6+\pi}$$

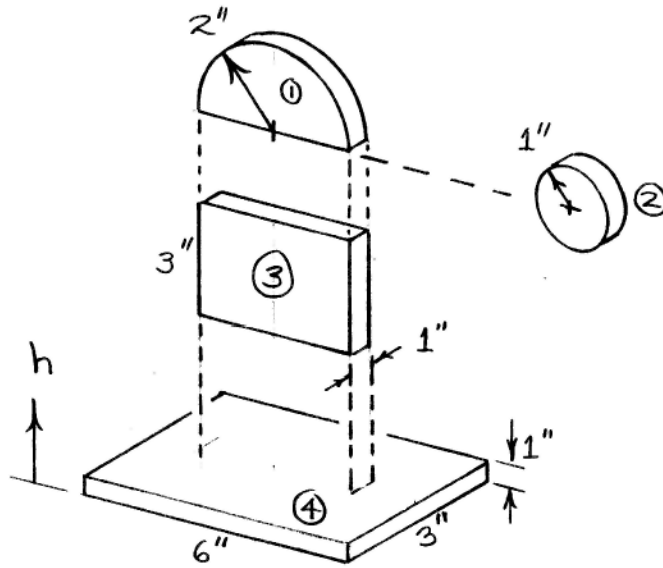
$$\bar{Z} = \frac{\sum m\bar{z}}{\sum m} = \frac{\frac{\pi}{2}\rho a^2}{\rho a(3+\frac{\pi}{2})} = \frac{\pi a}{6+\pi}$$

5/60 | 1 = semicircular rod  
 2 = two straight rods  
 3 = semicircular plate

Part	L, ft	A, ft <sup>2</sup>	W, lb	$\bar{x}$ , in.	$\bar{z}$ , in.	$W\bar{x}$	$W\bar{z}$
1	1.047	-	0.387	6	2.546	2.325	0.987
2	1	-	0.370	3	0	1.110	0
3	-	0.1745	1.396	0	1.698	0	2.370
$\Sigma$ 's			2.153			3.435	3.357

$$\bar{X} = \frac{\Sigma W\bar{x}}{\Sigma W} = \frac{3.435}{2.153} = 1.595 \text{ in.} \quad \bar{Z} = \frac{\Sigma W\bar{z}}{\Sigma W} = \frac{3.357}{2.153} = 1.559 \text{ in.}$$

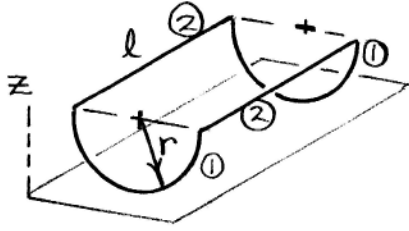
5/61



Part	$V, \text{in.}^3$	$\bar{h}, \text{in.}$	$V\bar{h}, \text{in.}^4$
①	6.28	4.85	30.5
②	-3.14	4	-12.57
③	12	2.5	30
④	18	0.5	9
Totals	33.1		56.9

$$\bar{H} = \frac{\sum V\bar{h}}{\sum V} = \frac{56.9}{33.1} = \underline{1.717 \text{ in.}}$$

5/62



Comp.	L	$\bar{z}$	$L\bar{z}$
①	$2(\pi r)$	$r - \frac{2r}{\pi}$	$2r^2(\pi - 2)$
②	$2l$	$r$	$2lr$

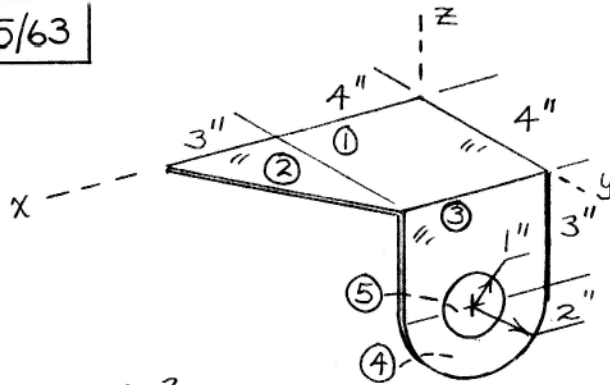
$\Sigma L = 2(\pi r + l)$                        $\Sigma L\bar{z} = 2r[r(\pi - 2) + l]$

Set  $\bar{z} = \frac{\Sigma L\bar{z}}{\Sigma L} = \frac{2r[r(\pi - 2) + l]}{2(\pi r + l)} = \frac{3r}{4}$

and solve for l as

$$\underline{l = (8 - \pi)r}$$

5/63



Comp.	in. <sup>2</sup>				in. <sup>3</sup>		
	A	$\bar{x}$	$\bar{y}$	$\bar{z}$	$A\bar{x}$	$A\bar{y}$	$A\bar{z}$
①	16	2	2	0	32	32	0
②	6	5	$\frac{4}{3}$	0	30	8	0
③	12	2	4	-1.5	24	48	-18
④	$\frac{\pi 2^2}{2}$	2	4	$-(3 + \frac{4(2)}{3\pi})$	12.57	25.1	-24.2
⑤	$-\pi 1^2$	2	4	-3	-6.28	-12.56	9.42

$$\Sigma A = 37.1$$

$$\Sigma = 92.3 \quad 100.6 \quad -32.8$$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{92.3}{37.1} = \underline{2.48 \text{ in.}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{100.6}{37.1} = \underline{2.71 \text{ in.}}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{-32.8}{37.1} = \underline{-0.882 \text{ in.}}$$

5/64



$$\left\{ \begin{aligned} V_1 &= \frac{\pi}{2} (30)^2 (35) = 49\,500 \text{ mm}^3 \\ \bar{x}_1 &= -\frac{4(30)}{3\pi} = -12.73 \text{ mm} \\ \bar{z}_1 &= 17.5 \text{ mm} \end{aligned} \right.$$

Length = 35 mm



$$V_2 = -\frac{\pi}{2} (20)^2 (25) = -15\,710 \text{ mm}^2$$

$$\bar{x}_2 = -\frac{4(20)}{3\pi} = -8.49 \text{ mm}$$

$$\bar{z}_2 = \frac{1}{2}(10 + 35) = 22.5 \text{ mm}$$

Length = 25 mm

$$\begin{aligned} \bar{X} &= \frac{\sum V \bar{x}}{\sum V} = \frac{49\,500(-12.73) - 15\,710(-8.49)}{49\,500 - 15\,710} \\ &= \underline{-14.71 \text{ mm}} \end{aligned}$$

$$\begin{aligned} \bar{Z} &= \frac{\sum V \bar{z}}{\sum V} = \frac{49\,500(17.5) - 15\,710(22.5)}{49\,500 - 15\,710} \\ &= \underline{15.17 \text{ mm}} \end{aligned}$$

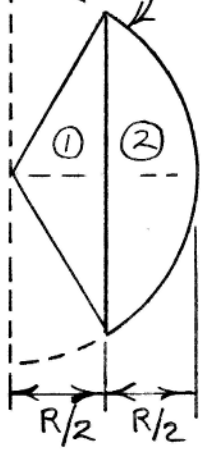
5/65

Interval	$A_{AV}$ ft <sup>2</sup>	Vol. $V$ ft <sup>3</sup>	$\bar{x}_{AV}$ ft	$V\bar{x}$ ft <sup>3</sup>
0-2	1.0	2.0	1.5	3.0
2-4	3.3	6.6	3.2	21.1
4-6	6.5	13.0	5.1	66.3
6-8	10.0	20.0	7.0	140.0
8-10	12.6	25.2	9.0	226.8
10-12	14.1	28.2	11.0	310.2
12-14	14.4	28.8	13.0	374.4
14-16	13.7	27.4	15.0	411.0
16-18	12.0	24.0	17.0	408.0
18-20	9.0	18.0	19.0	342.0
20-22	5.2	10.4	21.0	218.4
22-24	1.5	3.0	22.5	67.5
		206.6		2588.7

$$\bar{X} = \frac{\sum V\bar{x}}{\sum V} = \frac{2588.7}{206.6} = \underline{12.53 \text{ ft}}$$



5/66  
y



$$x^2 + y^2 = R^2, \quad y^2 = R^2 - x^2$$

$$\text{At } x = \frac{R}{2}, \quad y^2 = R^2 - \frac{R^2}{4} = \frac{3}{4}R^2$$

Conical volume (1):

$$V_1 = \frac{1}{3} \pi \left(\frac{3R^2}{4}\right) \left(\frac{R}{2}\right) = \frac{1}{8} \pi R^3$$

$$\bar{x}_1 = \frac{3}{4} \left(\frac{R}{2}\right) = \frac{3}{8} R$$

Spherical cap (2)

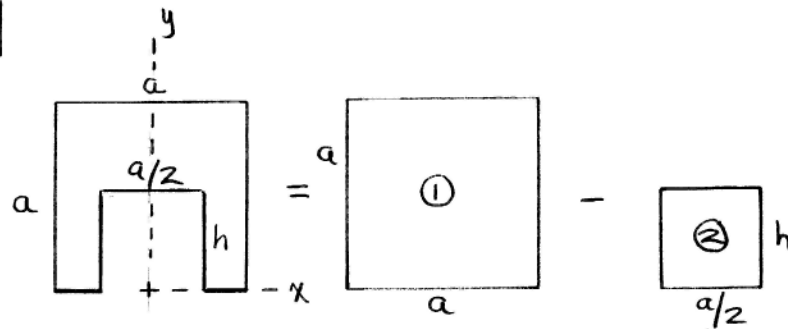
$$V_2 = \frac{5}{24} \pi R^3 \quad \left. \begin{array}{l} \text{from solution} \\ \text{to Prob. 5/27} \end{array} \right\}$$

$$\bar{x}_2 = \frac{27}{40} R$$

$$\bar{X} = \frac{\sum V \bar{x}}{\sum V} = \frac{\frac{1}{8} \pi R^3 \left(\frac{3R}{8}\right) + \frac{5}{24} \pi R^3 \left(\frac{27R}{40}\right)}{\frac{1}{8} \pi R^3 + \frac{5}{24} \pi R^3}$$

$$= \frac{9}{16} R \quad (\text{fairly close to } \frac{R}{2} !)$$

5/67



$$A_1 = a^2, \quad \bar{y}_1 = a/2; \quad A_2 = -ah/2, \quad \bar{y}_2 = h/2$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{a^2(\frac{a}{2}) - \frac{ah}{2}(\frac{h}{2})}{a^2 - \frac{ah}{2}} = \frac{1}{2} \frac{a^2 - h^2/2}{a - h/2}$$

$$\frac{d\bar{Y}}{dh} = \frac{1}{2} \frac{(a - h/2)(-h) - (a^2 - h^2/2)(-1/2)}{(a - h/2)^2}$$

$$= \frac{1}{2} \frac{\frac{h^2}{4} - ah + \frac{a^2}{2}}{(a - \frac{h}{2})^2} = 0 \quad \text{for max } \bar{Y}.$$

$$\text{So } \frac{1}{4}(h^2 - 4ah + 2a^2) = 0, \quad h = a(2 \pm \sqrt{2})$$

The plus sign is rejected because  $h$  must be less than  $a$ .

$$\text{Hence } h = a(2 - \sqrt{2}) = \underline{0.586a}$$

(Note that  $\bar{Y} = 0.586a$  for  $h = 0.586a$ )

$$\underline{5/68} \quad \text{Cube: } \begin{cases} V_1 = 350^3 = 42\,875\,000 \text{ mm}^3 \\ \bar{z}_1 = 175 \text{ mm} \end{cases}$$

$$\text{Hole: } \begin{cases} V_2 = -\pi(100)^2 h \\ \bar{z}_2 = \frac{h}{2} \end{cases}$$

$$\bar{z} = \frac{\sum V \bar{z}}{\sum V} = \frac{42\,875\,000(175) - \pi(100)^2 h \frac{h}{2}}{42\,875\,000 - \pi(100)^2 h}$$

For The maximum  $\bar{z}$ , set  $\frac{d\bar{z}}{dh} = 0$ :

$$\frac{(42\,875\,000 - \pi 100^2 h)(-\pi 100^2 h) - (42\,875\,000(175) - \pi 100^2 \frac{h^2}{2}) \times (-\pi 100^2)}{\text{(denominator)}} = 0$$

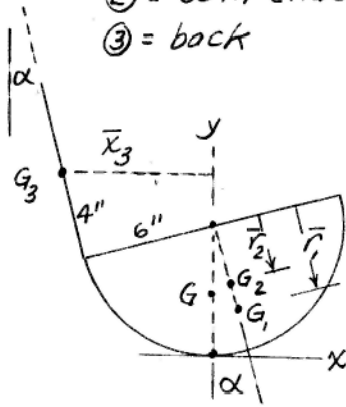
Set numerator equal to zero to obtain

$$\underline{h = 187.9 \text{ mm}}$$

► 5/69 | ① = semicircular shell

② = both ends

③ = back



$$A_1 = \pi(6)(16) = 301.6 \text{ in}^2$$

$$A_2 = 2(\pi \cdot 6^2/2) = 113.1 \text{ in}^2$$

$$A_3 = 8(16) = 128 \text{ in}^2$$

$$\Sigma A = 542.7 \text{ in}^2$$

$$\bar{r}_1 = \frac{2(6)}{\pi} = 3.82 \text{ in}$$

$$\bar{r}_2 = \frac{4(6)}{3\pi} = 2.55 \text{ in}$$

$$\bar{x}_1 = 3.82 \sin \alpha, \quad \bar{x}_2 = 2.55 \sin \alpha$$

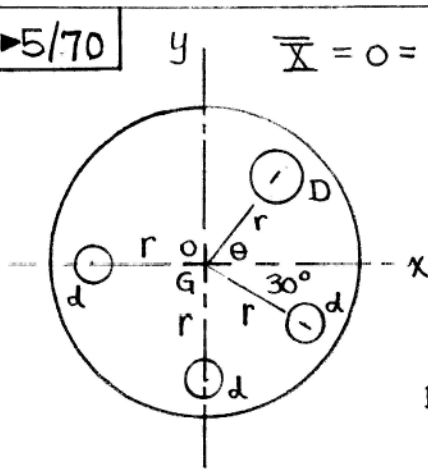
$$\bar{x}_3 = -6 \cos \alpha - 4 \sin \alpha$$

$$\bar{X} = \frac{\Sigma A \bar{x}}{\Sigma A}; \quad 0 = 301.6(3.82 \sin \alpha) + 113.1(2.55 \sin \alpha) + 128(-6 \cos \alpha - 4 \sin \alpha) = 0$$

$$928.0 \sin \alpha = 768 \cos \alpha, \quad \tan \alpha = \frac{768}{928} = 0.828$$

$$\alpha = 39.6^\circ$$

► 5/70



$$\bar{X} = 0 = \frac{\pi D^2}{4} r \cos \theta + \frac{\pi d^2}{4} r \cos 30^\circ - \frac{\pi d^2}{4} r, \quad D^2 \cos \theta = d^2 \left(1 - \frac{\sqrt{3}}{2}\right)$$

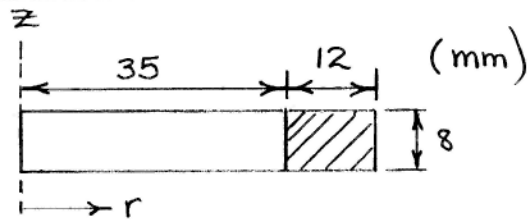
$$\bar{Y} = 0 = \frac{\pi D^2}{4} r \sin \theta - \frac{\pi d^2}{4} r - \frac{\pi d^2}{4} r \sin 30^\circ$$

$$D^2 \sin \theta = d^2 \left(1 + \frac{1}{2}\right)$$

Divide :  $\frac{\sin \theta}{\cos \theta} = \frac{3/2}{1 - \sqrt{3}/2}, \quad \theta = 84.9^\circ$

$$D^2 = \frac{3d^2/2}{\sin 84.9^\circ}, \quad D = 1.227 d$$

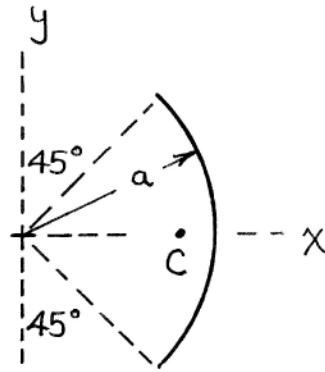
5/71



$$A = 2\pi r L = 2\pi \left(35 + \frac{12}{2}\right) (12 + 12 + 8 + 8)$$
$$= \underline{10\,300 \text{ mm}^2}$$

$$V = 2\pi r A = 2\pi \left(35 + \frac{12}{2}\right) (12 \cdot 8)$$
$$= \underline{24\,700 \text{ mm}^3}$$

5/72

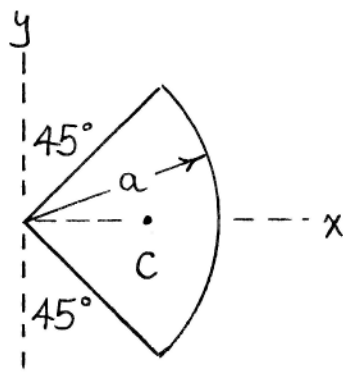


From Table D/3,

$$\bar{x} = \frac{a \sin \frac{\pi}{4}}{\pi/4} = a \frac{\frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{2\sqrt{2}}{\pi} a$$

$$A = 2\pi \bar{x} L = 2\pi \left( \frac{2\sqrt{2}}{\pi} a \right) \left( \frac{2\pi a}{4} \right) \\ = \underline{2\sqrt{2} \pi a^2}$$

5/73



From Table D/3,

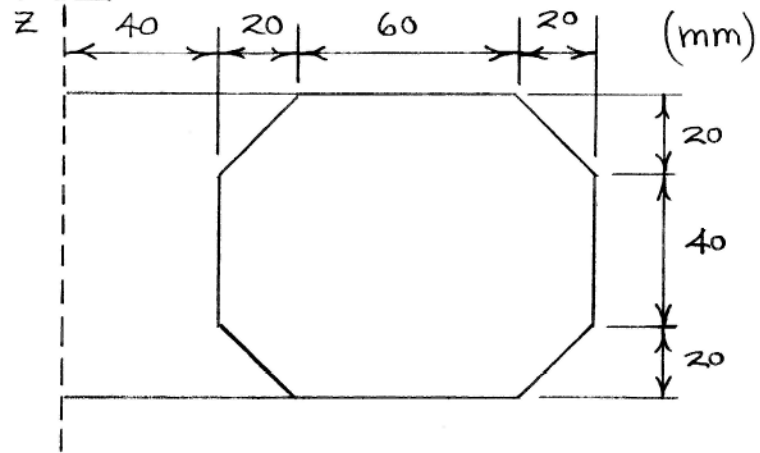
$$\bar{x} = \frac{2}{3} a \frac{\sin \pi/4}{\pi/4} = \frac{2}{3} a \left( \frac{\sqrt{2}}{2} \right) \left( \frac{4}{\pi} \right) = \frac{4\sqrt{2}}{3\pi} a$$

$$\begin{aligned} V &= 2\pi \bar{x} A = 2\pi \left( \frac{4\sqrt{2}}{3\pi} a \right) \left( \frac{\pi a^2}{4} \right) \\ &= \frac{2\pi\sqrt{2} a^3}{3} \end{aligned}$$



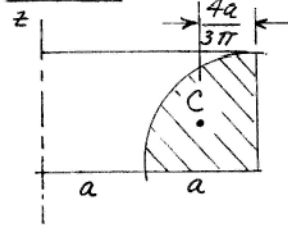
$$\underline{5/74} \quad V = \theta \bar{r} A = \pi \left( 8 + \frac{2}{3} 12 \right) \frac{1}{2} (12)(12) = \underline{3620 \text{ mm}^3}$$

5/75 |



$$A = 2\pi \bar{r} L = 2\pi \left[ 40 + 20 + \frac{60}{2} \right] \left[ 2(60) + 2(40) + 4 \cdot 20\sqrt{2} \right]$$
$$= \underline{177\,100 \text{ mm}^2}$$

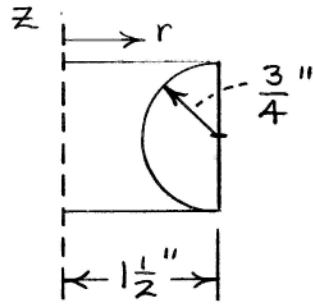
5/76



$$V = \bar{r} \theta A$$

$$= \left(2a - \frac{4a}{3\pi}\right) \frac{\pi}{2} \frac{\pi a^2}{4} = \frac{\pi a^3}{12} (3\pi - 2)$$

5/77



$$V = 2\pi \bar{r} A = 2\pi \left[ 1.5 - \frac{4\left(\frac{3}{4}\right)}{3\pi} \right] \left[ \frac{\pi \left(\frac{3}{4}\right)^2}{2} \right]$$
$$= \underline{6.56 \text{ in.}^3}$$

Inner area

$$A_i = 2\pi \bar{r} L = 2\pi \left[ 1.5 - \frac{2\left(\frac{3}{4}\right)}{\pi} \right] \left[ \frac{2\pi \left(\frac{3}{4}\right)}{2} \right]$$
$$= 15.14 \text{ in.}^2$$

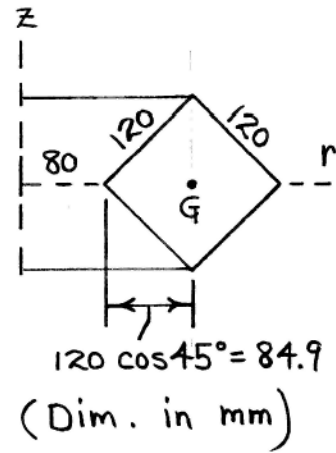
$$\text{Outer area } A_o = 2\left(\frac{3}{4}\right)(2\pi \cdot 1.5)$$
$$= 14.14 \text{ in.}^2$$

$$\text{So } A = A_i + A_o = \underline{29.3 \text{ in.}^2}$$

5/78

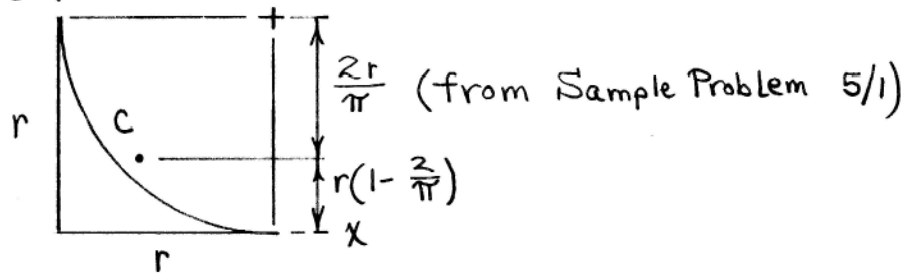
$$\begin{aligned} V &= 2\pi \bar{r} A \\ &= 2\pi (80 + 84.9) (120)^2 \\ &= \underline{14.92 (10^6) \text{ mm}^3} \end{aligned}$$

$$\begin{aligned} \text{Surface area } A &= 2\pi \bar{r} L \\ &= 2\pi (80 + 84.9) (4 \times 120) \\ &= \underline{497 (10^3) \text{ mm}^2} \end{aligned}$$



5/79

y :



"Lateral" surface area is

$$A_L = 2\pi \bar{y} h = 2\pi r \left(1 - \frac{2}{\pi}\right) \left(\frac{\pi r}{2}\right) = \pi^2 r^2 \left(1 - \frac{2}{\pi}\right)$$

"Bottom" (left end) surface area is

$$A_B = \pi r^2$$

Total body surface area is

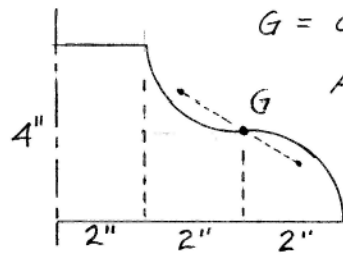
$$A = A_L + A_B = \underline{\pi r^2 [\pi - 1]}$$

$$\frac{5}{80} \quad A = 2\pi rL + \pi dh$$

$$= 2\pi(8.2)34 + \pi(8)(18) = 2204 \text{ ft}^2$$

$$\text{No. of gal. for 2 coats} = \frac{2204}{500} \times 2 = \underline{8.82 \text{ gal}}$$

5/81



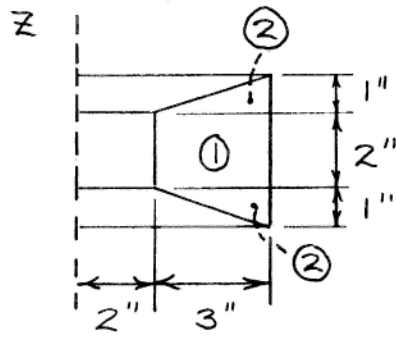
$G = \text{centroid by symmetry}$

$$A = 2\pi \bar{r} L = 2\pi (4) \pi (2)$$

$$= 16\pi^2 = \underline{157.9 \text{ in.}^2}$$



5/82



$$V_1 = \pi \bar{r}_1 A_1 = \pi \left(2 + \frac{3}{2}\right) (3 \cdot 2) = 66.0 \text{ in.}^3$$

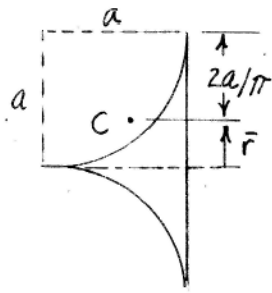
$$V_2 = 2 \left( \pi \bar{r}_2 A_2 \right) = 2 \pi \left(5 - \frac{3}{3}\right) \left(\frac{1}{2} (3) (1)\right) = 37.7 \text{ in.}^3$$

$$V = V_1 + V_2 = 103.7 \text{ in.}^3$$

$$W = \mu V = \left(168 \frac{\text{lb}}{\text{ft}^3}\right) (103.7 \text{ in.}^3) \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= \underline{10.08 \text{ lb}}$$

5/83



$$\begin{aligned} A &= 2\pi \bar{r} L \\ &= 2\pi \left(a - \frac{2a}{\pi}\right) \frac{\pi a}{2} \\ &= \underline{\underline{\pi a^2 (\pi - 2)}} \end{aligned}$$

5/84  $m = \rho V$ , where  $\rho = 7830 \frac{\text{kg}}{\text{m}^3}$  (Appendix D)

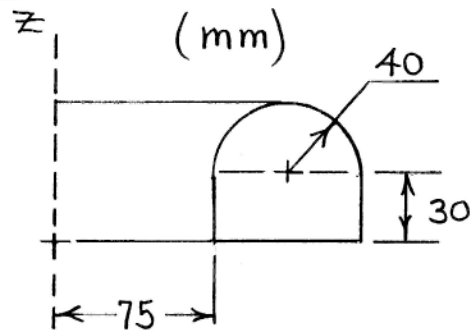
$$V = 2\pi \bar{r} A, \quad \bar{r} A = 200(100) \left( \frac{60+160}{2} \right) - \frac{\pi(60^2)}{2} \left( 60 + \frac{4(60)}{3\pi} \right)$$
$$= 1.717 (10^6) \text{ mm}^3$$

$$V = 2\pi (1.717 \times 10^6) = 10.79 (10^6) \text{ mm}^3$$

or  $V = 0.01079 \text{ m}^3$

$\therefore m = \rho V = 7830(0.01079) = \underline{84.5 \text{ kg}}$

5/85



$$A_1 = \pi \bar{r} L = \pi (75 + 40) (2 \cdot 30 + 80 + \pi 40) \\ = 96000 \text{ mm}^2$$

$$\text{End areas } A_2 = 2 \left( \frac{\pi}{2} \cdot 40^2 + 80(30) \right) \\ = 9830 \text{ mm}^2$$

$$\text{Total area } A = A_1 + A_2 = \underline{105800 \text{ mm}^2}$$

$$V = \pi \bar{r} A = \pi (75 + 40) (30 \cdot 80 + \pi 40^2 / 2) \\ = \underline{1.775(10^6) \text{ mm}^3}$$

$$\frac{5}{86} \quad V = 2\pi\bar{r}A, \quad m = V\rho$$

$$\text{where } m = 10.0 \text{ kg}, \quad \rho = 2.69 \times 10^3 \text{ kg/m}^3$$

$$A = \frac{1}{2} 15,200 \times 10^{-6} = 7.600 \times 10^{-3} \text{ m}^2$$

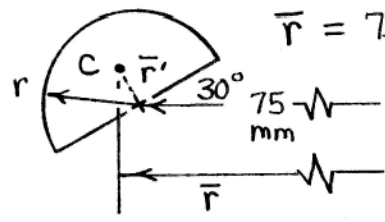
$$\text{Thus } \bar{r} = \frac{V}{2\pi A} = \frac{m}{2\pi\rho A} = \frac{10.0}{2\pi(2.69 \times 10^3)(7.6 \times 10^{-3})}$$

$$= 0.0778 \text{ m}$$

$$\text{or } \underline{\bar{r} = 77.8 \text{ mm}}$$



$$\frac{5}{87} \quad \bar{r}' = \frac{4r}{3\pi} = \frac{4(30)}{3\pi} = 12.73 \text{ mm}$$

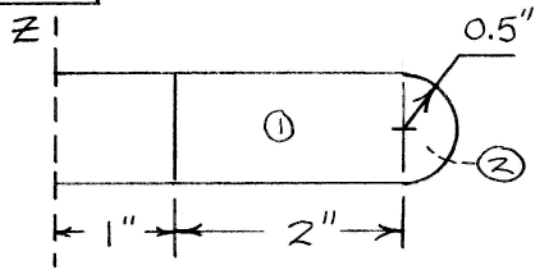


$$\bar{r} = 75 + 12.73 \sin 30^\circ = 81.4 \text{ mm}$$

$$A = \frac{\pi r^2}{2} = \frac{\pi (30)^2}{2} = 1414 \text{ mm}^2$$

$$V = \pi \bar{r} A = \pi (81.4)(1414) = \underline{361\,000 \text{ mm}^3}$$

5/88



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi (1+1)(2 \cdot 1) = 25.1 \text{ in.}^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi \left[1+2 + \frac{4(0.5)}{3\pi}\right] \left[\frac{\pi (0.5)^2}{2}\right]$$
$$= 7.93 \text{ in.}^3$$

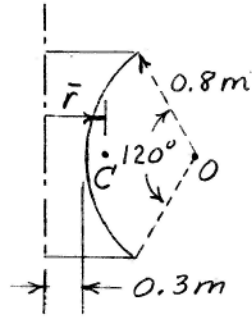
$$V = V_1 + V_2 = \underline{33.1 \text{ in.}^3}$$

$$A_1 = 1(2\pi \cdot 1) + 2\pi (3^2 - 1^2) = 56.5 \text{ in.}^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi \left[1+2 + \frac{2(0.5)}{\pi}\right] [\pi (0.5)]$$
$$= 32.8 \text{ in.}^2$$

$$A = A_1 + A_2 = \underline{89.3 \text{ in.}^2}$$

5/89 |  $A = 2\pi \bar{r} L$  ;  $L = \frac{120}{360} 2\pi(0.8) = 1.68 \text{ m}$



From Sample Prob. 5/1,  
 $\bar{OC} = \frac{r \sin \alpha}{\alpha}$  where  $\alpha = 60^\circ$   
 or  $\alpha = \pi/3 \text{ rad}$

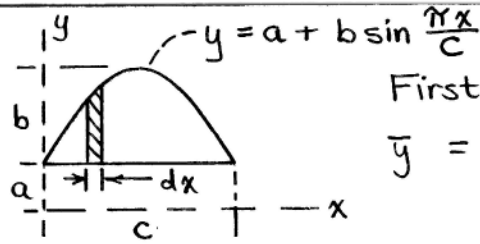
$$= \frac{0.8(\sqrt{3}/2)}{\pi/3} = 0.662 \text{ m}$$

$$\bar{r} = (0.3 + 0.8 - 0.662) = 0.438 \text{ m}$$

$$\text{So } A = 2\pi(0.438)(1.68) = \underline{4.62 \text{ m}^2}$$



5/90

First, find  $\bar{y}$  by

$$\bar{y} = \frac{\int y_c dA}{\int dA}$$

$$A = \int dA = \int_0^c b \sin \frac{\pi x}{c} dx = -\frac{bc}{\pi} \cos \frac{\pi x}{c} \Big|_0^c = \frac{2bc}{\pi}$$

$$\int y_c dA = \int_0^c \left[ a + \frac{b}{2} \sin \frac{\pi x}{c} \right] \left[ b \sin \frac{\pi x}{c} dx \right]$$

$$= \int_0^c ab \sin \frac{\pi x}{c} dx + \int_0^c \frac{b^2}{2} \sin^2 \frac{\pi x}{c} dx$$

$$= -ab \frac{c}{\pi} \cos \frac{\pi x}{c} \Big|_0^c + \frac{b^2}{2} \left[ \frac{x}{2} - \frac{1}{4} \sin \frac{2\pi x}{c} \right]_0^c$$

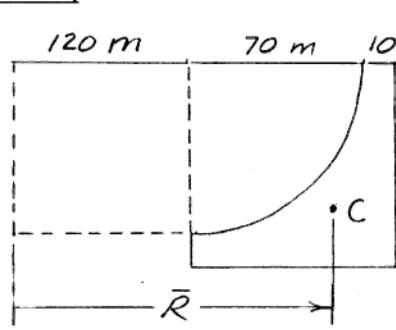
$$= \frac{2abc}{\pi} + \frac{b^2 c}{4} = bc \left[ \frac{2a}{\pi} + \frac{b}{4} \right]$$

$$\therefore \bar{y} = \frac{bc \left[ \frac{2a}{\pi} + \frac{b}{4} \right]}{2bc/\pi} = a + \frac{b}{8} \pi$$

$$V = 2\pi \bar{y} A = 2\pi \left( a + \frac{b}{8} \pi \right) \left( \frac{2bc}{\pi} \right)$$

$$= \underline{4bc \left( a + \frac{b\pi}{8} \right)}$$

5/91



$$\text{Square: } A = 80^2 = 6400 \text{ m}^2$$

$$\frac{1}{4} \text{ circle: } A = \frac{1}{4} \pi (70^2)$$

$$= 3848 \text{ m}^2$$

$$\text{Net area} = 2552 \text{ m}^2$$

$$\bar{r}_{\text{square}} = 120 + 40 = 160 \text{ m}$$

$$\bar{r}_{\frac{1}{4} \text{ circle}} = 120 + \frac{4(70)}{3\pi} = 149.7 \text{ m}$$

$$\bar{R} = \frac{\sum A \bar{r}}{\sum A} = \frac{6400(160) - 3848(149.7)}{2552}$$

$$= 175.5 \text{ m}$$

$$V = \theta \bar{R} A = \frac{\pi}{3} (175.5)(2552) = 469000 \text{ m}^3$$

$$m = \rho V = 2.40 (469000) = \underline{1.126 \times 10^6 \text{ Mg}}$$

5/92 | From the solution to Prob. 5/7 ,

$$\bar{r} = 8 - \frac{2}{3} \frac{2(1.5) + 2}{1.5 + 2} = 7.05 \text{ m}$$

$$A = \frac{2 + 1.5}{2} (2) = 3.5 \text{ m}^2$$

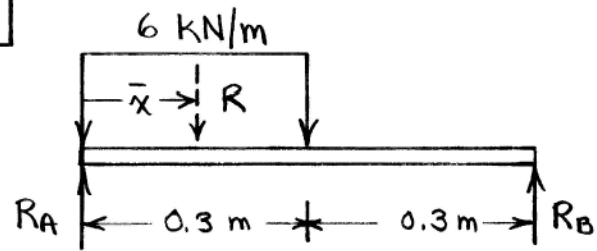
$$\theta = \frac{\pi}{3}$$

$$\text{So } V = \theta \bar{r} A = \frac{\pi}{3} (7.05) (3.5) = 25.8 \text{ m}^3$$

$$W = \rho g V = 2400 (9.81) (25.8) = 608 (10^3) \text{ N}$$

$$\text{or } \underline{W = 608 \text{ kN}}$$

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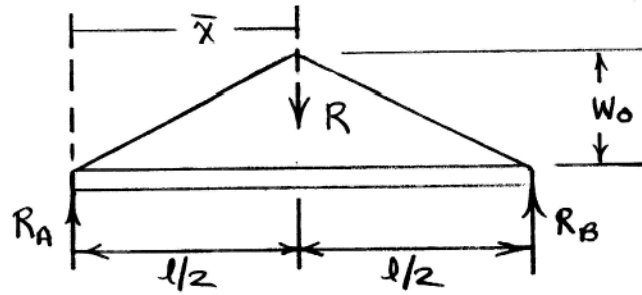


$$R = 6(0.3) = 1.8 \text{ kN @ } \bar{x} = \frac{1}{2}(0.6) = 0.3 \text{ m}$$

$$\sum M_A = 0: R_B(0.6) - 1.8(0.3) = 0, \quad \underline{R_B = 0.9 \text{ kN}}$$

$$\sum F = 0: 0.9 - 1.8 + R_A = 0, \quad \underline{R_A = 0.9 \text{ kN}}$$

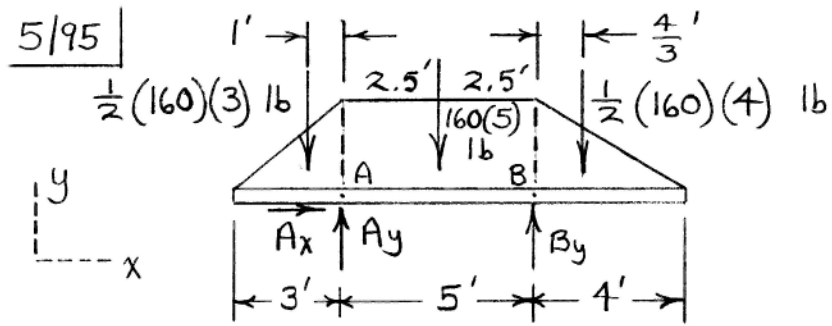
5/94



$$R = 2 \cdot \frac{1}{2} (w_0) \left(\frac{l}{2}\right) = \frac{1}{2} w_0 l \quad @ \quad \bar{x} = \frac{l}{2}$$

$$\circlearrowleft + \sum M_A = 0: R_B(l) - \frac{1}{2} w_0 l \left(\frac{l}{2}\right) = 0, \quad \underline{R_B = \frac{1}{4} w_0 l}$$

$$+\uparrow \sum F = 0: \frac{1}{4} w_0 l - \frac{1}{2} w_0 l + R_A = 0, \quad \underline{R_A = \frac{1}{4} w_0 l}$$



$$\curvearrowright \sum M_A = 0 : 240(1) - 800(2.5) - 320(6.33) + B_y(5) = 0$$

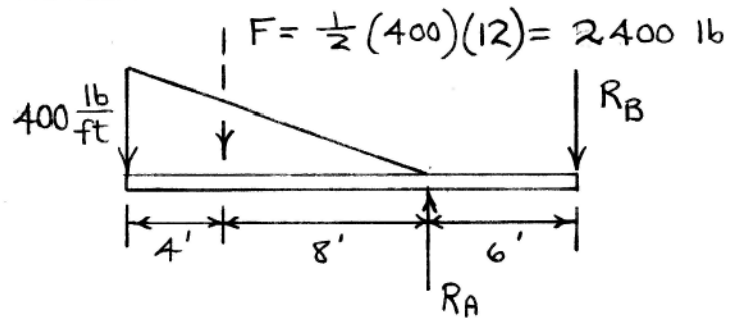
$$\underline{B_y = 757 \text{ lb}}$$

$$\sum F_y = 0 : A_y + 757 - 240 - 800 - 320 = 0$$

$$\underline{A_y = 603 \text{ lb}}$$

$$\sum F_x = 0 \Rightarrow \underline{A_x = 0}$$

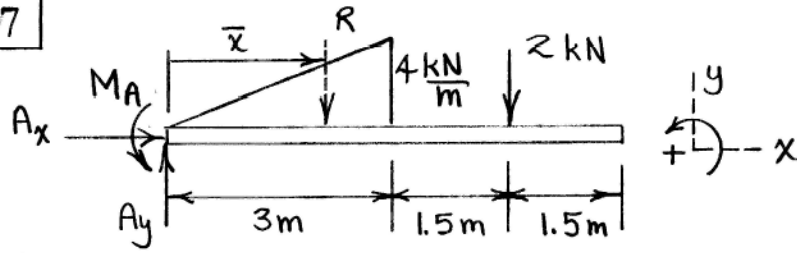
5/96



$$\curvearrowright \sum M_A = 0: 2400(8) - 6R_B = 0, \quad \underline{R_B = 3200 \text{ lb}}$$

$$+\uparrow \sum F = 0: R_A - 2400 - 3200 = 0, \quad \underline{R_A = 5600 \text{ lb}}$$

5/97



$$R = \frac{1}{2}(3)(4) = 6 \text{ kN} @ \bar{x} = \frac{2}{3}(3) = 2 \text{ m}$$

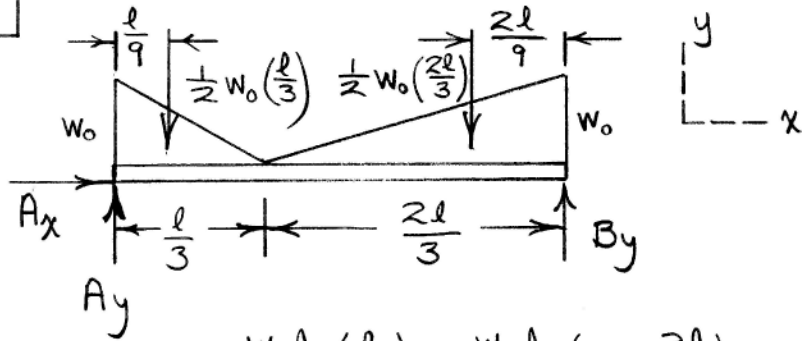
$$\sum M_A = 0: M_A - 6(2) - 2(4.5) = 0, \quad M_A = 21 \text{ kN}\cdot\text{m}$$

$$\sum F_y = 0: A_y - 6 - 2 = 0, \quad A_y = 8 \text{ kN}$$

$$\sum F_x = 0: A_x = 0$$



5/98



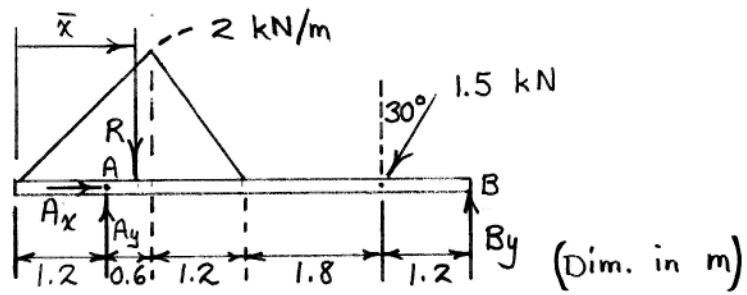
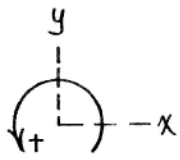
$$\sqrt{+} \sum M_A = 0 : -\frac{w_0 l}{6} \left(\frac{l}{9}\right) - \frac{w_0 l}{3} \left(l - \frac{2l}{9}\right) + B_y l = 0, \quad \underline{B_y = \frac{5}{18} w_0 l}$$

$$\sum F_y = 0 : A_y + \frac{5}{18} w_0 l - \frac{w_0 l}{6} - \frac{w_0 l}{3} = 0$$

$$\underline{A_y = \frac{2}{9} w_0 l}$$

$$\sum F_x = 0 \Rightarrow \underline{A_x = 0}$$

5/99



$$R = \frac{1}{2} (1.2 + 0.6 + 1.2) (2) = 3 \text{ kN}$$

$$\bar{x} = \frac{(1.2 + 0.6) + (1.2 + 0.6 + 1.2)}{3} = 1.6 \text{ m}$$

$$\sum F_x = 0 : A_x - 1.5 \sin 30^\circ = 0, \quad \underline{A_x = 750 \text{ N}}$$

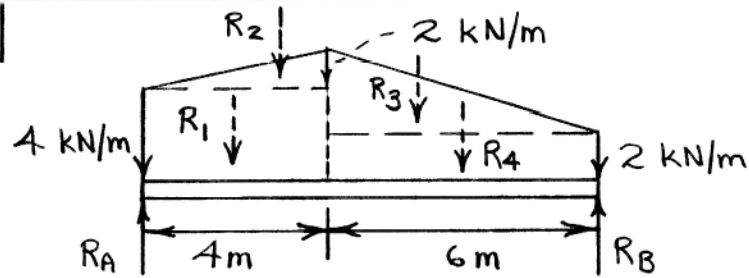
$$\sum M_A = 0 : -3(1.6 - 1.2) - 1.5 \cos 30^\circ (3.6) - B_y (4.8) = 0$$

$$\underline{B_y = 1.224 \text{ kN}}$$

$$\sum F_y = 0 : A_y - 3 - 1.5 \cos 30^\circ + 1.224 = 0$$

$$\underline{A_y = 3.07 \text{ kN}}$$

5/100



$$R_1 = 4(4) = 16 \text{ kN}, \quad R_2 = \frac{1}{2}(2)(4) = 4 \text{ kN}$$

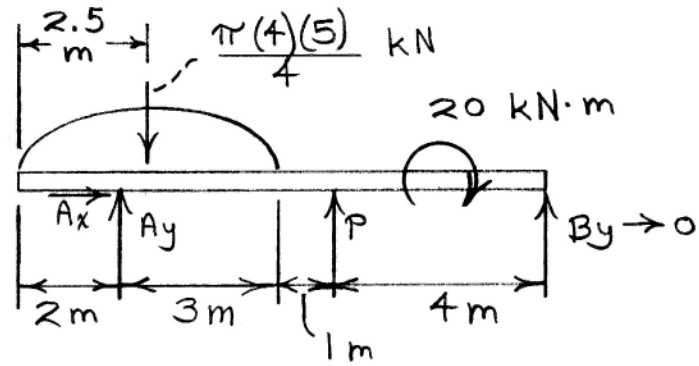
$$R_3 = \frac{1}{2}(4)(6) = 12 \text{ kN}, \quad R_4 = 2(6) = 12 \text{ kN}$$

$$\begin{aligned} \uparrow \sum M_A = 0: & 16(2) + 4\left(\frac{2}{3}4\right) + 12\left(4 + \frac{1}{3}6\right) \\ & + 12(4+3) - 10R_B = 0, \quad \underline{R_B = 19.87 \text{ kN}} \end{aligned}$$

$$\uparrow \sum F = 0: R_A + 19.87 - (16 + 4 + 12 + 12) = 0$$

$$\underline{R_A = 24.1 \text{ kN}}$$

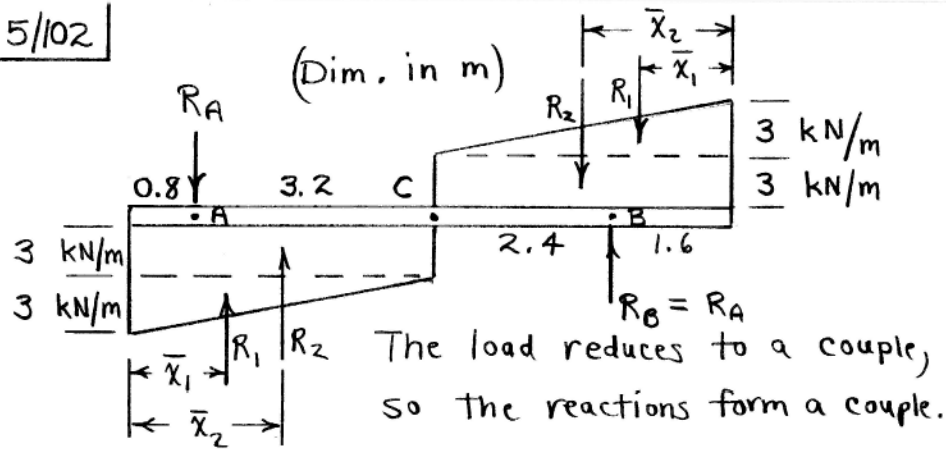
5/101



$$\uparrow + \sum M_A = 0 : - 5\pi(0.5) + P(4) - 20 = 0$$

$$\underline{P = 6.96 \text{ kN}}$$

5/102

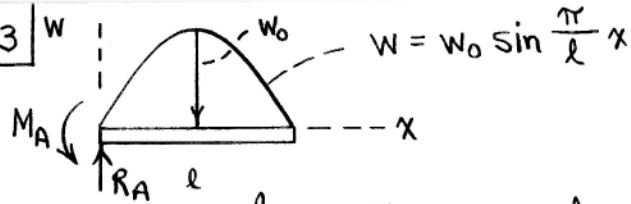


$$R_1 = \frac{1}{2}(4)(3) = 6 \text{ kN} ; \quad \bar{x}_1 = \frac{1}{3}(4) = \frac{4}{3} \text{ m}$$

$$R_2 = 4(3) = 12 \text{ kN} ; \quad \bar{x}_2 = \frac{4}{2} = 2 \text{ m}$$

$$\begin{aligned} \uparrow + \sum M_C = 0 : & R_A(3.2 + 2.4) - 12(2 + 2) \\ & - 6(2 \cdot (4 - \frac{4}{3})) = 0, \quad R_A = 14.29 \text{ kN} \\ & R_B = 14.29 \text{ kN} \end{aligned}$$

5/103



$$R = \int w dx = \int_0^l w_0 \sin \frac{\pi}{l} x = -w_0 \frac{l}{\pi} \cos \frac{\pi}{l} x \Big|_0^l = \frac{2w_0 l}{\pi}$$

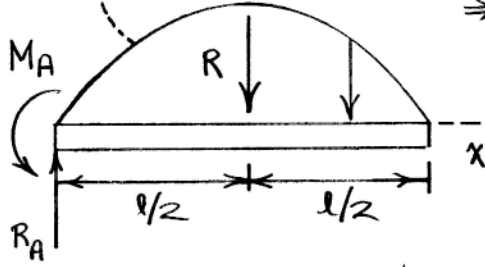
$$\bar{x} = \frac{l}{2}, \text{ by inspection}$$

$$\uparrow + \sum M_A = 0: M_A - \frac{2w_0 l}{\pi} \left(\frac{l}{2}\right) = 0, \quad M_A = \frac{w_0 l^2}{\pi}$$

$$\uparrow + \sum F = 0: R_A - \frac{2w_0 l}{\pi} = 0, \quad R_A = \frac{2w_0 l}{\pi}$$

5/104

$$W = W_0 - kx^2$$



$$\text{At } x = \frac{l}{2}, W = 0 = W_0 - k\left(\frac{l}{2}\right)^2$$

$$\Rightarrow k = \frac{4W_0}{l^2}$$

$$W = W_0 \left(1 - \frac{4}{l^2} x^2\right)$$

$$\bar{x} = 0, \text{ by inspection}$$

$$R = \int W dx = 2 \int_0^{l/2} W_0 \left(1 - \frac{4}{l^2} x^2\right) dx$$

$$= 2W_0 \left[ x - \frac{4}{3l^2} x^3 \right]_0^{l/2} = \frac{2}{3} W_0 l$$

$$+\uparrow \Sigma F = 0: R_A - \frac{2}{3} W_0 l = 0, \quad \underline{R_A = \frac{2}{3} W_0 l}$$

$$\curvearrow + \Sigma M_A = 0: M_A - \frac{2}{3} W_0 l \left(\frac{l}{2}\right) = 0, \quad \underline{M_A = \frac{1}{3} W_0 l^2}$$

5/105

$$dF = w dx$$

$$w = k_1 x - k_2 x^2$$

$$\text{At } x = 0:$$

$$\frac{dw}{dx} = k_1 = 50 \text{ lb/ft}^2$$

$$\text{At } x = 10', \quad w = 50(10) - k_2(10)^2 = 300$$

$$k_2 = 2 \text{ lb/ft}^3$$

$$\text{So } w = 50x - 2x^2, \quad dF = (50x - 2x^2) dx$$

$$\sum M_A = 0: \int x dF - 10R_B = 0$$

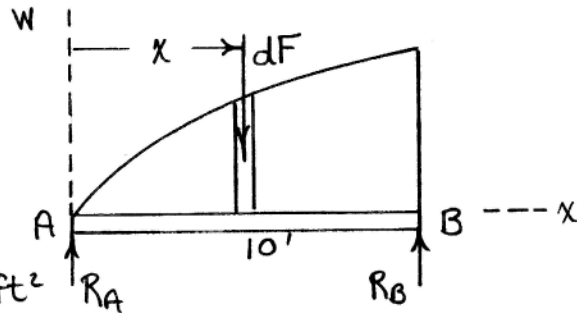
$$\int_0^{10} x(50x - 2x^2) dx = \frac{7}{6} 10^4 = 10R_B$$

$$\underline{R_B = 1167 \text{ lb}}$$

$$\sum F = 0: R_A - \int dF + 1167 = 0$$

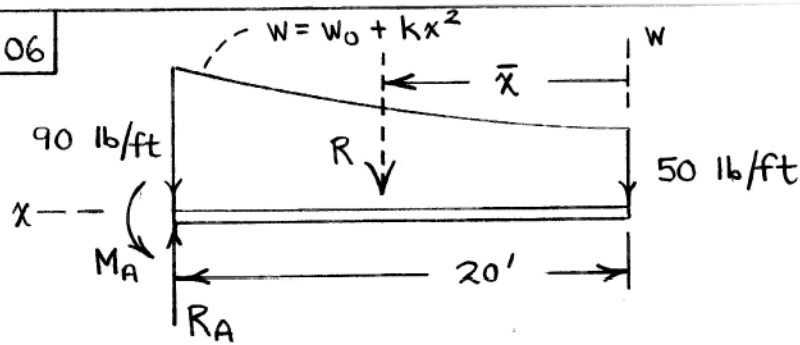
$$R_A - \int_0^{10} (50x - 2x^2) dx + 1167 = 0$$

$$R_A - 1833 + 1167 = 0, \quad \underline{R_A = 667 \text{ lb}}$$





5/106



$$\text{At } x=0, w=50 = w_0$$

$$\text{At } x=20', w=90 = 50 + k(20)^2, k = \frac{1}{10} \text{ lb/ft}^3$$

$$\text{So } w = 50 + \frac{x^2}{10} \quad (\text{lb/ft})$$

$$R = \int w dx = \int_0^{20} \left(50 + \frac{x^2}{10}\right) dx = \left[50x + \frac{x^3}{30}\right]_0^{20} = 1267 \text{ lb}$$

$$\bar{x} = \frac{\int x w dx}{R} = \frac{\int_0^{20} x \left(50 + \frac{x^2}{10}\right) dx}{1267} = \frac{\left[\frac{50}{2}x^2 + \frac{x^4}{40}\right]_0^{20}}{1267}$$

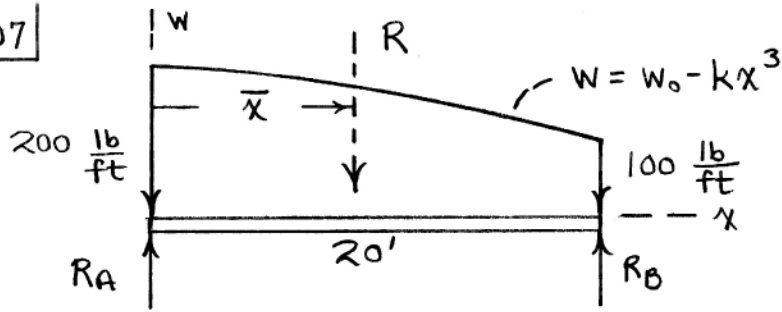
$$= 11.05 \text{ ft}$$

$$\uparrow \sum F = 0: R_A - 1267 = 0,$$

$$\underline{R_A = 1267 \text{ lb}}$$

$$\curvearrowright \sum M_A = 0: M_A - 1267(20 - 11.05) = 0, \underline{M_A = 11.33(10^3) \text{ lb-ft}}$$

5/107



$$\text{At } x=0, w = w_0 = 200 \text{ lb/ft}$$

$$\text{At } x=20', w = 200 - k(20)^3 = 100, \quad k = \frac{1}{80} \frac{\text{lb}}{\text{ft}^4}$$

$$\text{So } w = 200 - \frac{x^3}{80} \quad (\text{lb/ft})$$

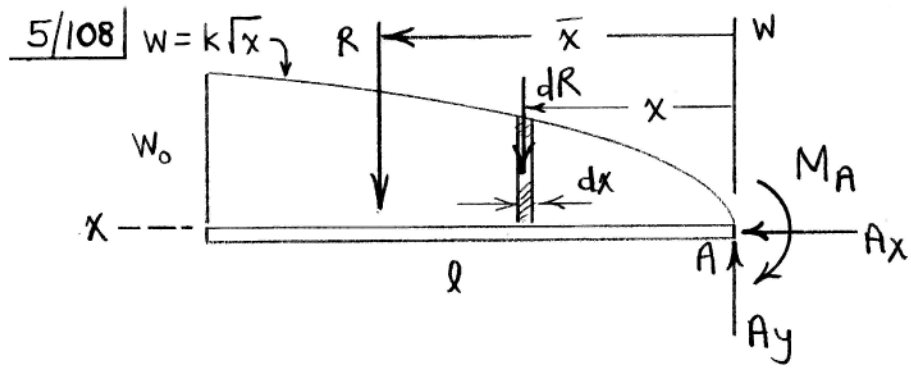
$$R = \int w dx = \int_0^{20} \left(200 - \frac{x^3}{80}\right) dx = \left[200x - \frac{x^4}{320}\right]_0^{20} = 3500 \text{ lb}$$

$$\bar{x} = \frac{\int x w dx}{R} = \frac{\int_0^{20} \left(200 - \frac{x^3}{80}\right) x dx}{3500} = \frac{\left[100x^2 - \frac{x^5}{400}\right]_0^{20}}{3500}$$

$$= 9.14'$$

$$\curvearrowleft \sum M_A = 0: 20 R_B - 3500(9.14) = 0, \quad \underline{R_B = 1600 \text{ lb}}$$

$$+\uparrow \sum F = 0: R_A + 1600 - 3500 = 0, \quad \underline{R_A = 1900 \text{ lb}}$$



$$w = k\sqrt{x} \text{ @ left end : } w_0 = k\sqrt{l}, \quad k = \frac{w_0}{\sqrt{l}}$$

$$\text{So } w = \frac{w_0}{\sqrt{l}} \sqrt{x}$$

$$R = \int dR = \int w dx = \int_0^l \frac{w_0}{\sqrt{l}} \sqrt{x} dx = \frac{2}{3} w_0 l$$

$$\bar{x} = \frac{\int x w dx}{\int w dx} = \frac{\int_0^l x \frac{w_0}{\sqrt{l}} \sqrt{x} dx}{\frac{2}{3} w_0 l} = \frac{3}{5} l$$

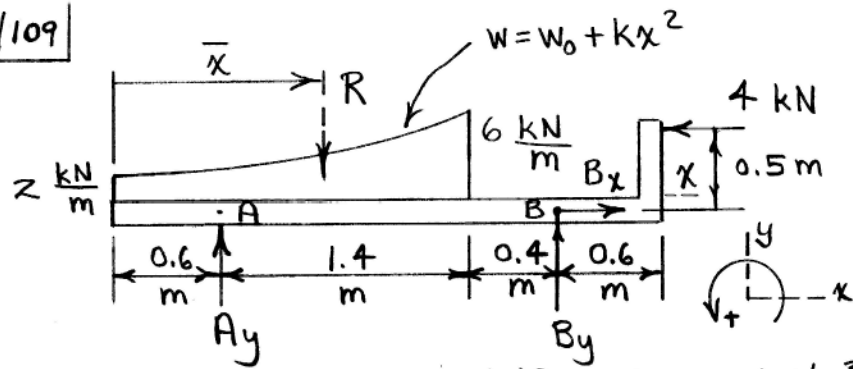
$$\uparrow \sum F = 0 : A_y - \frac{2}{3} w_0 l = 0, \quad \underline{A_y = \frac{2}{3} w_0 l}$$

$$\curvearrowright \sum M_A = 0 : \frac{2}{3} w_0 l \left( \frac{3}{5} l \right) - M_A = 0$$

$$\underline{M_A = \frac{2}{5} w_0 l^2 \text{ CW}}$$

$$\rightarrow \sum F = 0 \Rightarrow \underline{A_x = 0}$$

5/109



$$w = 2 + kx^2 : 6 = 2 + k(2)^2, \quad k = 1 \text{ kN/m}^3,$$

$$R = \int w dx = \int_0^2 (2 + x^2) dx = 2x + \frac{x^3}{3} \Big|_0^2 = 6.67 \text{ kN}$$

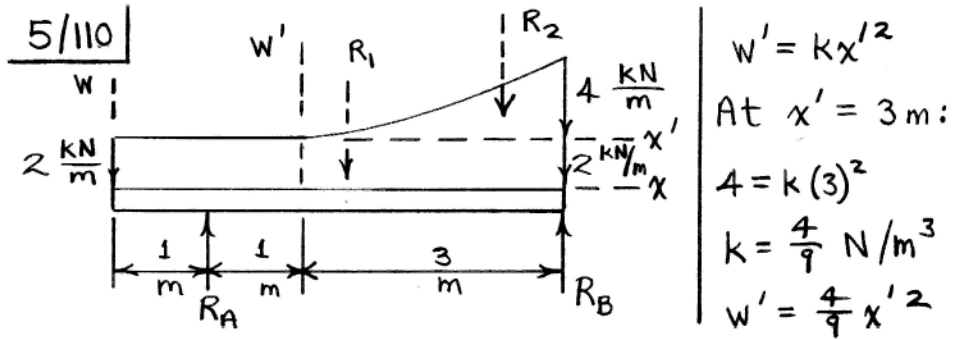
$$\bar{x} = \frac{\int x w dx}{R} = \frac{\int_0^2 (2x + x^3) dx}{6.67} = \frac{x^2 + \frac{x^4}{4}}{6.67} \Big|_0^2 = 1.2 \text{ m}$$

$$\sum F_x = 0 : B_x - 4 = 0, \quad \underline{B_x = 4 \text{ kN}}$$

$$\sum M_B = 0 : 4(0.5) + 6.67(2.4 - 1.2) - A_y(1.8) = 0$$

$$\underline{A_y = 5.56 \text{ kN}}$$

$$\sum F_y = 0 : 5.56 + B_y - 6.67 = 0, \quad \underline{B_y = 1.11 \text{ kN}}$$



$$R_2 = \int_0^3 w' dx' = \int_0^3 \frac{4}{9} x'^2 = \frac{4}{9} \frac{x'^3}{3} \Big|_0^3 = 4 \text{ kN}$$

$$\bar{x}'_2 = \frac{\int_0^3 x' w' dx'}{R_2} = \frac{1}{4} \int_0^3 \frac{4}{9} x'^3 = \frac{1}{4} \frac{4}{9} \frac{x'^4}{4} \Big|_0^3 = 2.25 \text{ m}$$

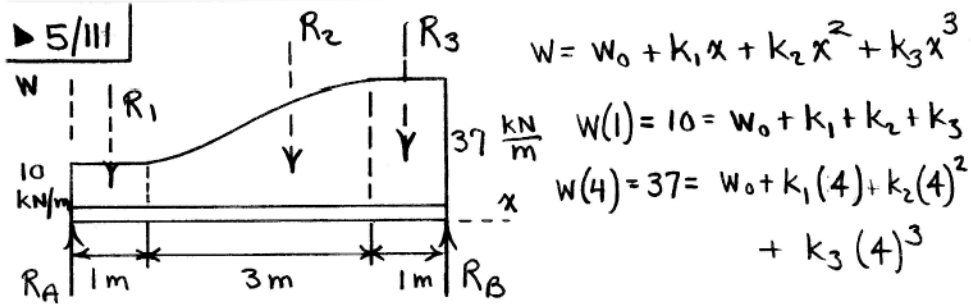
$$R_1 = 2(5) = 10 \text{ kN @ } \bar{x}_1 = 2.5 \text{ m} \quad \bar{x}_2 = 4.25 \text{ m}$$

$$\uparrow + \sum M_A = 0: -10(1.5) - 4(3.25) + 4R_B = 0$$

$$\underline{R_B = 7 \text{ kN}}$$

$$\uparrow + \sum F = 0: 7 + R_A - 4 - 10 = 0, \quad \underline{R_A = 7 \text{ kN}}$$

► 5/III



$$\frac{dw}{dx} = k_1 + 2k_2x + 3k_3x^2 : \begin{cases} 0 = k_1 + 2k_2(1) + 3k_3(1)^2 \\ 0 = k_1 + 2k_2(4) + 3k_3(4)^2 \end{cases}$$

Solve simultaneously to get  $w = 21 - 24x + 15x^2 - 2x^3$

$$R_2 = \int w dx = \int_1^4 (21 - 24x + 15x^2 - 2x^3) dx$$

$$= \left[ 21x - 12x^2 + 5x^3 - \frac{1}{2}x^4 \right]_1^4 = 70.5 \text{ kN}$$

$$\bar{x}_2 = \frac{1}{R_2} \int x w dx = \frac{1}{70.5} \int_1^4 (21 - 24x + 15x^2 - 2x^3) x dx$$

$$= \frac{1}{70.5} \left[ \frac{21}{2}x^2 - 8x^3 + \frac{15}{4}x^4 - \frac{2}{5}x^5 \right]_1^4 = 2.84 \text{ m}$$

$$R_1 = 10(1) = 10 \text{ kN @ } \bar{x}_1 = 0.5 \text{ m}$$

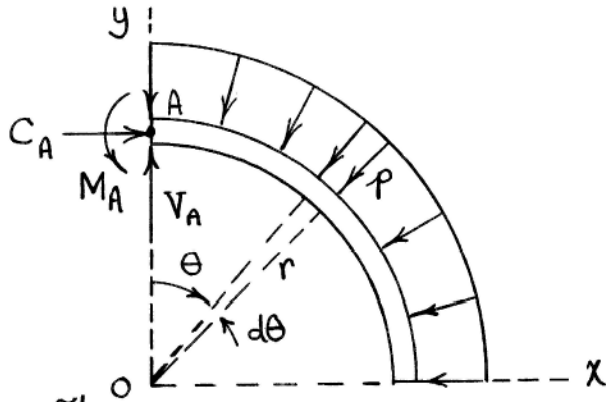
$$R_3 = 37(1) = 37 \text{ kN @ } \bar{x}_3 = 4.5 \text{ m}$$

$$\sum M_A = 0 : 5R_B - 10(0.5) - 70.5(2.84) - 37(4.5) = 0$$

$$R_B = 74.4 \text{ kN}$$

$$\sum F = 0 : 74.4 - 10 - 70.5 - 37 + R_A = 0, \quad R_A = 43.1 \text{ kN}$$

► 5/112



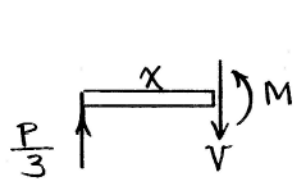
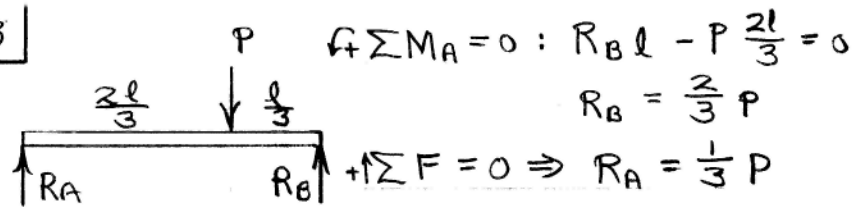
$$\Sigma F_x = 0: C_A - \int_0^{\pi/2} p r d\theta \sin\theta = 0, \quad \underline{C_A = pr}$$

$$\Sigma F_y = 0: V_A - \int_0^{\pi/2} p r d\theta \cos\theta = 0, \quad \underline{V_A = pr}$$

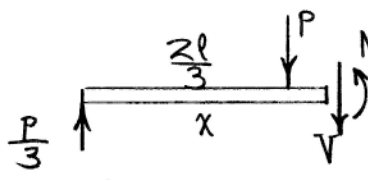
$$\Sigma M_A = 0: M_A - \int_0^{\pi/2} p r d\theta (r \sin\theta) = 0, \quad \underline{M_A = pr^2}$$

(Alternatively,  $\Sigma M_o = 0: M_A - C_A r = 0, M_A = pr^2$ )

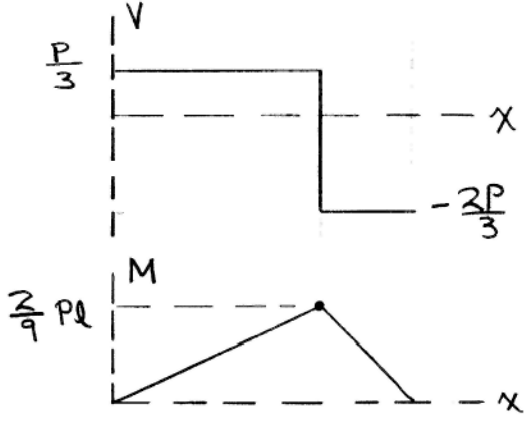
5/113



$0 < x < \frac{2l}{3} :$   
 $\uparrow \sum F = 0 \Rightarrow V = \frac{P}{3}$   
 $\sum M = 0 \Rightarrow M = \frac{P}{3} x$



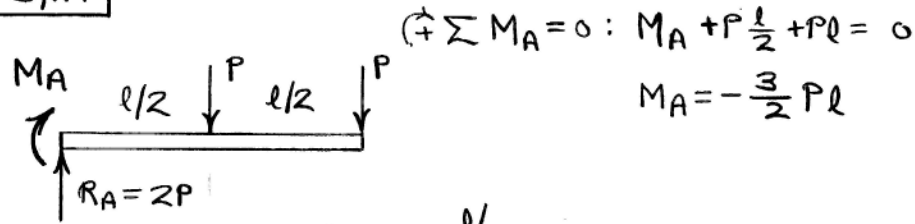
$\uparrow \sum F = 0 : \frac{P}{3} - P - V = 0, V = -\frac{2P}{3}$   
 $\uparrow \sum M = 0 : M + P(x - \frac{2l}{3}) - \frac{P}{3} x = 0$   
 $M = \frac{2P}{3} (l - x)$



At  $x = l/2,$   
 $V = \frac{P}{3}$   
 $M = \frac{P}{3} (\frac{l}{2}) = \frac{Pl}{6}$



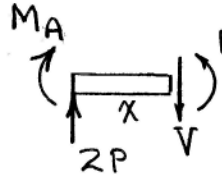
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$$\uparrow \sum M_A = 0 : M_A + P \frac{l}{2} + Pl = 0$$

$$M_A = -\frac{3}{2} Pl$$

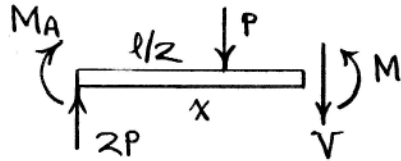
$0 < x < \frac{l}{2}$  :



$$\uparrow \sum F = 0 \Rightarrow V = 2P$$

$$\downarrow \sum M_A = 0 : \frac{3}{2} Pl + M - 2Px = 0$$

$$M = 2Px - \frac{3}{2} Pl$$

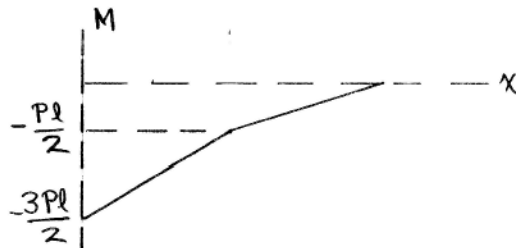
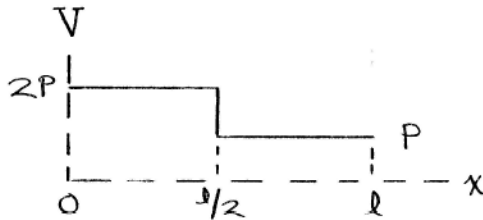


$\frac{l}{2} < x < l$  :

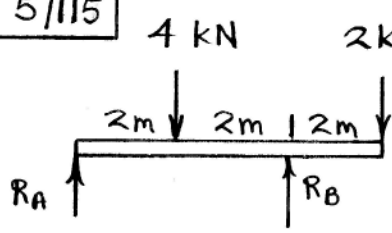
$$\uparrow \sum F = 0 : 2P - P - V = 0, V = P$$

$$\downarrow \sum M_A = 0 : \frac{3}{2} Pl - P \frac{l}{2} - Px + M = 0$$

$$M = -P(l-x)$$



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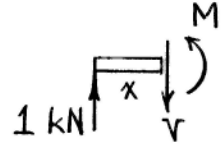


$\sum M_A = 0: 4(2) + 2(6) - 4R_B = 0$

$R_B = 5 \text{ kN}$

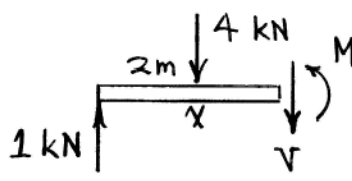
$\uparrow \sum F = 0: R_A + 5 - 6 = 0$

$R_A = 1 \text{ kN}$



$0 < x < 2 \text{ m} :$

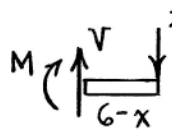
$V = 1 \text{ kN}, M = 1x$



$2 < x < 4 \text{ m} :$

$V = -3 \text{ kN}$

$\uparrow \sum M_A = 0: -8 + 3x + M = 0, M = 8 - 3x$

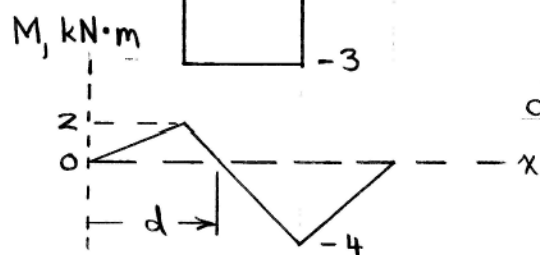
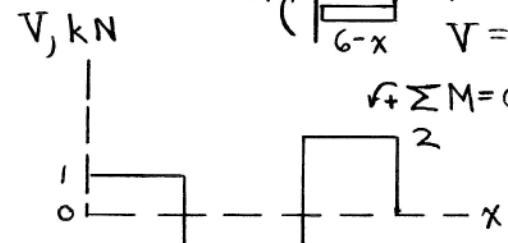


$4 < x < 6 \text{ m} :$

$V = 2 \text{ kN}$

$\uparrow \sum M = 0: -M - 2(6-x) = 0$

$M = -2(6-x)$



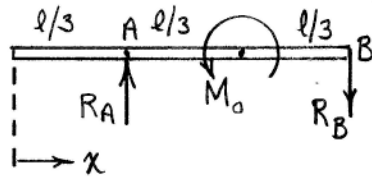
$d = 2 + \frac{1}{3}(2) = \underline{2.67 \text{ m}}$

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$R_A = R_B$  (load is a couple)

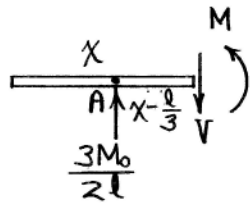
$$\sum M_A = 0: M_0 - R_B \left(\frac{2l}{3}\right) = 0$$

$$R_B = \frac{3M_0}{2l}$$



$$x < \frac{l}{3}:$$

By inspection,  $V = M = 0$

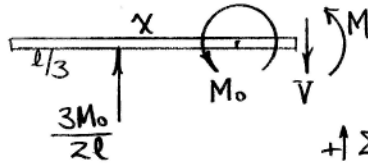


$$\frac{l}{3} < x < \frac{2l}{3}:$$

$$+\uparrow \sum F = 0 \Rightarrow V = \frac{3M_0}{2l}$$

$$+\curvearrowright \sum M = 0: M - \frac{3M_0}{2l} \left(x - \frac{l}{3}\right) = 0$$

$$M = \frac{3M_0}{2l} \left(x - \frac{l}{3}\right)$$

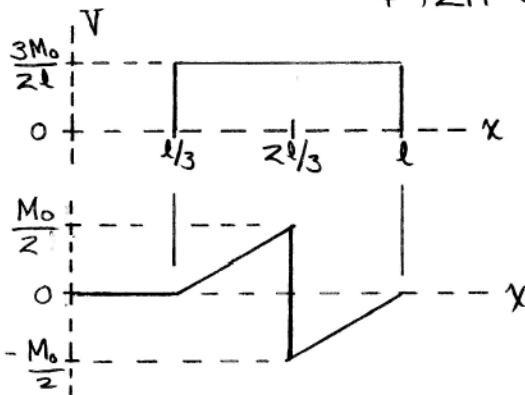


$$\frac{2l}{3} < x < l:$$

$$+\uparrow \sum F = 0 \Rightarrow V = \frac{3M_0}{2l}$$

$$+\curvearrowright \sum M = 0: M + M_0 - \frac{3M_0}{2l} \left(x - \frac{l}{3}\right) = 0$$

$$M = \frac{3M_0}{2} \left(\frac{x}{l} - 1\right)$$



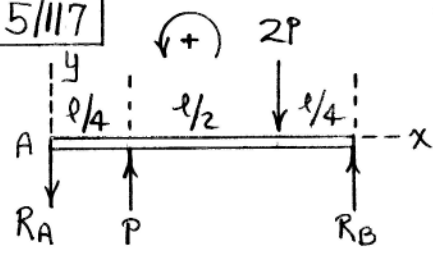
At  $x = \frac{l}{2}$ :

$$V = \frac{3M_0}{2l}$$

$$M = \frac{3M_0}{2l} \left(\frac{l}{2} - \frac{l}{3}\right)$$

$$= \frac{M_0}{4}$$

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$$\sum M_A = 0: P\left(\frac{l}{4}\right) - 2P\left(\frac{3l}{4}\right) + R_B(l) = 0$$

$$R_B = \frac{5P}{4}$$

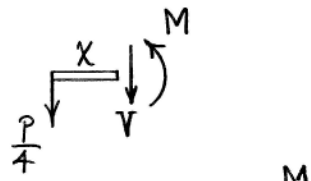
$$\sum F_y = 0: -R_A + P - 2P + \frac{5}{4}P = 0$$

$$R_A = \frac{P}{4}$$

$x < \frac{l}{4}$ :

$$\sum F_y = 0: -\frac{P}{4} - V = 0, \quad V = -\frac{P}{4}$$

$$\sum M = 0: M + \frac{P}{4}x = 0, \quad M = -\frac{P}{4}x$$

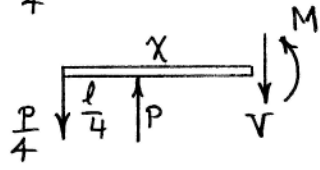


$\frac{l}{4} < x < \frac{3l}{4}$ :

$$\sum F_y = 0: -\frac{P}{4} + P - V = 0, \quad V = \frac{3P}{4}$$

$$\sum M = 0: M - P\left(x - \frac{l}{4}\right) + \frac{P}{4}x = 0$$

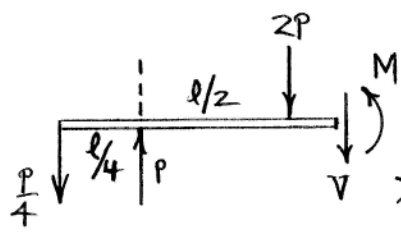
$$M = \frac{P}{4}(3x - l)$$



$\frac{3l}{4} < x < l$ :

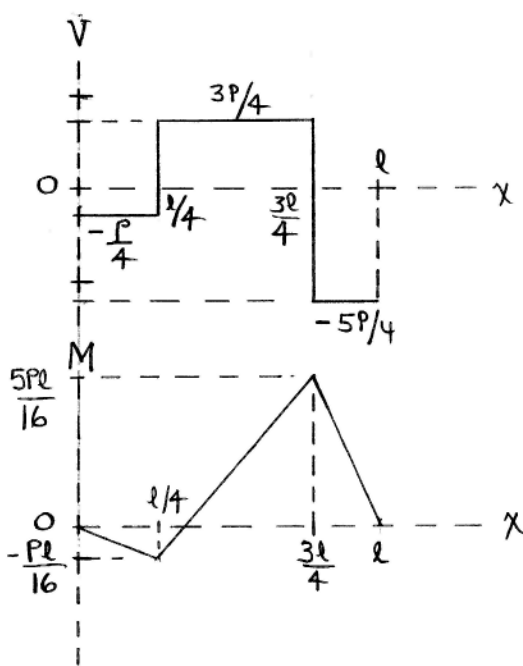
$$\sum F_y = 0: -\frac{P}{4} + P - 2P - V = 0$$

$$V = -\frac{5P}{4}$$



$$\sum M = 0: M + 2P\left(x - \frac{3l}{4}\right) - P\left(x - \frac{l}{4}\right) + \frac{P}{4}x = 0$$

$$M = \frac{5}{4}P(l - x)$$



(Note:  $V_{max} = \frac{3P}{4}$   
on  $\frac{l}{4} < x < \frac{3l}{4}$ .)

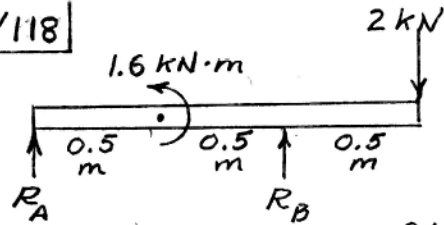
$$M_{max} = \frac{5Pl}{16}$$

$$@ x = \frac{3l}{4}$$



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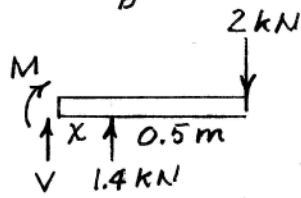
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$$\sum M_A = 0; 1.6 + 1.0R_B - 2(1.5) = 0$$

$$R_B = 1.4 \text{ kN}$$

$$\sum F = 0; R_A = 0.6 \text{ kN}$$



$$\sum F = 0; V + 1.4 - 2 = 0$$

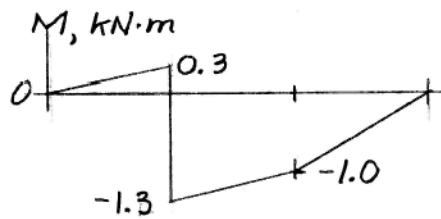
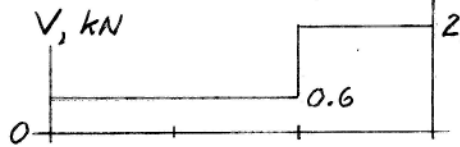
$$V = 0.6 \text{ kN}$$

$$\sum M = 0; M + 2(0.5 + x) - 1.4x = 0$$

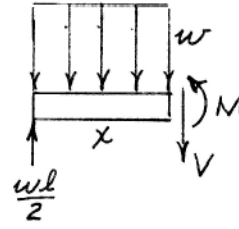
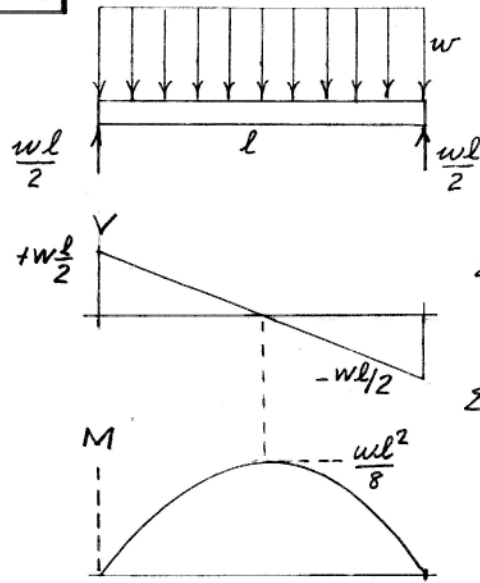
$$M = -(1 + 0.6x)$$

$$(0 < x < 0.5 \text{ m})$$

(x measured to the left of B)



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$$\Sigma F = 0; -V - wx + \frac{wl}{2} = 0$$

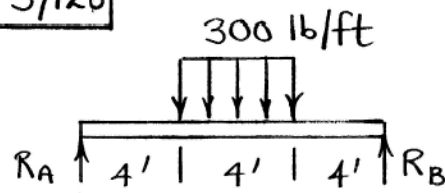
$$V = w\left(\frac{l}{2} - x\right)$$

$$\Sigma M = 0; M + wx\left(\frac{x}{2}\right) - \frac{wlx}{2} = 0$$

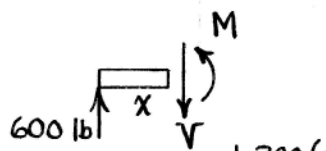
$$M = \frac{wx}{2}(l - x)$$

$$M_{max} = \frac{wl^2}{8}$$

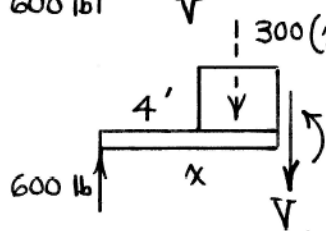
5/120



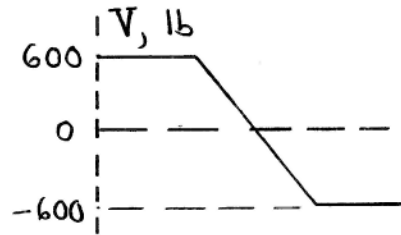
By symmetry,  
 $R_A = R_B = \frac{1}{2}(300)(8) = 600 \text{ lb}$



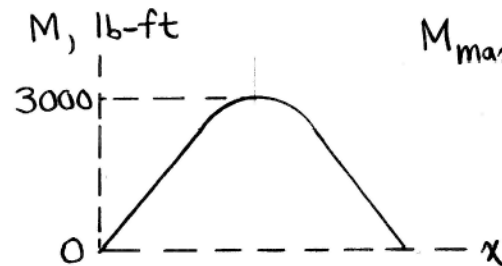
$0 < x < 4 \text{ ft} :$   
 $V = 600 \text{ lb}, M = 600x$



$4 < x < 8 \text{ ft} :$   
 $\uparrow \sum F = 0 : 600 - 300(x-4) - V = 0$   
 $V = 1800 - 300x$   
 $\curvearrowright \sum M = 0 : M + 300(x-4) \frac{x-4}{2} - 600x = 0$   
 $M = -150x^2 + 1800x - 2400$

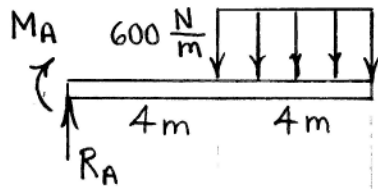


For  $M_{\max}$ ,  
 $\frac{dM}{dx} = -300x + 1800 = 0$   
 $x = 6 \text{ ft}$

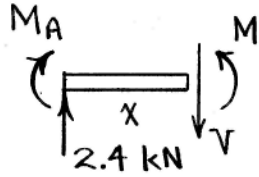


$M_{\max} = -150(6^2) + 1800(6) - 2400$   
 $= \underline{3000 \text{ lb-ft}}$

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From FBD of entire beam,  
 $R_A = 2.4 \text{ kN}$ ,  $M_A = -14.4 \text{ kN}\cdot\text{m}$

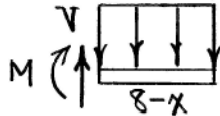


$0 < x < 4 \text{ m}$  :

$$+\uparrow \sum F = 0 \Rightarrow V = 2.4 \text{ kN}$$

$$\curvearrowright \sum M_A = 0 : 14.4 + M - 2.4x = 0$$

$$M = 2.4x - 14.4$$



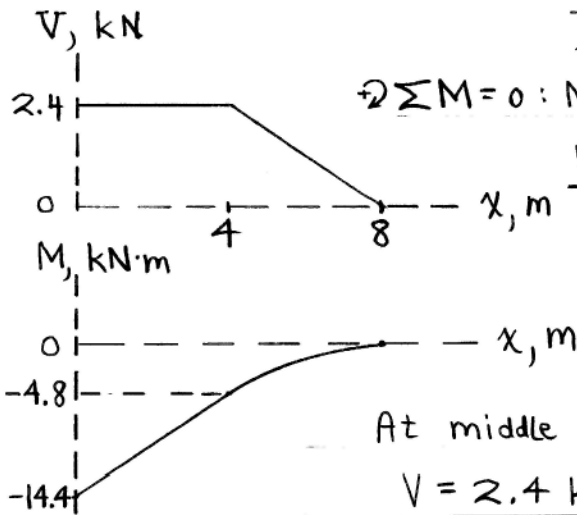
$4 < x < 8 \text{ m}$  :

$$+\uparrow \sum F = 0 : V - 0.6(8-x) = 0$$

$$V = 4.8 - 0.6x \text{ kN}$$

$$\curvearrowright \sum M = 0 : M + 0.6(8-x) \frac{8-x}{2} = 0$$

$$M = -0.3(8-x)^2 \text{ kN}\cdot\text{m}$$



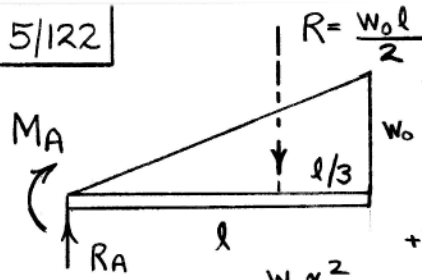
At middle of beam :

$$V = 2.4 \text{ kN}$$

$$M = -4.8 \text{ kN}\cdot\text{m}$$



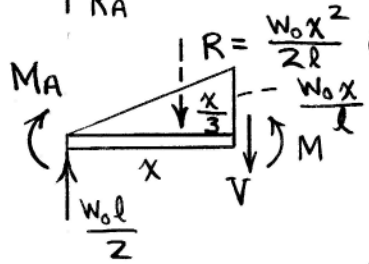
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$$\sum M_A = 0: M_A + \frac{w_0 l}{2} \frac{2l}{3} = 0$$

$$M_A = -\frac{w_0 l^2}{3}$$

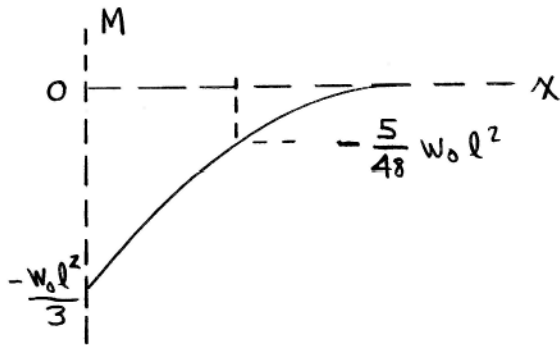
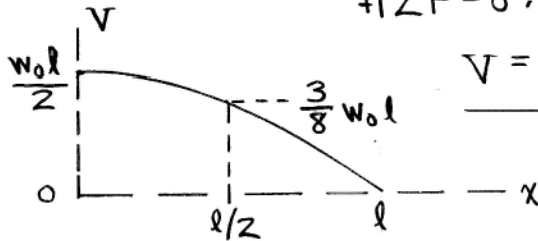
$$+\uparrow \sum F = 0 \Rightarrow R_A = \frac{w_0 l}{2}$$



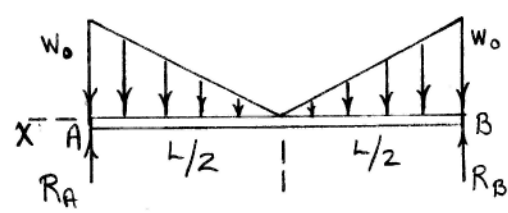
$$\sum M = 0: M + \frac{w_0 x^2}{2l} \frac{x}{3} + \frac{w_0 l^2}{3} - \frac{w_0 l}{2} x = 0, \quad M = \frac{w_0}{6} \left( -2l^2 + 3lx - \frac{x^3}{l} \right)$$

$$+\uparrow \sum F = 0: \frac{w_0 l}{2} - \frac{w_0 x^2}{2l} - V = 0$$

$$V = \frac{w_0 l}{2} \left( 1 - \frac{x^2}{l^2} \right)$$



5/123 (See beam element, lower left)



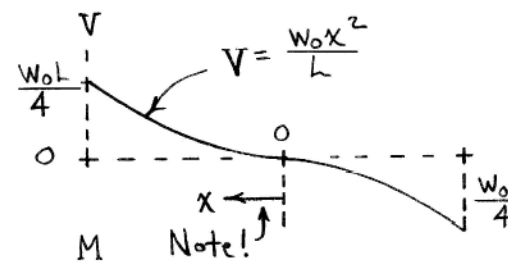
$$R_A = R_B = \frac{w_0 L}{4}$$

For  $x = \frac{L}{2}$ ,  $M=0$ ,  $V=R_A$

Element (lower left):

$$\uparrow \sum F = 0: V - \frac{w_0 x^2}{L} = 0$$

$$V = \frac{w_0 x^2}{L}$$

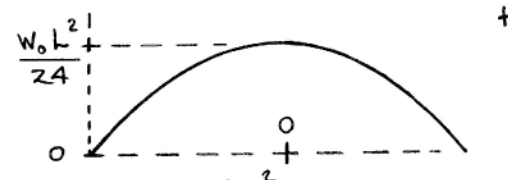


Consider  $x = L/2$ :

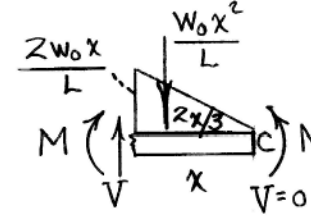
$$\uparrow \sum M_C = 0:$$

$$+M_0 - \frac{w_0 L}{4} \left(\frac{L}{2}\right) + \frac{w_0 \left(\frac{L}{2}\right)^2}{L} \left(\frac{2L}{3}\right) = 0$$

$$M_0 = \frac{w_0 L^2}{24}$$



Consider arbitrary  $x$ :



$$\uparrow \sum M_0 = 0:$$

$$-M - \frac{w_0 x^2}{L} x + \frac{w_0 x^2}{L} \frac{2x}{3} + \frac{w_0 L^2}{24} = 0$$

$$M = \frac{w_0}{3L} \left( \frac{L^3}{8} - x^3 \right)$$

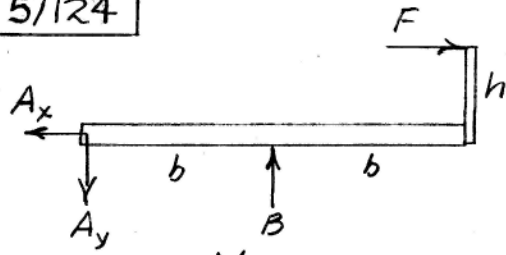
$$M = M_{\max} @ x=0 : M_{\max} = \frac{w_0}{3L} \left( \frac{L^3}{8} - 0 \right)$$

$$= \frac{w_0 L^2}{24}$$



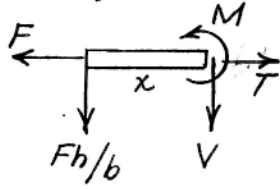
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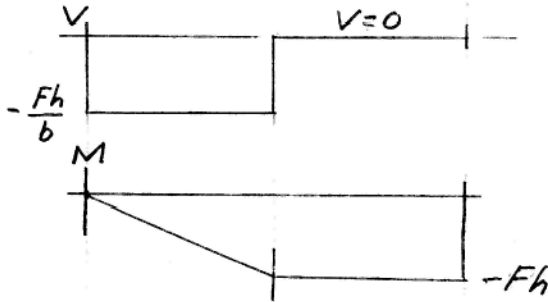
$$\Sigma M_A = 0; Fh - Bb = 0$$
$$B = Fh/b$$

$$\Sigma F_y = 0; A_y = B = Fh/b$$

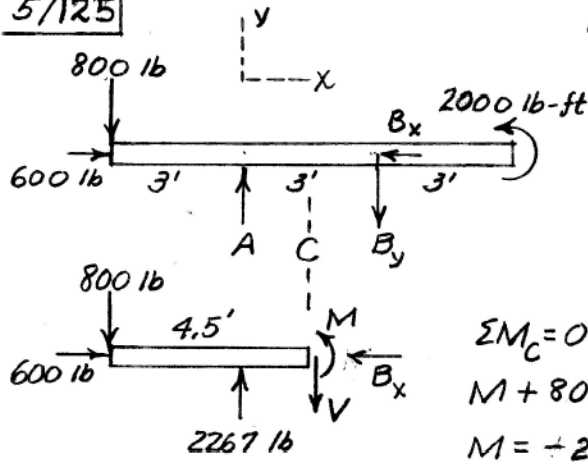


$$\Sigma F_y = 0; V = -Fh/b$$

$$\Sigma M = 0; M + \frac{Fh}{b}x = 0, M = -\frac{Fh}{b}x$$



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$$\Sigma M_A = 0;$$

$$2000 - 3B_y + 3(800) = 0$$

$$B_y = 1467 \text{ lb}$$

$$\Sigma F_y = 0;$$

$$A - 1467 - 800 = 0$$

$$A = 2267 \text{ lb}$$

$$\Sigma M_C = 0;$$

$$M + 800(4.5) - 2267(1.5) = 0$$

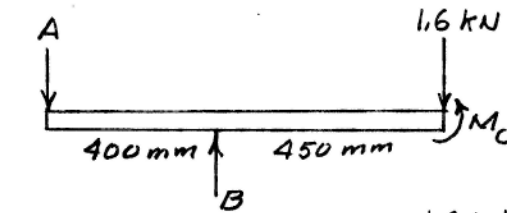
$$M = +200 \text{ lb-ft}$$

$$\Sigma F_y = 0;$$

$$2267 - 800 - V = 0$$

$$V = 1467 \text{ lb}$$

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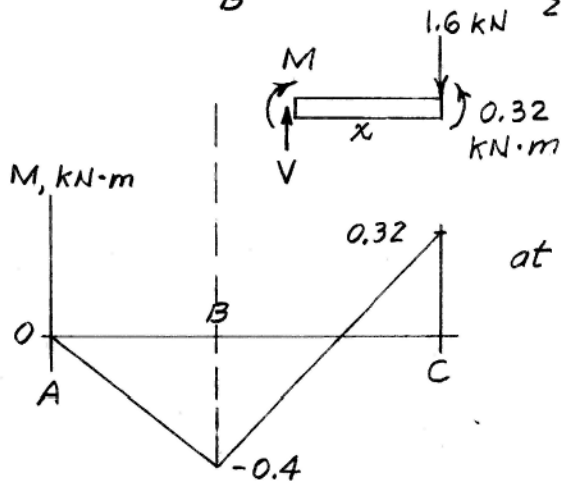
$$M_C = 1.6(0.200) = 0.32 \text{ kN}\cdot\text{m}$$

$$\sum M_A = 0; 0.4B + 0.32 - 0.85(1.6) = 0$$

$$B = 2.6 \text{ kN}$$

$$\sum F = 0; A + 1.6 - 2.6 = 0$$

$$A = 1.0 \text{ kN}$$



$$\sum M = 0$$

$$0.32 - 1.6x - M = 0$$

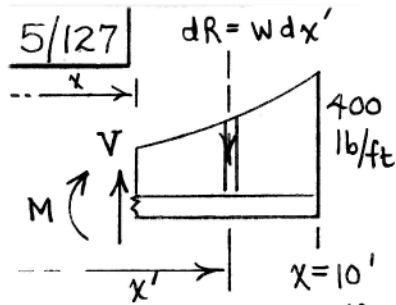
$$M = 0.32 - 1.6x$$

at B,  $x = 0.45 \text{ m}$

$$M_B = -0.40 \text{ kN}\cdot\text{m}$$

$M = 0$  when

$$x = \frac{0.32}{1.6} = 0.2 \text{ m}$$



From  $w = w_0 + kx^2 = 100 + kx^2$ :

$$400 = 100 + k(10)^2$$

$$k = 3 \text{ lb/ft}^3, \quad w = 100 + 3x^2$$

$$\uparrow \Sigma F = 0 : V - \int_x^{10} w dx' = 0$$

$$V = \int_x^{10} (100 + 3x'^2) dx' = 100x' + x'^3 \Big|_x^{10}$$

$$V = 2000 - 100x - x^3 \quad (\text{in lb if } x \text{ is in ft})$$

$$\curvearrowright \Sigma M = 0 : -M - \int_x^{10} (x' - x) w dx' = 0$$

$$M = - \int_x^{10} (-100x + 100x' + 3x'^3 - 3xx'^2) dx'$$

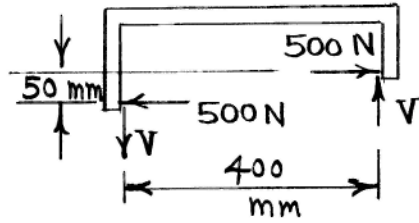
$$= - \left[ -100xx' + 50x'^2 - xx'^3 + \frac{3}{4}x'^4 \right] \Big|_x^{10}$$

$$= -12,500 + 2000x - 50x^2 - \frac{1}{4}x^4$$

(in lb-ft if  $x$  is in ft)

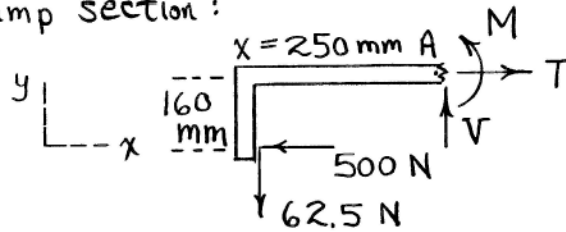
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FBD of clamp:



$$\Sigma M = 0 : 500 (0.050) - V (0.400) = 0, V = 62.5 \text{ N}$$

FBD of clamp section:



$$\Sigma F_x = 0 : T - 500 = 0,$$

$$T = 500 \text{ N}$$

$$\Sigma F_y = 0 : V - 62.5 = 0,$$

$$V = 62.5 \text{ N}$$

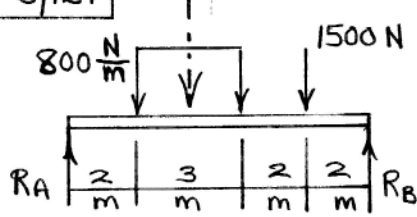
$$\Sigma M_A = 0 : M + 62.5 (0.250) - 500 (0.160) = 0$$

$$M = 64.4 \text{ N}\cdot\text{m}$$

M is the only quantity which depends on x.

5/129

$$R = 800(3) = 2400 \text{ N}$$

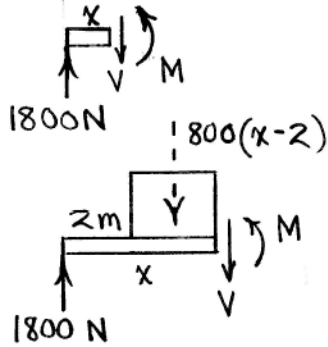


$$\sum M_A = 0: R_B(9) - 1500(7)$$

$$- 2400(3.5) = 0, R_B = 2100 \text{ N}$$

$$\sum F = 0: R_A - 2400 - 1500 + 2100 = 0$$

$$R_A = 1800 \text{ N}$$



$$0 < x < 2 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = 1800 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 1800x$$

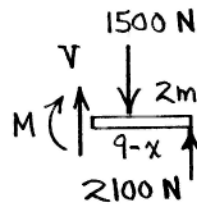
$$2 < x < 5 \text{ m}:$$

$$\sum F = 0: 1800 - 800(x-2) - V = 0$$

$$V = 3400 - 800x$$

$$\sum M = 0: M + 800(x-2) \frac{x-2}{2} + 1800x = 0$$

$$M = -400x^2 + 3400x - 1600$$



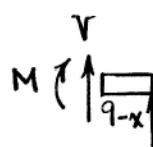
$$5 < x < 7:$$

$$\sum F = 0: 2100 - 1500 + V = 0$$

$$V = -600 \text{ N}$$

$$\sum M = 0: -M - 1500(7-x) + 2100(9-x)$$

$$= 0, M = 8400 - 600x$$

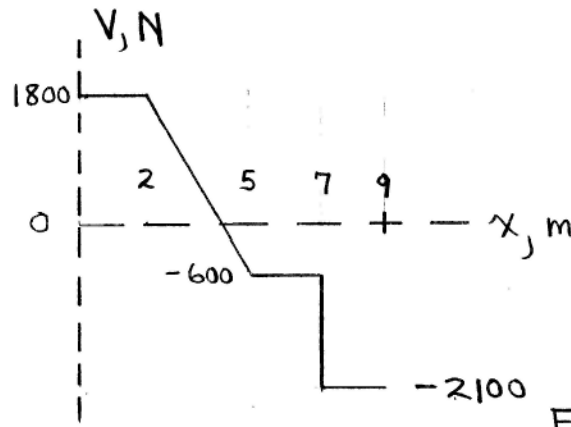


$$7 < x < 9 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = -2100 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 18900 - 2100x$$





At  $x = 6 \text{ m}$ :

$$V = -600 \text{ N}$$

$$M = 8400 - 600(6)$$

$$= \underline{4800 \text{ N}\cdot\text{m}}$$

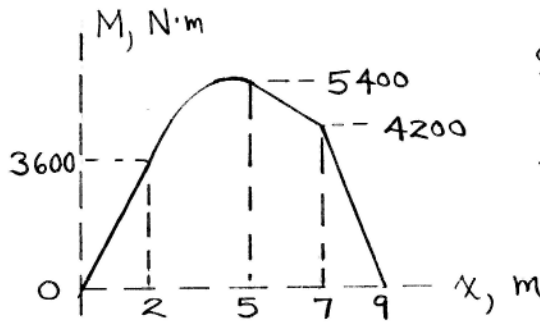
For  $M_{\max}$ ,

$$\frac{dM}{dx} = 0$$

$$\frac{d}{dx} (-400x^2 + 3400x - 1600)$$

$$= -800x + 3400 = 0$$

$$\underline{x = 4.25 \text{ m}}$$

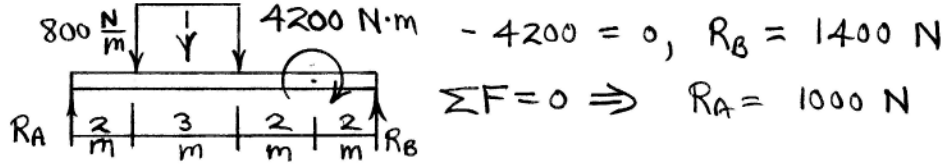


$$M_{\max} = -400(4.25)^2 + 3400(4.25) - 1600 = \underline{5620 \text{ N}\cdot\text{m}}$$

5/130

$$R = 800(3) = 2400 \text{ N}$$

$$\sum M_A = 0: R_B(9) - 2400(3.5)$$



$$\sum F = 0 \Rightarrow R_B = 1400 \text{ N}$$

$$0 < x < 2 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = 1000 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 1000x$$

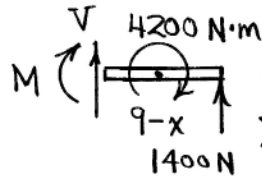
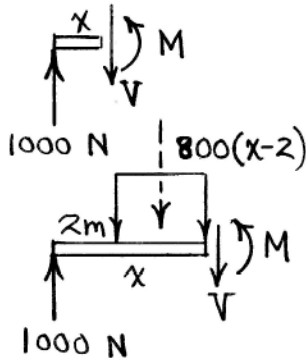
$$2 < x < 5 \text{ m}:$$

$$\sum F = 0: 1000 - 800(x-2) - V = 0$$

$$V = 2600 - 800x$$

$$\sum M = 0: M + 800(x-2)\frac{x-2}{2} - 1000x = 0$$

$$M = -400x^2 + 2600x - 1600$$

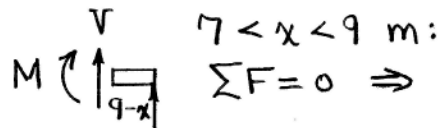


$$5 < x < 7 \text{ m}:$$

$$\sum F = 0: V + 1400 = 0, V = -1400 \text{ N}$$

$$\sum M = 0: -M - 4200 + 1400(9-x) = 0$$

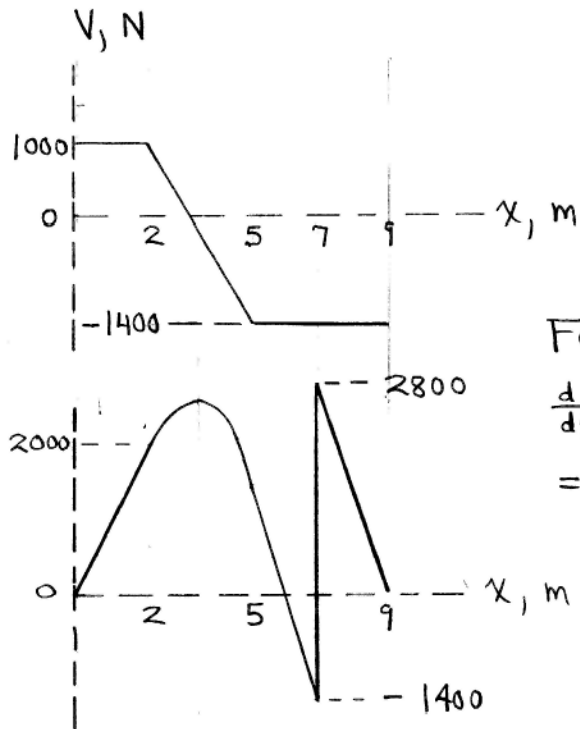
$$M = 8400 - 1400x$$



$$7 < x < 9 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = -1400 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 12600 - 1400x$$



$$\text{At } x = 6 \text{ m,}$$

$$V = -1400 \text{ N}$$

$$M = 8400 - 1400(6)$$

$$= 0$$

$$\text{For } M_{\max}, \frac{dM}{dx} = 0$$

$$\frac{d}{dx} (-400x^2 + 2600x - 1600)$$

$$= -800x + 2600 = 0$$

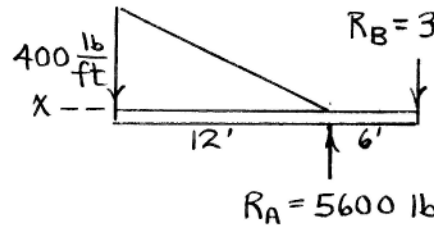
$$x = 3.25 \text{ m}$$

$$M_{x=3.25} = -400(3.25)^2 + 2600(3.25) - 1600 = 2625 \text{ N}\cdot\text{m}$$

$$\text{So } \underline{M_{\max} = 2800 \text{ N}\cdot\text{m}}$$

5/131

( $R_A$  &  $R_B$  from Prob. 5/96)

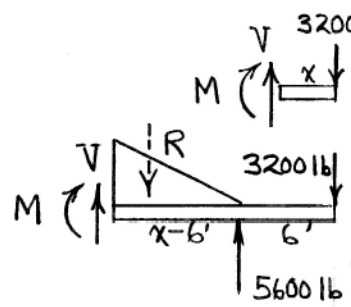


$0 < x < 6'$  :

$$+\uparrow \Sigma F = 0 \Rightarrow V = 3200 \text{ lb}$$

$$\Sigma M = 0 : M + 3200x = 0$$

$$M = -3200x$$



$6' < x < 18'$  :

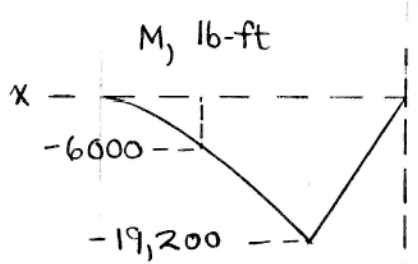
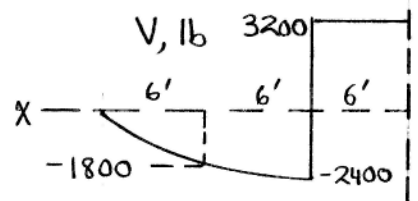
$$R = \frac{1}{2} \left[ \frac{100}{3}(x-6) \right] (x-6) = \frac{100}{6}(x-6)^2$$

$$+\uparrow \Sigma F = 0 : V + 5600 - 3200 - \frac{100}{6}(x-6)^2 = 0$$

$$V = \frac{100}{6}(x-6)^2 - 2400$$

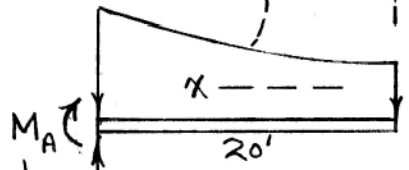
$$\Sigma M = 0 : M + \frac{100}{6}(x-6)^2 \frac{x-6}{3} + 3200(6+x-6) - 5600(x-6) = 0$$

$$M = -\frac{50}{9}(x-6)^3 + 2400x - 33,600$$



At  $x = 12'$  ,  $V = -1800 \text{ lb}$   
 $M = -6000 \text{ lb-ft}$

5/132  $W = 50 + \frac{x^2}{10}$   $\left\{ W, R_A, \text{ and } M_A \text{ from Prob. 5/106} \right.$

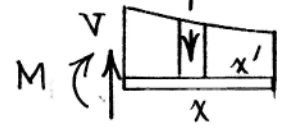


$R_A = 1267 \text{ lb}$   
 $M_A = -11.33(10^3) \text{ lb-ft}$

$+\uparrow \Sigma F = 0: V - \int W dx' = 0$

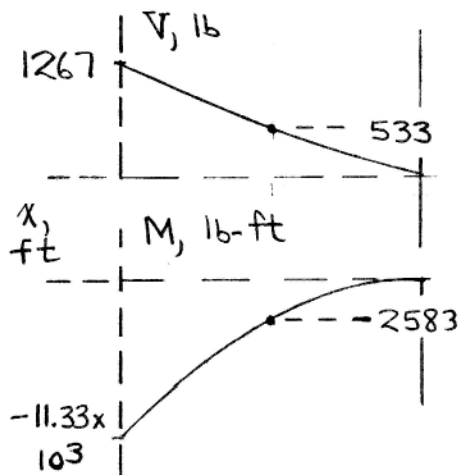
$V = \int_0^x (50 + \frac{x'^2}{10}) dx'$   
 $= 50x + \frac{x^3}{30}$

$dR = w dx'$



$\Sigma M = 0: M + \int_0^x w dx' (x - x'), M = - \int_0^x (50 + \frac{x'^2}{10}) (x - x') dx'$

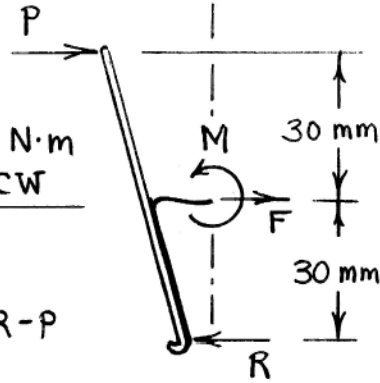
$M = -50xx' + 25x'^2 - \frac{xx'^3}{30} + \frac{x'^4}{40} \Big|_0^x$   
 $= -25x^2 - \frac{x^4}{120}$



$\left\{ \begin{array}{l} \text{At } x = 20', \\ V = 533 \text{ lb} \\ M = -2580 \text{ lb-ft} \end{array} \right.$

5/133

For  $P=0$ ,  $R=40\text{ N}$  and  
 $F=40\text{ N}$ ,  $M=40(0.030)=1.2\text{ N}\cdot\text{m}$   
 CCW



For  $0 < P < 40\text{ N}$

$$\Rightarrow \sum F_x = 0: F - R + P = 0, \quad F = R - P$$

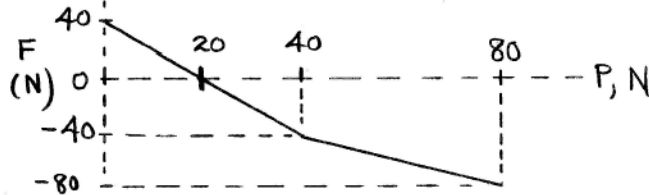
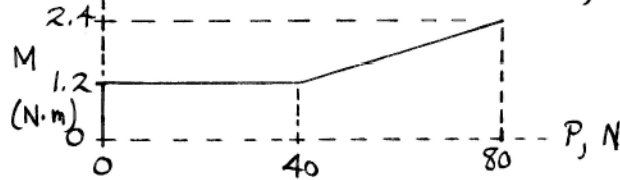
$$\circlearrowleft \sum M_c = 0: \underline{M = 1.2\text{ N}\cdot\text{m}}, \text{ so}$$

$$(P+R)(0.030) - 1.2 = 0, \quad P+R = 40\text{ N}$$

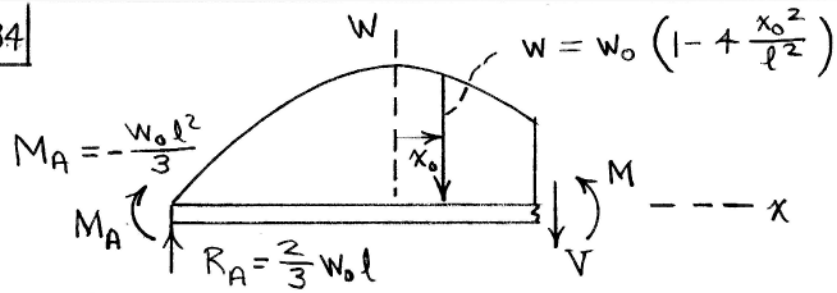
$$\text{So } \underline{F = 40 - 2P}$$

For  $P > 40\text{ N}$ ,  $R=0$ ,  $F=-P$

$$\sum M_c = 0: P(0.030) - M = 0, \quad \underline{M = 0.030P}$$



5/134



( $R_A$  and  $M_A$  from Prob. 5/104)

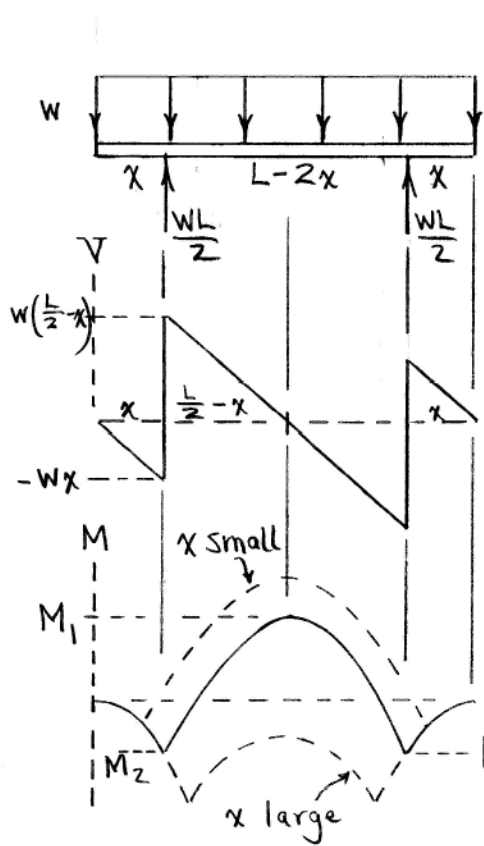
$$\uparrow \sum F = 0: \frac{2}{3} w_0 l - \int_{-l/2}^x w_0 \left(1 - 4 \frac{x_0^2}{l^2}\right) dx_0 - V = 0$$

$$V = w_0 \left(\frac{l}{3} - x + \frac{4x^3}{3l^2}\right)$$

$$\curvearrowright \sum M = 0: M + \int_{-l/2}^x w_0 \left(1 - 4 \frac{x_0^2}{l^2}\right) (x - x_0) dx_0$$

$$+ \frac{w_0 l^2}{3} - \frac{2}{3} w_0 l \left(\frac{l}{2} + x\right) = 0, \quad M = w_0 \left(-\frac{l^2}{16} + \frac{x l}{3} - \frac{x^2}{2} + \frac{x^4}{3l^2}\right)$$

► 5/135 | From areas under shear diagram :



$$|M_2| = \frac{1}{2} wx^2$$

$$|M_1| = \frac{w}{2} \left(\frac{L}{2} - x\right)^2 - \frac{1}{2} wx^2$$

$$= \frac{wL}{2} \left(\frac{L}{4} - x\right)$$

$|M|_{\min}$  occurs when

$$|M_1| = |M_2| \text{ . So}$$

$$\frac{wL}{2} \left(\frac{L}{4} - x\right) = \frac{1}{2} wx^2$$

$$\text{or } x^2 + Lx - \frac{L^2}{4} = 0$$

$$\Rightarrow x = \frac{L}{2} (-1 \pm \sqrt{2})$$

(-) sign : irrelevant

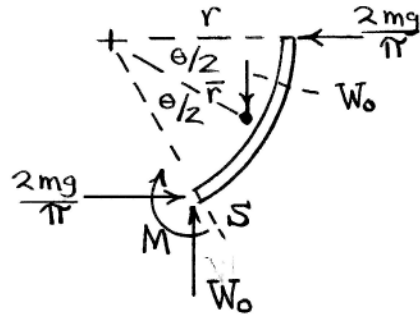
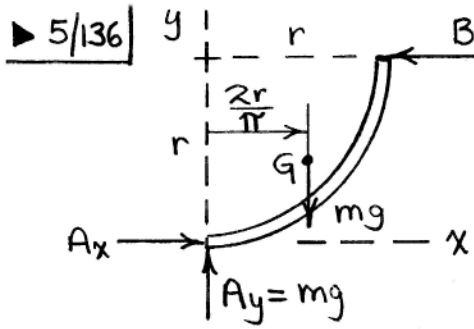
(+) sign :  $x = 0.207L$

$$x \quad M_{\max} = |M_1|$$

$$= \frac{1}{2} w (0.207L)^2$$

$$= \underline{0.0214wL^2}$$





As a whole :  $\sum M_A = 0 : Br - mg \frac{2r}{\pi} = 0, B = \frac{2mg}{\pi}$

Section :  $W_0 = \frac{\theta}{\pi/2} mg = \frac{2\theta}{\pi} mg, \bar{r} = r \frac{\sin \frac{\theta}{2}}{\theta/2}$

$\sum M_S = 0 : M + \frac{2\theta}{\pi} mg \left( r \frac{\sin \frac{\theta}{2}}{\theta/2} \cos \frac{\theta}{2} - r \cos \theta \right) - \frac{2mg}{\pi} r \sin \theta = 0, M = \frac{2mgr}{\pi} \theta \cos \theta$

$C = W_0 \cos \theta + \frac{2mg}{\pi} \sin \theta, C = \frac{2mg}{\pi} (\theta \cos \theta + \sin \theta)$

$V = W_0 \sin \theta - \frac{2mg}{\pi} \cos \theta, V = \frac{2mg}{\pi} (\theta \sin \theta - \cos \theta)$



5/137 | Given :  $\begin{cases} 2s = 50 \text{ ft}, & s = 25 \text{ ft} \\ \mu = \frac{0.1}{50} = 0.002 \text{ lb/ft} \\ T = 10 \text{ lb} \end{cases}$

$$T^2 = T_0^2 + \mu^2 s^2 : 10^2 = T_0^2 + (0.002 \cdot 25)^2$$
$$T_0 = 9.999875 \text{ lb}$$

Eq. 5/22 :  $T = T_0 + \mu y$

$$10 = 9.999875 + 0.002 y$$

$$y = 0.0625 \text{ ft (0.750 in.)}$$

\*5/138 | Eq. 5/19 at B:

$$5 = \frac{T_0}{\mu} \left[ \cosh \frac{\mu(65)}{T_0} - 1 \right]$$

Numerical solution with  $\mu = 14(9.81) = 137.3 \text{ N/m}$ :

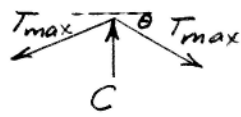
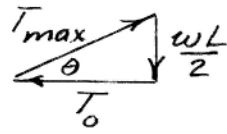
$$T_0 = \underline{58\,100 \text{ N}} = T_A$$

Then Eq. 5/22 gives

$$T_B = 58\,100 + 137.3(5) = \underline{58\,800 \text{ N}}$$

5/139 |  $L = 4200 \text{ ft}$ ,  $h = 470 \text{ ft}$ ,  $w = \frac{21,300}{2} = 10650 \text{ lb/ft}$   
 for each cable

$$T_0 = \frac{wL^2}{8h} = \frac{10650(4200)^2}{8(470)} = \underline{50.0(10^6) \text{ lb}}$$



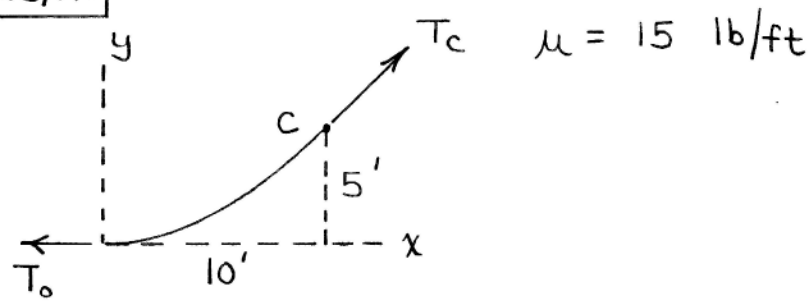
$$C = 2T_{\max} \sin \theta$$

$$= 2\left(\frac{wL}{2}\right) = wL$$

$$= 10650(4200)$$

$$= \underline{44.7(10^6) \text{ lb}}$$

\*5/140



Eq. 5/19 evaluated at point C:

$$5 = \frac{T_0}{\mu} \cosh \left[ \frac{10}{T_0/\mu} - 1 \right]$$

Numerical solution:  $\frac{T_0}{\mu} = 10.74 \text{ ft}$

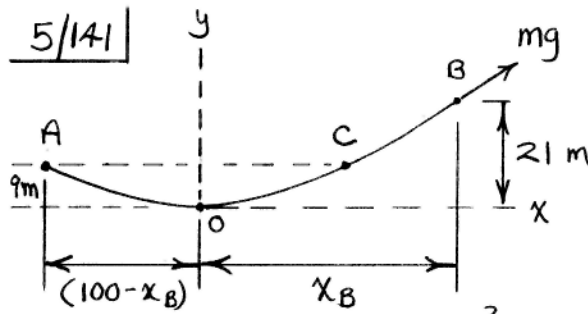
Then  $T_0 = 10.74(15) = 161.1 \text{ lb}$

Eq. 5/22 evaluated at C:

$$T_c = 161.1 + 15(5) = \underline{236 \text{ lb}}$$

Eq. 5/20:  $s = 10.74 \sinh \frac{10}{10.74} = 11.51 \text{ ft}$

$$L = 2s = \underline{23.0 \text{ ft}}$$



Eq. 5/14 :  $y = \frac{Wx^2}{2T_0}$

At B:  $21 = \frac{Wx_B^2}{2T_0}$  } Eliminate  $T_0$  to obtain

At A:  $9 = \frac{W(100-x_B)^2}{2T_0}$  }  $x_B^2 - 350x_B + 17500 = 0$

Quadratic solution:  $x_B = 290 \text{ m (reject)}$ ,  $60.4 \text{ m}$

From Eq. 5/14 @ B:  $21 = \frac{25(60.4)^2}{2T_0}$ ,  $T_0 = 2170 \text{ N}$

For section OB:  $(mg)^2 = T_0^2 + (Wx_B)^2$

$m^2(9.81)^2 = 2170^2 + (25 \cdot 60.4)^2$ ,  $m = 270 \text{ kg}$

Parabolic equation  $y = \frac{25x^2}{2(2170)} = 0.00575x^2$

At C,  $y_C = 9 \text{ m}$ :  $9 = 0.00575x_C^2$ ,  $x_C = 39.6 \text{ m}$

$\overline{AC} = 2x_C = 79.1 \text{ m}$

\*5/142 Please refer to the diagram in the solution to Prob. 5/141. Eq. 5/19:

$$y = \frac{T_0}{\mu} \left[ \cosh \frac{\mu x}{T_0} - 1 \right]$$

$$\text{At B: } 21 = \frac{T_0}{25} \left[ \cosh \frac{25 x_B}{T_0} - 1 \right]$$

$$\text{At A: } 9 = \frac{T_0}{25} \left[ \cosh \frac{25}{T_0} (100 - x_B) - 1 \right]$$

Solve these two equations numerically to obtain

$$T_0 = 2240 \text{ N}, \quad x_B = 60.2 \text{ m}$$

$$\text{Eq. 5/20: } s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$$

$$\text{At B: } s_B = \frac{2240}{25} \sinh \left( \frac{25 \cdot 60.2}{2240} \right) = 64.8 \text{ m}$$

Equilibrium of section OB:

$$(mg)^2 = T_0^2 + (\mu s_B)^2: m^2 (9.81)^2 = 2240^2 + (25 \cdot 64.8)^2$$

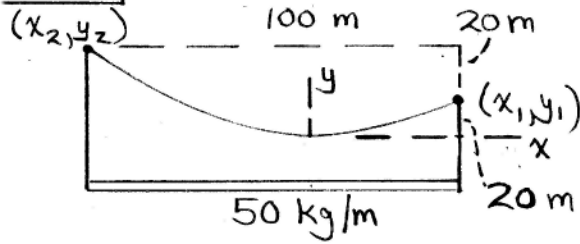
$$m = 282 \text{ kg}$$

$$\text{Eq. 5/19 @ C: } 9 = \frac{2240}{25} \left[ \cosh \frac{25 x_C}{2240} - 1 \right]$$

Numerical solution:  $x_C = 39.8 \text{ m}$

$$\overline{AC} = 2 x_C = \underline{79.6 \text{ m}}$$

5/143



Eq. 5/14:

$$y = \frac{wx^2}{2T_0}$$

$$20 = \frac{50(9.81)x_1^2}{2T_0} \quad , \quad 40 = \frac{50(9.81)x_2^2}{2T_0}$$

$$\text{Also, } x_1 + (-x_2) = 100 \text{ m}$$

$$\text{Solve simultaneously: } x_1^2 + 200x_1 - 10000 = 0$$

$$x_1 = 41.4 \text{ m (or } -241 \text{ m)}$$

$$x_2 = x_1 - 100 = 41.4 - 100 = -58.6 \text{ m}$$

$$T_0 = \frac{wx^2}{2y} = \frac{50(9.81)(41.4)^2}{2(20)} = 21.0(10^3) \text{ N}$$

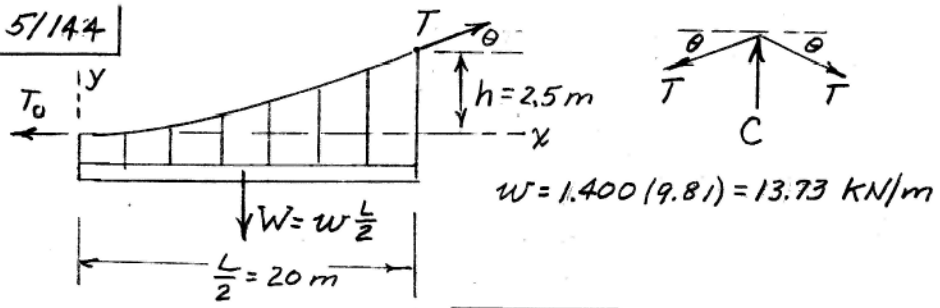
$$\text{Maximum tension is } T_{\max} = \sqrt{T_0^2 + (wx_2)^2}$$

$$= \sqrt{[21.0(10^3)]^2 + [50(9.81)(58.6)]^2} = 35.6(10^3) \text{ N}$$

$$\text{or } \underline{T_{\max} = 35.6 \text{ kN}}$$



5/144



From Eq. 5/15b,  $T = \frac{wL}{2} \sqrt{1 + (L/4h)^2}$

$$= \frac{13.73(40)}{2} \sqrt{1 + \left[\frac{40}{4(2.5)}\right]^2} = 1133 \text{ kN}$$

$$T^2 = W^2 + T_0^2, \quad T_0 = \sqrt{(1133)^2 - [(13.73)(20)]^2} = 1099 \text{ kN}$$

$$\frac{dy}{dx} = \tan \theta = \frac{w(L/2)}{T_0} = \frac{13.73(20)}{1099} = 0.250, \quad \theta = 14.04^\circ$$

$\sum F_y = 0$  at support;  $2T \sin \theta - C = 0$

$$C = 2(1133) \sin 14.04^\circ = \underline{549 \text{ kN}}$$

$$\underline{5/145} \quad w = w_0 \left(1 - \frac{x}{l}\right)$$

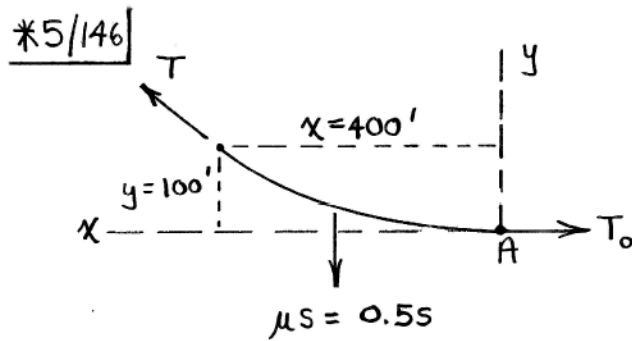
$$\text{From Eq. 5/13, } \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{w_0}{T_0} \left(1 - \frac{x}{l}\right)$$

$$\text{so } \frac{dy}{dx} = \frac{w_0}{T_0} \left(x - \frac{x^2}{2l}\right) + C_1, \quad C_1 = 0 \text{ since } \frac{dy}{dx} = 0 \text{ at } x=0$$

$$\& \quad y = \frac{w_0}{T_0} \left(\frac{x^2}{2} - \frac{x^3}{6l}\right) + C_2, \quad C_2 = 0 \text{ since } y=0 \text{ at } x=0$$

$$\text{For } y=h \text{ \& } x=l, \quad T_0 h = w_0 \left(\frac{l^2}{2} - \frac{l^2}{6}\right), \quad T_0 = \frac{w_0 l^2}{3h}$$

$$\text{Thus } \underline{y = \frac{3hx^2}{2l^2} \left(1 - \frac{x}{3l}\right)}$$



Catenary Eq. 5/19:  $y = \frac{T_0}{\mu} \left( \cosh \frac{\mu x}{T_0} - 1 \right)$

$$100 = \frac{T_0}{0.5} \left( \cosh \frac{0.5(400)}{T_0} - 1 \right)$$

Solve numerically to obtain  $T_0 = 408 \text{ lb}$

Parabolic Eq. 5/14:  $y = \frac{wx^2}{2T_0} \approx \frac{\mu x^2}{2T_0}$

$$\text{So } T_0 \approx \frac{\mu x^2}{2y} = \frac{0.5(400)^2}{2(100)} = \underline{400 \text{ lb}}$$

$$\underline{5/147} \quad w = a + bx^2, \text{ when } x=0, w = w_0$$

$$x=L/2, w = w_1$$

$$\text{So } a = w_0 \text{ \& } b = \frac{4}{L^2}(w_1 - w_0); \text{ Thus } w = w_0 + \frac{4(w_1 - w_0)}{L^2} x^2$$

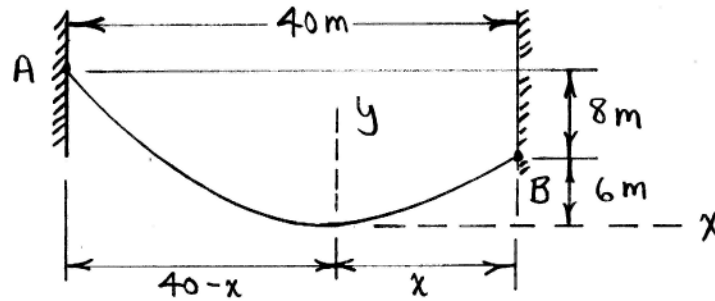
$$\text{From Eq. 5/13, } \frac{dy}{dx} = \frac{1}{T_0} \int_0^x w dx$$

$$= \frac{1}{T_0} \left[ w_0 x + \frac{4(w_1 - w_0)}{L^2} \frac{x^3}{3} \right]$$

$$\& y = \frac{w_0 x^2}{2T_0} + \frac{w_1 - w_0}{3T_0 L^2} x^4; \text{ Thus for } x=L/2, y=h$$

$$\& h = \frac{w_0 L^2}{8T_0} + \frac{w_1 - w_0}{3T_0 L^2} \frac{L^4}{16} = \frac{L^2}{48T_0} (5w_0 + w_1)$$

\*5/148



$$\text{Eq. 5/19 : } y = \frac{T_0}{\mu} \left( \cosh \frac{\mu x}{T_0} - 1 \right)$$

$$\text{At B: } 6 = \frac{T_0}{\mu} \left( \cosh \frac{x}{T_0/\mu} - 1 \right) \quad (1)$$

$$\text{At A: } 14 = \frac{T_0}{\mu} \left( \cosh \frac{x-40}{T_0/\mu} - 1 \right) \quad (2)$$

Solve Eqs. (1) & (2) numerically to obtain

$$x = 16.07 \text{ m, } \frac{T_0}{\mu} = 22.5 \text{ m}$$

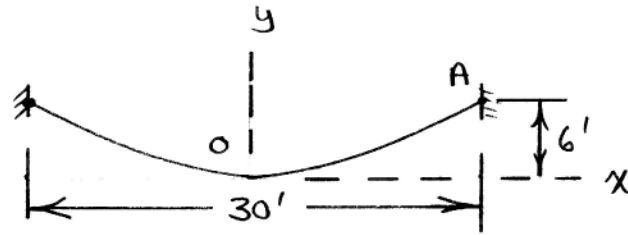
$$\text{Eq. 5/20: } S = \frac{T_0}{\mu} \sinh \frac{x}{T_0/\mu}$$

$$\text{At B: } S_B = 22.5 \sinh \frac{16.07}{22.5} = 17.48 \text{ m}$$

$$\text{At A: } S_A = 22.5 \sinh \frac{40-16.07}{22.5} = 28.7 \text{ m}$$

$$L = S_A + S_B = 28.7 + 17.48 = \underline{46.2 \text{ m}}$$

\*5/149



Eq. 5/19, from o to A:  $6 = \frac{T_0}{\mu} \left[ \cosh \frac{\mu}{T_0} (15) - 1 \right]$

By numerical or graphical means,  $\frac{T_0}{\mu} = 19.68 \text{ m}$

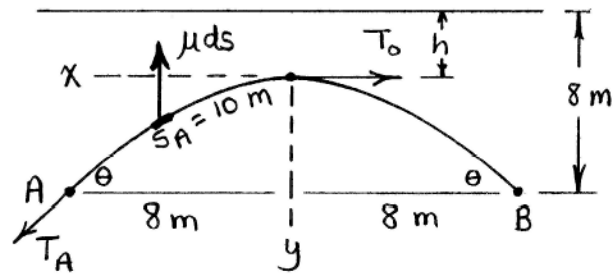
Eq. 5/20:  $s = \frac{L}{2} = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$\therefore L = 2 \left( \frac{T_0}{\mu} \right) \sinh \frac{\mu}{T_0} 15$

$= 2(19.68) \sinh \frac{15}{19.68} = \underline{\underline{33.0 \text{ ft}}}$

\*5/150

$$\begin{aligned}\mu &= 560 - 100 \\ &= 460 \text{ N/m}\end{aligned}$$



$$\text{Eq. 5/20: } s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} : 10 = \frac{T_0}{460} \sinh \frac{460(8)}{T_0}$$

Numerical or graphical solution:  $T_0 = 3110 \text{ N}$

$$\text{Eq. 5/21: } T = T_0 \cosh \frac{\mu x}{T_0} = 3110 \cosh \frac{460(8)}{3110}$$

$$T_A = 5550 \text{ N}$$

$$\begin{aligned}\text{Eq. 5/22: } T &= T_0 + \mu y \text{ at A: } 5550 = 3110 + 460y \\ y &= 5.31 \text{ m}\end{aligned}$$

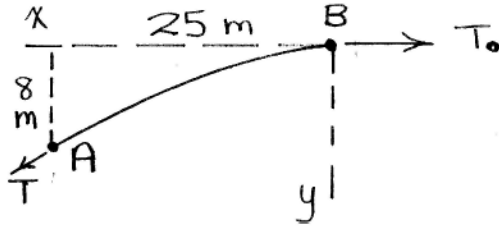
$$\text{Then } h = 8 - 5.31 = \underline{2.69 \text{ m}}$$

$$\text{From Eq. 5/19, } \frac{dy}{dx} = \tan \theta = \sinh \frac{\mu x}{T_0}$$

$$\tan \theta = \sinh \frac{460(8)}{3110} = \underline{55.9^\circ}$$

\*5/151

$$\mu = 30 \text{ N/m}$$



$$\text{Eq. 5/19: } y = \frac{T_0}{\mu} \left[ \cosh \frac{\mu x}{T_0} - 1 \right]$$

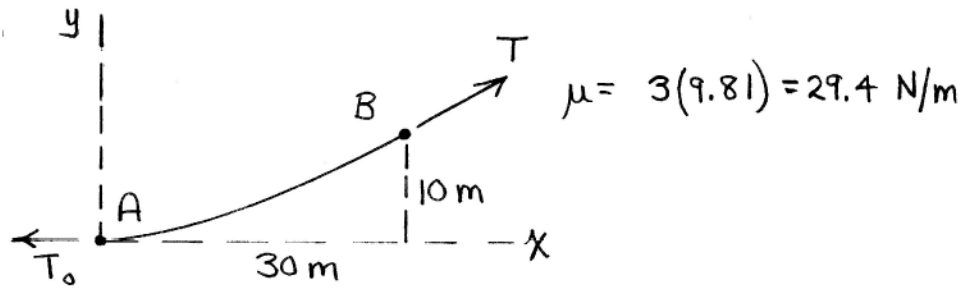
$$\text{At A: } 8 = \frac{T_0}{\mu} \left[ \cosh \frac{25\mu}{T_0} - 1 \right]$$

$$\text{Numerical solution: } \frac{T_0}{\mu} = 40.3 \text{ m}$$

$$T_0 = 40.3 (30) = \underline{1210 \text{ N}}$$



\*5/152



$$\text{Eq. 5/19: } 10 = \frac{T_0}{\mu} \left[ \cosh \frac{\mu x}{T_0} - 1 \right]$$

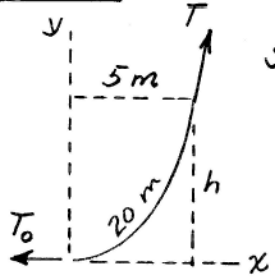
Solve numerically to obtain  $\frac{T_0}{\mu} = 46.6 \text{ N}$

$$T_0 = 46.6 (29.4) = 1371 \text{ N}$$

$$\text{Eq. 5/22: } T = T_0 + \mu y = 1371 + 29.4(10) \\ = \underline{1665 \text{ N}}$$

$$\text{Eq. 5/20: } s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} = 46.6 \sinh \frac{30}{46.6} \\ = \underline{32.1 \text{ m}}$$

\* 5/153



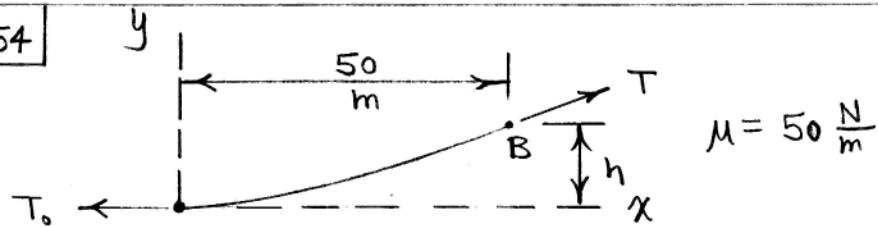
$$\text{Eq. 5/20, } 20 = \frac{T_0}{\mu} \sinh \frac{5\mu}{T_0}$$

Solve by computer or graphically  
& get  $T_0/\mu = 1.532 \text{ m}$

$$\text{Eq. 5/19, } y = 1.532 (\cosh \frac{5}{1.532} - 1)$$

$$h = y = 1.532 (13.09 - 1) = \underline{18.53 \text{ m}}$$

\*5/154



Eliminate  $T_0$  between Eqs. 5/21 & 5/22  
to get  $T = (T - \mu y) \cosh \frac{\mu x}{T - \mu y}$

$$\text{At B: } T = (T - 50h) \cosh \frac{50(50)}{T - 50h}$$

$$\text{or } T = \beta \cosh \frac{2500}{\beta} \quad (\beta = T - 50h)$$

$$\text{For minimum } T, \quad \frac{dT}{d\beta} = \cosh \frac{2500}{\beta} - \frac{1}{\beta} \sinh \frac{2500}{\beta}$$

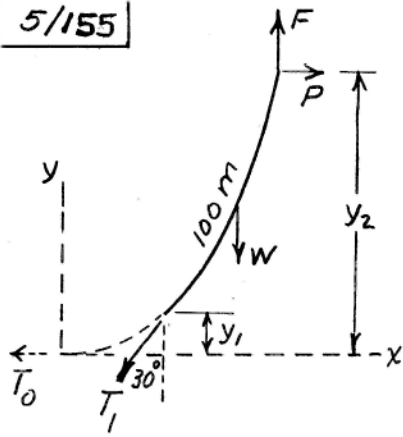
$$\text{Numerically solve: } \beta = 2084 \text{ N} \quad = 0$$

$$T = 2084 \cosh \frac{2500}{2084} = 3772 \text{ N}$$

$$\beta = T - 50h: \quad 2084 = 3772 - 50h, \quad \underline{h = 33.8 \text{ m}}$$

(Note:  $\beta = T_0$  !)

5/155



$$W = 0.51(9.81)(100) = 500.3 \text{ N}$$

$$T_1 = M/r = 400/0.5 = 1600 \text{ N}$$

$$\Sigma F_x = 0; P - 1600 \sin 30^\circ = 0$$
$$P = 800 \text{ N } (= T_0)$$

$$\Sigma F_y = 0; F - 500.3 - 1600 \cos 30^\circ = 0$$
$$F = 1886 \text{ N}$$

$$T = \sqrt{F^2 + P^2} = \sqrt{(1886)^2 + (800)^2}$$
$$= 2049 \text{ N}$$

From Eq. 5/22,  $T = T_0 + \mu y$ ,  $2049 = 800 + 5.003 y_2$

$$y_2 = 249.6 \text{ m}$$

Also,  $1600 = 800 + 5.003 y_1$

$$y_1 = 159.9 \text{ m}$$

$$H = y_2 - y_1 = \underline{89.7 \text{ m}}$$

\*5/156 (a) Use  $w = \mu = 1.2(9.81) = 11.77 \text{ N/m}$

Eq. 5/14:  $y = \frac{wx^2}{2T_0}$  @ A:  $2.4 = \frac{11.77(5)^2}{2T_0}$

So  $y_p = \frac{11.77x^2}{2(61.3)} = 0.096x^2$  (see plots below)  $T_0 = 61.3 \text{ N}$

Eq. 5/16:  $S_A = l_A \left[ 1 + \frac{2}{3} \left( \frac{h_A}{l_A} \right)^2 - \frac{2}{5} \left( \frac{h_A}{l_A} \right)^4 + \dots \right]$   
 $= 5 \left[ 1 + \frac{2}{3} \left( \frac{2.4}{5} \right)^2 - \frac{2}{5} \left( \frac{2.4}{5} \right)^4 + \dots \right] = 5.66 \text{ m}$

So the required length is  $L_p = 2S_A = \underline{11.32 \text{ m}}$

(b) Eq. 5/9:  $y = \frac{T_0}{\mu} \left( \cosh \frac{\mu x}{T_0} - 1 \right)$

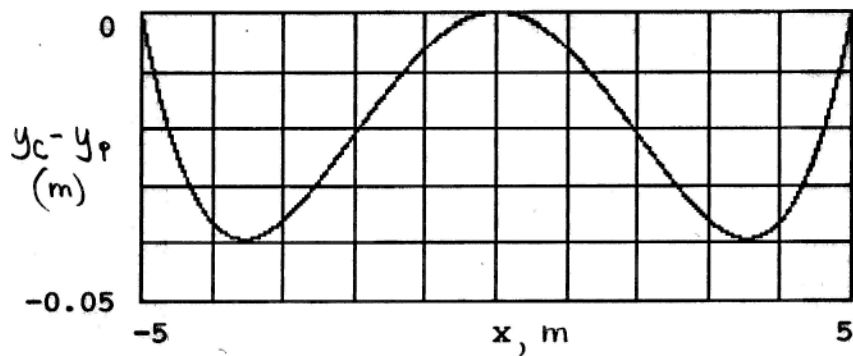
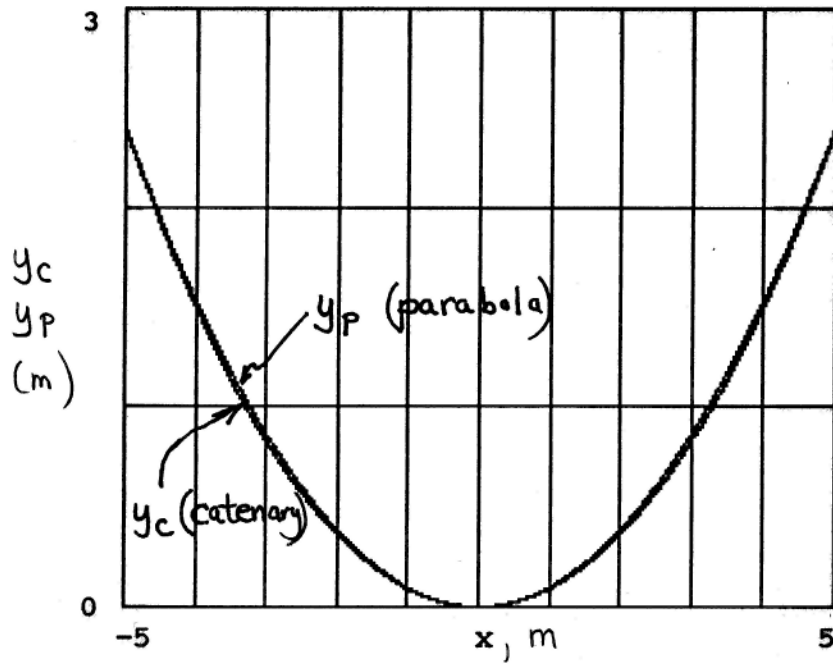
At A:  $2.4 = \frac{T_0}{11.77} \left[ \cosh \frac{11.77(5)}{T_0} - 1 \right]$

Numerical solution:  $T_0 = 65.5 \text{ N}$

So  $y_c = 5.57 \left[ \cosh(0.1796x) - 1 \right]$  (see plots)

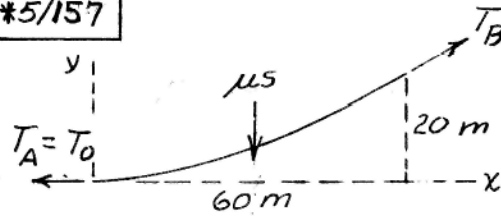
Eq. 5/20:  $S_A = \frac{65.5}{11.77} \sinh \frac{11.77(5)}{65.6} = 5.70 \text{ m}$

The required length is  $L_c = 2S_A = \underline{11.40 \text{ m}}$



Even with an expanded vertical scale,  $y_p$  and  $y_c$  are nearly indistinguishable from each other. Note that  $y_p$  is above  $y_c$  except at the end points (A and B) and the center. So the fact that  $L_c > L_p$  makes sense!

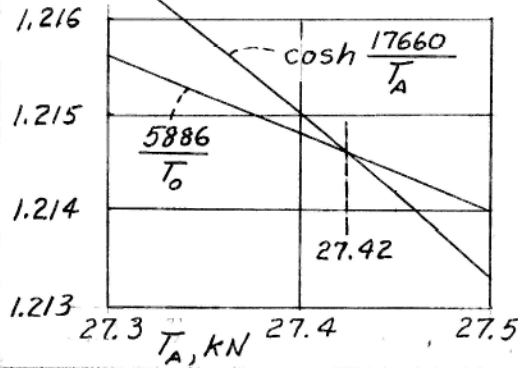
\*5/157



$$\mu = (10 + 20) 9.81 = 294 \text{ N/m}$$

Eg. 5/19,  $y = \frac{T_0}{\mu} (\cosh \frac{\mu x}{T_0} - 1)$ ,  $20 = \frac{T_A}{294} (\cosh \frac{294(60)}{T_A} - 1)$

$\frac{5886}{T_A} + 1 = \cosh \frac{17660}{T_A}$  solution by graphical or computer analysis  $T_A = 27420 \text{ N}$  or  $T_A = 27.4 \text{ kN}$



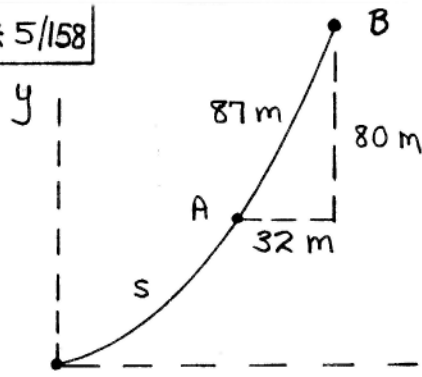
Eg. 5/22,  $T = T_0 + \mu y$

$$T_B = 27.4 + 0.294(20) = 33.3 \text{ kN}$$

Eg. 5/20,  $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$$s = \frac{27.4(10^3)}{294} \sinh \frac{294(60)}{27.4(10^3)} = 64.2 \text{ m}$$

\* 5/158



$$\mu = 2(9.81) = 19.62 \text{ N/m}$$

$$\text{Eq. 5/19: } y = \frac{T_0}{\mu} \left( \cosh \frac{\mu x}{T_0} - 1 \right)$$

$$\text{A: } y_A = \frac{T_0}{\mu} \left( \cosh \frac{\mu x_A}{T_0} - 1 \right)$$

$$\text{B: } y_A + 80 = \frac{T_0}{\mu} \left( \cosh \frac{\mu}{T_0} (x_A + 32) - 1 \right)$$

$$\text{Eq. 5/20: } s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$$

$$\text{A: } s = \frac{T_0}{\mu} \sinh \frac{\mu x_A}{T_0}$$

$$\text{B: } s + 87 = \frac{T_0}{\mu} \sinh \frac{\mu}{T_0} (x_A + 32)$$

$$\text{Numerical solution: } x_A = 24.0 \text{ m, } y_A = 12.30 \text{ m}$$
$$\frac{T_0}{\mu} = 25.2 \text{ m, } s = 27.8 \text{ m}$$

$$T_0 = 25.2 (19.62) = 495 \text{ N}$$

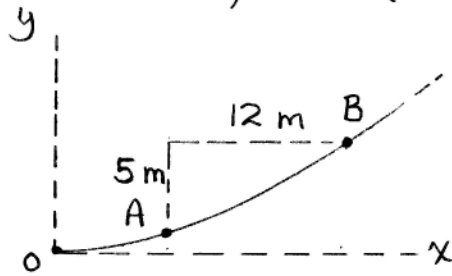
$$T_A = T_0 + \mu y_A = 495 + 19.62(12.30) = \underline{736 \text{ N}}$$

$$T_B = T_0 + \mu y_B = 495 + 19.62(92.3) = \underline{2310 \text{ N}}$$



\*5/159

$$\mu = 0.6(9.81) = 5.89 \text{ N/m}$$



$$\text{Eq. 5/19 @ A: } y_A = \frac{T_0}{\mu} \left[ \cosh \frac{\mu x_A}{T_0} - 1 \right]$$

$$\text{Eq. 5/19 @ B: } y_A + 5 = \frac{T_0}{\mu} \left[ \cosh \frac{\mu(x_A + 12)}{T_0} - 1 \right]$$

$$\text{Eq. 5/22 @ A: } 200 = T_0 + \mu y_A$$

Numerical solution of above three equations:

$$x_A = 7.37 \text{ m, } y_A = 0.823 \text{ m, } T_0 = 195.2 \text{ N}$$

From  $\frac{dy}{dx} = \sinh \frac{\mu x}{T_0}$ , we have, at A

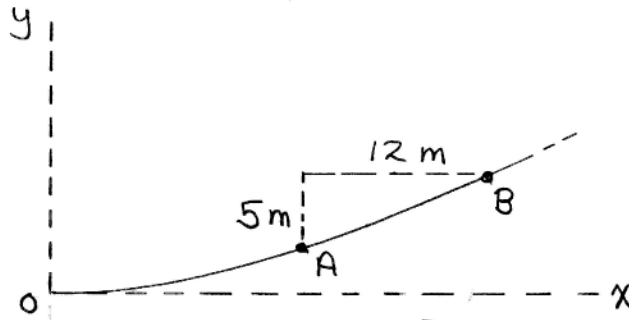
$$\theta_A = \tan^{-1} \left[ \sinh \left( \frac{5.89(7.37)}{195.2} \right) \right] = 12.64^\circ$$

$$\text{From 5/20, } s_B - s_A = \frac{195.2}{5.89} \left[ \sinh \frac{5.89(7.37+12)}{195.2} - \sinh \frac{5.89(7.37)}{195.2} \right] = 13.06 \text{ m} = L$$

$$\text{5/22: } T_B = 195.2 + 5.89(0.823 + 5) = 229 \text{ N}$$

\*5/160

$$\mu = 0.6(9.81) = 5.89 \text{ N/m}$$



$$\text{Eq. 5/19 @ A : } y_A = \frac{T_0}{\mu} \left[ \cosh \frac{\mu x_A}{T_0} - 1 \right]$$

$$\text{Eq. 5/19 @ B : } y_A + 5 = \frac{T_0}{\mu} \left[ \cosh \frac{\mu(x_A + 12)}{T_0} - 1 \right]$$

From Eq. 5/20

$$s_B - s_A = 13.02 = \frac{T_0}{\mu} \left[ \sinh \frac{\mu(x_A + 12)}{T_0} - \sinh \frac{\mu x_A}{T_0} \right]$$

Numerical solution of above three equations:

$$x_A = 17.34 \text{ m, } y_A = 2.63 \text{ m, } T_0 = 339 \text{ N}$$

$$5/22: T_A = 339 + 5.89(2.63) = \underline{355 \text{ N}}$$

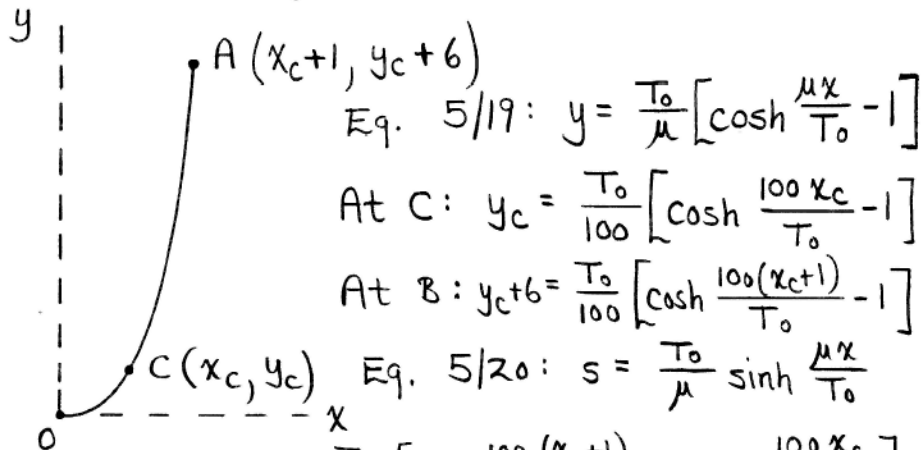
$$\theta_A = \tan^{-1} \left[ \sinh \frac{\mu x_A}{T_0} \right] = \tan^{-1} \left[ \sinh \frac{5.89(17.34)}{339} \right]$$
$$= \underline{16.98^\circ}$$

$$\text{Similarly, } T_B = 339 + 5.89(2.63 + 5) = \underline{384 \text{ N}}$$

$$\theta_B = \tan^{-1} \left[ \sinh \frac{5.89(17.34 + 12)}{339} \right] = \underline{28.0^\circ}$$

\*5/161 | Architect's plan :  $(T_A)_{\text{arch}} = 6(100) = 600 \text{ N}$

Builder's arrangement :



$$\text{So } s_A - s_c = 6.1 = \frac{T_0}{100} \left[ \sinh \frac{100(x_c+1)}{T_0} - \sinh \frac{100 x_c}{T_0} \right]$$

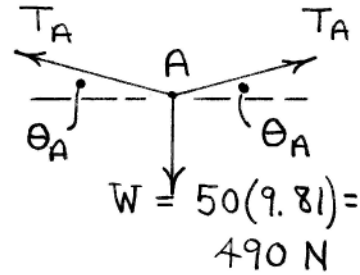
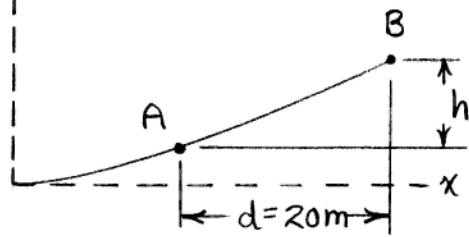
Numerical solution of three equations :

$$x_c = 1.071 \text{ m}, \quad y_c = 1.088 \text{ m}, \quad T_0 = 65.5 \text{ N}$$

$$\text{Eq. 5/22: } T = T_0 + \mu y, \text{ so } T_A = 65.5 + 100(1.088 + 6) = 774 \text{ N}$$

$$\text{Percent increase } n = \frac{774 - 600}{600} (100) = \underline{29.0\%} (!)$$

$$\frac{*5/162}{y} \begin{cases} \mu = 1.2 (9.81) = 11.77 \text{ N/m} \\ L = 21 \text{ m} \end{cases}$$



From FBD of junction ring at A,

$$\uparrow \Sigma F = 0: 2T_A \sin \theta_A - W = 0$$

$$\text{or } [T_0 + \mu y_A] \sin \left[ \tan^{-1} \left( \sinh \frac{\mu x_A}{T_0} \right) \right] - \frac{W}{2} = 0 \quad (1)$$

$$\text{Eq. 5/19 @ A: } y_A = \frac{T_0}{\mu} \left[ \cosh \frac{\mu x_A}{T_0} - 1 \right] \quad (2)$$

$$\text{Eq. 5/19 @ B: } y_A + h = \frac{T_0}{\mu} \left[ \cosh \frac{\mu (x_A + d)}{T_0} - 1 \right] \quad (3)$$

$$\text{Eq. 5/20: } s_B - s_A = \frac{T_0}{\mu} \left[ \sinh \frac{\mu (x_A + d)}{T_0} - \sinh \frac{\mu x_A}{T_0} \right] = L \quad (4)$$

Solution of (1)-(4) with  $W=0$ :  $h = 5.57 \text{ m}$

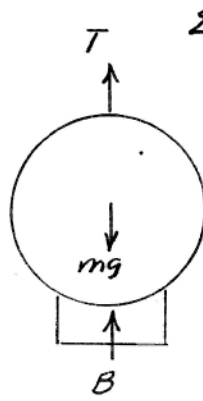
With  $W \neq 0$ :  $h = 6.30 \text{ m}$

$$\text{So } \delta = 6.30 - 5.57 = \underline{0.724 \text{ m}}$$

5/163

$$T = 8 \text{ kN}$$

$$mg = 6.7(9.81) = 65.7 \text{ kN}$$



$$\Sigma F = 0; 8 - 65.7 + B = 0$$

$$B = 57.7 \text{ kN}$$

$$B = \rho g V; V = \frac{57.7}{1.03(9.81)} = 5.71 \text{ m}^3$$

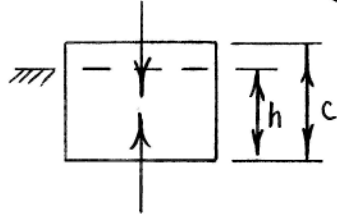
$$\rho_{\text{salt water}} = 1.03(10^3) \text{ kg/m}^3$$

$$\begin{aligned} \underline{5/164} \quad \text{Force on bottom} &= \text{weight of water} \\ &= \rho g V = (1000 \frac{\text{kg}}{\text{m}^3})(9.81 \frac{\text{m}}{\text{s}^2})(0.3\text{m})(0.7\text{m})(0.4\text{m}) \\ &= \underline{824 \text{ N}} \quad (\text{down, at center of bottom}) \end{aligned}$$

$$\begin{aligned} \text{Force on front \& back} &= P_{av} A_f = \frac{\rho g h}{2} A_f \\ &= \frac{1000(9.81)(0.4)}{2} (0.7)(0.4) = \underline{549 \text{ N}} \quad (\text{outward, at } \frac{2}{3} \text{ depth}) \end{aligned}$$

$$\begin{aligned} \text{Force on each end glass} &= P_{av} A_e = \frac{\rho g h}{2} A_e \\ &= \frac{1000(9.81)(0.4)}{2} (0.3)(0.4) = \underline{235 \text{ N}} \quad (\text{outward, at } \frac{2}{3} \text{ depth}) \\ &(\text{All side forces centered horizontally}) \end{aligned}$$

$$\frac{5/165}{W = mg = \rho_1 V g = \rho_1 abc g}$$



$$B = \rho_2 V_{\text{sub}} g = \rho_2 abhg$$

$$+\uparrow \Sigma F = 0: \rho_2 abhg - \rho_1 abcg = 0, \quad h = \frac{\rho_1}{\rho_2} c$$

$$r = \frac{h}{c} = \frac{\rho_1}{\rho_2}$$

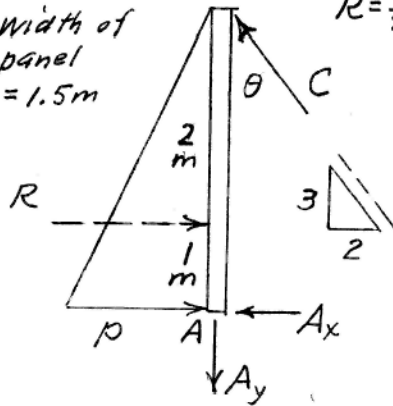
$$\text{Oak in water: } r = \frac{800}{1000} = \underline{0.8}$$

$$\text{Steel in mercury: } r = \frac{7830}{13570} = \underline{0.577}$$

5/166

$$p = \rho g h = 2400(9.81)(3) = 70.6 \text{ kPa}$$

width of  
panel  
= 1.5m



$$R = \frac{1}{2} p A = \frac{1}{2} (70.6 \times 10^3) (3 \times 1.5) \\ = 158.9 (10^3) \text{ N}$$

$$\sum M_A = 0;$$

$$(C \sin 33.7^\circ) 3 - 158.9 (10^3) (1) = 0$$

$$C = \frac{158.9 (10^3)}{3 \sin 33.7^\circ} = 95.5 (10^3) \text{ N}$$

$$\text{or } C = 95.5 \text{ kN}$$

$$\theta = \tan^{-1} 2/3 = 33.7^\circ$$



5/167

$$F = pA = \rho g h A = \rho g h \pi r^2$$

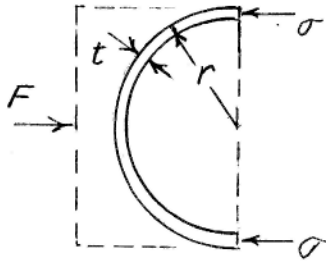
$$\Sigma F = 0; \rho g h \pi r^2 - \sigma (\pi r^2 - \pi [r-t]^2)$$

$$\sigma = \frac{\rho g h r}{2t} \frac{1}{1 - t/2r}$$

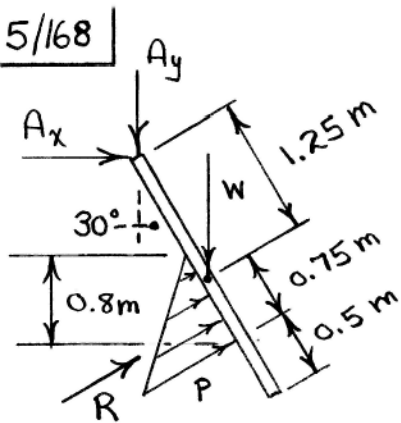
$$= \frac{1.03(10^3)(9.81) \frac{1.500}{2} 3(10^3)}{2(0.025) \left(1 - \frac{25}{1500}\right)}$$

$$= 454.7(10^6)(1.0169)$$

$$\text{or } \sigma = \underline{463 \text{ MPa}}$$



5/168



$$p = \rho g h = 1000(9.81)(0.8)$$

$$= 7850 \text{ Pa}$$

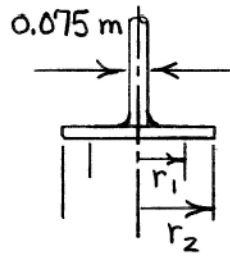
$$R = \frac{1}{2} (7850) \left( \frac{0.8}{\cos 30^\circ} \right)$$

$$= 3620 \text{ N/m}$$

$$\sum M_A = 0 : W(1.25 \sin 30^\circ) - 3620 \left( 2.5 - 0.5 - \frac{1}{3} \frac{0.8}{\cos 30^\circ} \right)$$

$$= 0 ; \quad \underline{W = 9810 \text{ N/m}}$$

5/169



$$\begin{cases} r_1 = 0.2 \text{ m} \\ r_2 = 0.3 \text{ m} \end{cases}$$

$$p = \rho g h = 1030 (9.81) (0.6) = 6060 \text{ Pa}$$

$$R = pA = 6060 \pi \left(0.3^2 - \frac{0.075^2}{4}\right) = 1687 \text{ N}$$

Pressure supported by seal

$$\sigma = \frac{R}{\pi(r_2^2 - r_1^2)} = \frac{1687}{\pi(0.3^2 - 0.2^2)}$$

$$= 10740 \text{ Pa or } \underline{10.74 \text{ kPa}}$$

Force to lift plunger

$$P = R = 1687 \text{ N or } \underline{1.687 \text{ kN}}$$

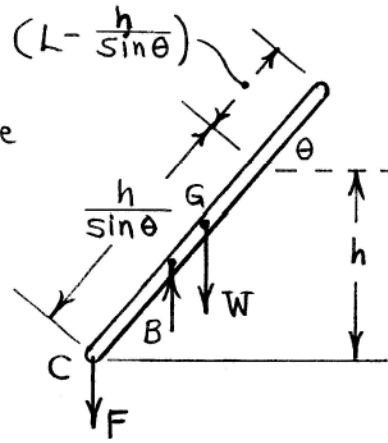
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Let  $A$  = cross-sectional area of the pole

Buoyancy force  $B$  is

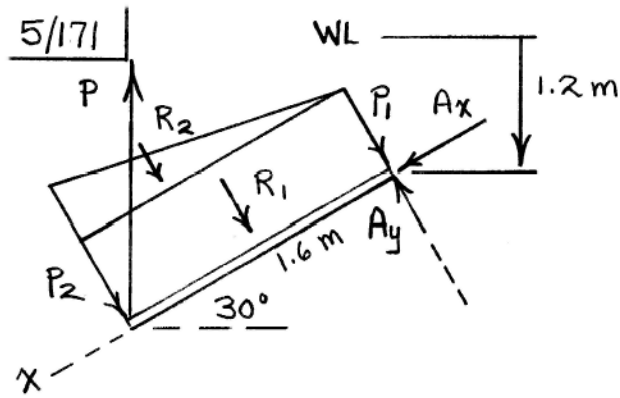
$$B = \rho_g \bar{V} = \rho_g \frac{hA}{\sin \theta}$$

Weight  $W = \rho'_g LA$



$$\sum M_C = 0: \rho_g \frac{hA}{\sin \theta} \left( \frac{1}{2} \frac{h}{\sin \theta} \cos \theta \right) - \rho'_g LA \left( \frac{L}{2} \cos \theta \right) = 0$$

$$\theta = \sin^{-1} \left( \frac{h}{L} \sqrt{\frac{\rho}{\rho'}} \right) \quad \left( \frac{h^2 \rho}{L^2 \rho'} \leq 1 \right)$$



$$p_1 = \rho g h_1 = 1.000 (9.81) (1.2) = 11.77 \text{ kPa}$$

$$p_2 = \rho g h_2 = 1.000 (9.81) (2) = 19.62 \text{ kPa}$$

$$R_1 = 11.77 (1.6)(0.8) = 15.07 \text{ kN}$$

$$R_2 = (19.62 - 11.77) \frac{1}{2} (1.6)(0.8) = 5.02 \text{ kN}$$

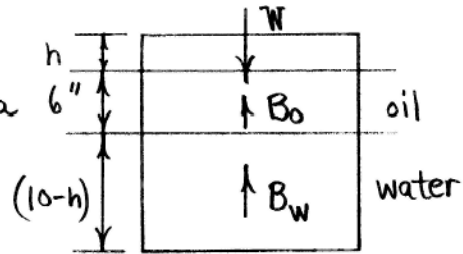
$$\sum M_A = 0 : P (1.6 \cos 30^\circ) - 15.07(0.8)$$

$$- 5.02 \left( \frac{2}{3} 1.6 \right) = 0, \quad \underline{P = 12.57 \text{ kN}}$$

5/172

Let  $A$  be the area of a horizontal slice of block

$$A = \frac{16^2}{144} \text{ ft}^2$$



$$\text{Weight of block } W = \frac{16}{12} A (50) = 66.7A \text{ (lb)}$$

$$\text{Buoyancy of oil } B_o = \frac{6}{12} A (56) = 28A \text{ (lb)}$$

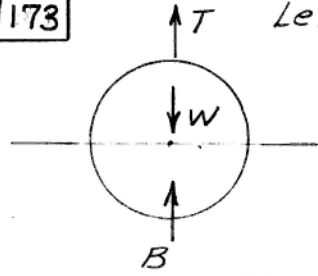
$$\text{Buoyancy of water } B_w = \frac{10-h}{12} A (64) = 5.33(10-h)A$$

(in lb,  $h$  in inches)

$$\Sigma F = 0: W - B_o - B_w = 0: 66.7A - 28A - 5.33(10-h)A = 0$$

$$\underline{h = 2.75 \text{ in.}}$$

5/173



Let  $\gamma_c$  = wt. density of concrete  
= 150 lb/ft<sup>3</sup>

$\gamma_w$  = wt. density of fresh  
water = 62.4 lb/ft<sup>3</sup>

$L$  = length of cylinder = 6 ft

$r$  = radius of cylinder = 2 ft

For equil.,  $T = W - B$

$$= \gamma_c \pi r^2 L - \gamma_w \frac{\pi r^2}{2} L$$

$$= \pi r^2 L \left( \gamma_c - \frac{1}{2} \gamma_w \right)$$

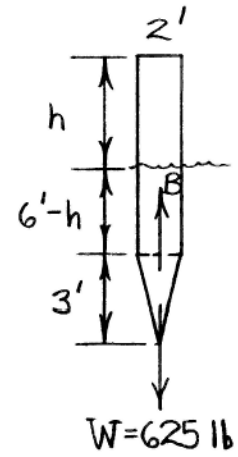
$$= \pi (2^2) (6) \left( 150 - \frac{62.4}{2} \right) = \underline{8960 \text{ lb}}$$

5/174

$$\begin{aligned} B = \mu V &= 64 \left[ \pi \frac{z^2}{4} (6-h) + \frac{1}{3} \pi \frac{z^2}{4} (3) \right] \\ &= 64 \pi \frac{z^2}{4} [6-h+1] \\ &= 64 \pi [7-h] \end{aligned}$$

$$W = B : 625 = 64 \pi [7-h]$$

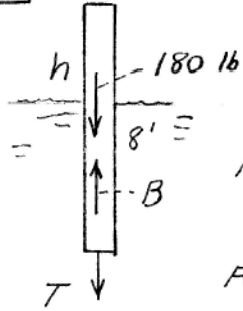
$$\underline{h = 3.89 \text{ ft}}$$





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→ | ← 1' Dia.



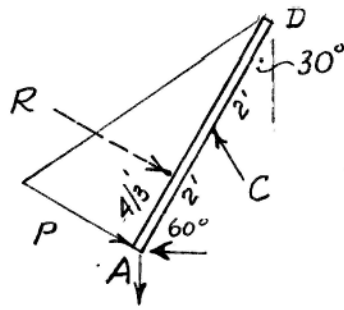
$$B = \rho g V = 64 \left( \frac{\pi \times 1^2}{4} [8-h] \right)$$
$$= 50.27 (8-h)$$

$$\Sigma F = 0; T + 180 - 50.27(8-h) = 0$$

$$\text{For } h = 2 \text{ ft, } T = -180 + 50.27(8-2)$$
$$= \underline{121.6 \text{ lb}}$$

$$\text{For } T = 0, \quad 8-h = 180/50.27$$
$$h = \underline{4.42 \text{ ft}}$$

5/176



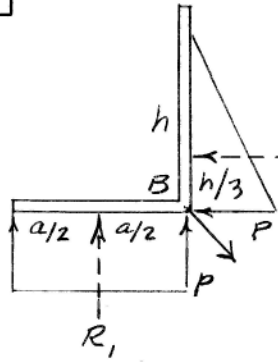
$$p = \rho g h = 62.4 (4 \cos 30^\circ) = 216 \text{ lb/ft}^2$$

$$R = \frac{p}{2} \text{ Area per panel}$$
$$= \frac{216}{2} (4)(2) = 865 \text{ lb}$$

$$\sum M_A = 0; 2C \cos 30^\circ - 865 (4/3) = 0$$

$$C = \frac{865(2)}{3 \cos 30^\circ} = \underline{666 \text{ lb}}$$

5/177



$$R_1 = \rho g h a b$$

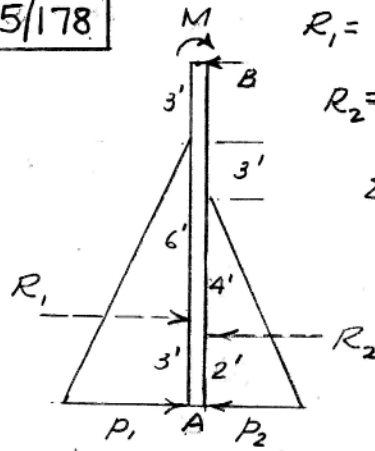
$$R_2 = \frac{\rho g h}{2} h b$$

$$\sum M_B = 0; R_1 \frac{a}{2} = R_2 \frac{h}{3}$$

$$\rho g h a b \frac{a}{2} = \frac{\rho g h}{2} h b \frac{h}{3}$$

$$h^2 = 3a^2, \quad \underline{h = a\sqrt{3}}$$

5/178



Horiz. length of gate is 10 ft

$$R_1 = \frac{p_1 A_1}{2} = \frac{62.4(9)(10)}{2} = 25,270 \text{ lb}$$

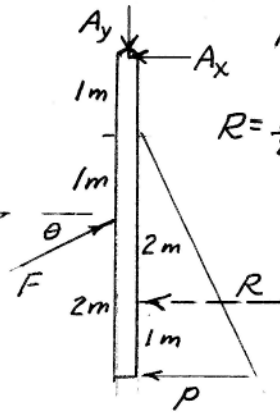
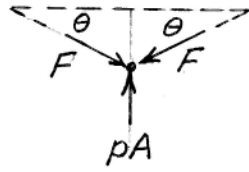
$$R_2 = \frac{p_2 A_2}{2} = \frac{64(6)(10)}{2} = 11,520 \text{ lb}$$

$$\sum M_B = 0; M + 11,520(10) - 25,270(9) = 0$$

$$M = 11.22(10^4) \text{ lb-ft}$$

5/1.79

$$\theta = \tan^{-1} \frac{0.5}{1} \\ = 26.6^\circ$$



$$p = \rho g h = 1.0(9.81)(3)$$

$$= 29.4 \text{ kPa}$$

$$R = \frac{p}{2} \text{ Area} = \frac{29.4}{2} (3)(2)$$

$$= 88.3 \text{ kN}$$

Gate:

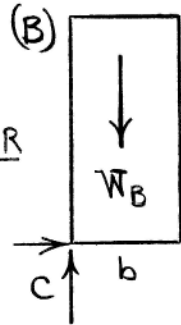
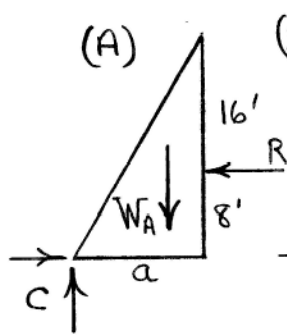
$$\sum M_A = 0; (F \cos 26.6^\circ) 2 - 88.3 (3) = 0, F = 148.1 \text{ kN}$$

Toggle:

$$\sum F = 0; pA - 2F \sin \theta = 0; \frac{\pi(0.150)^2}{4} p = 2(148.1)(10^3) \sin 26.6^\circ$$

$$p = 7.49(10^6) \text{ Pa or } p = \underline{7.49 \text{ MPa}}$$

5/180 | Per foot of length



$$W_A = \frac{1}{2} (24a) (150) \\ = 1800a \text{ (lb/ft)}$$

$$W_B = (24b) (150) \\ = 3600b \text{ (lb/ft)}$$

$$R = \frac{24}{2} (62.4) (24) = 17,970 \frac{\text{lb}}{\text{ft}}$$

$$(A) \sum M_c = 0 : 17,970 (8) - \frac{2}{3} a (1800a) = 0$$

$$a = 10.95 \text{ ft}, \quad W_A = 1800 (10.95) = 19,700 \frac{\text{lb}}{\text{ft}}$$

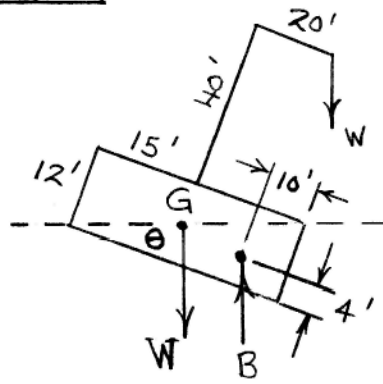
$$(B) \sum M_c = 0 : 17,970 (8) - \frac{b}{2} (3600b) = 0$$

$$b = 8.94 \text{ ft}, \quad W_B = 3600 (8.94) = 32,200 \frac{\text{lb}}{\text{ft}}$$

$$\text{So A requires } 32,200 - 19,700 = 12,470 \frac{\text{lb}}{\text{ft}}$$

less than B.

5/181



$$\theta = \tan^{-1} \frac{12}{30} = 21.8^\circ$$

Moment arm of B about G

$$(15-10) \cos \theta - (6-4) \sin \theta = 3.90 \text{ ft}$$

Moment arm of W about G

$$(40+6) \sin \theta + 20 \cos \theta = 35.7 \text{ ft}$$

$$B = \rho g V = 64(12)(15)(80) = 922,000 \text{ lb}$$

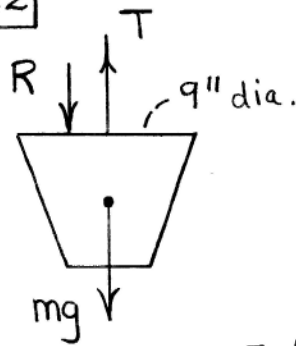
$$\sum M_G = 0 : 35.7 W - 3.90 (922,000) = 0$$

$$W = 100,800 \text{ lb}$$

$$W = B - w = 922,000 - 100,800 = 821,000 \text{ lb}$$

$$\text{or } W = \frac{821,000}{2240} = \underline{366 \text{ long tons}}$$

5/182



$$p = \rho g h = 62.4(20) = 1248 \frac{\text{lb}}{\text{ft}^2}$$

$$R = pA = 1248 \left[ \pi \left( \frac{4.5}{12} \right)^2 \right] \\ = 551 \text{ lb}$$

$$mg = \rho V g = \rho g \left[ \frac{7\pi r^2 h}{24} \right] \quad (\text{from Prob. 5/30})$$

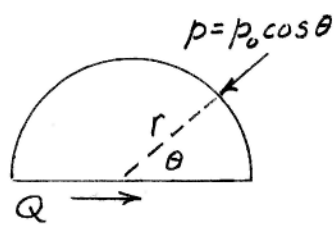
$$= 450 \left[ \frac{7\pi \left( \frac{4.5}{12} \right)^2 \left( \frac{12}{12} \right)}{24} \right] = 58.0 \text{ lb}$$

$$+\uparrow \Sigma F = 0 : T - 551 - 58.0 = 0, \quad \underline{T = 609 \text{ lb}}$$



5/183

$$Q = \int_0^{\pi} (p_0 \cos \theta) (\cos \theta) r d\theta$$



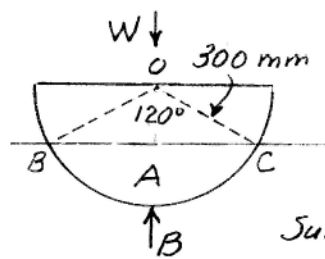
$$= p_0 r \int_0^{\pi} \cos^2 \theta d\theta$$

$$= p_0 r \left[ \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= p_0 r \left[ \frac{\pi}{2} \right]$$

$$\underline{Q = \frac{1}{2} \pi r p_0}$$

5/184 | Submerged area A of end = area of



120° sector minus area of triangle OBC

$$A = \frac{120}{360} \pi (0.3)^2 - (0.150)(0.3 \cos 30^\circ)$$

$$= 0.05528 \text{ m}^2$$

Submerged volume is

$$V = 0.05528 (0.600) = 33.17 (10^{-3}) \text{ m}^3$$

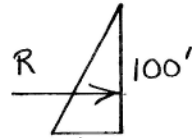
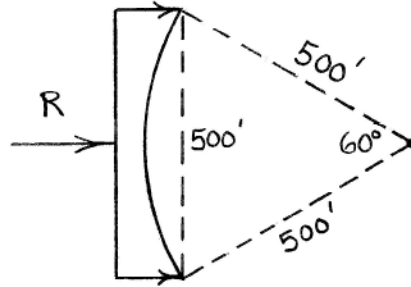
$$B = \rho g V = 1000 (9.81) (33.17) (10^{-3}) = 325.4 \text{ N}$$

$$W = (26.6 + m) 9.81 \text{ N}$$

For equilibrium  $B = W$ , so  $(26.6 + m) 9.81 = 325.4$

$$m = \underline{6.57 \text{ kg}}$$

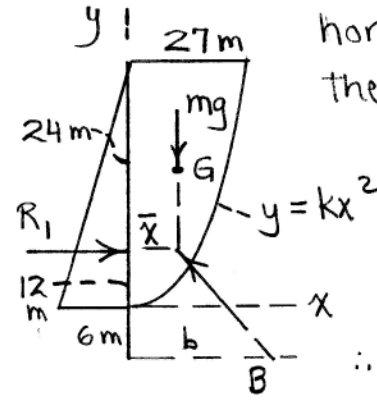
5/185



$$P_{100} = \mu h = 62.4(100) = 6240 \frac{\text{lb}}{\text{ft}^2}$$

$$R = P_{av} (\text{Area}) = \frac{6240}{2} (500)(100) = \underline{156.0(10^6) \text{ lb}}$$

5/186 | Take a vertical section of water of unit horizontal length. Let  $\rho$  be the water density in  $t/m^3$ .



$$y = kx^2: 36 = k(27)^2, k = \frac{4}{81} m^{-1}$$

$$\bar{x} = \frac{\int x dA}{\int dA}, dA = x dy = 2 \frac{4}{81} x^2 dx$$

$$\therefore \bar{x} = \frac{\int_0^{27} \frac{x}{2} \frac{8}{81} x^2 dx}{\int_0^{27} \frac{8}{81} x^2 dx} = 10.12 m$$

$$A = \int dA = 648 m^2, mg = 648 \rho g$$

$$R_1 = \frac{1}{2} 36 \rho g (36)(1) = 648 \rho g$$

Resultant of  $mg$  &  $R_1$  passes through B, so

$$\sum M_B = 0. \text{ Thus } 648 \rho g (18) = 648 \rho g (b - 10.12)$$

$$\underline{b = 28.1 m}$$



ارائه دهنده خدمات متنوع مهندسی

$$5/187 \quad B_1 = \rho_w g V_1 = 1.03(9.81) \frac{\pi(0.9)^2}{4} (3) \quad (3)$$

$$= 19.28 \text{ kN}$$

$$B_2 = \rho_w g V_2 = 1.03(9.81) \frac{\pi(1.8)^2}{4} 2.4$$

$$= 61.7 \text{ kN}$$

$V_3 =$  volume of ballast

$$B_3 = \rho_w g V_3 = \rho_w g \frac{m}{\rho_L}$$

$$= 1.03(9.81) \frac{m}{11.37} = 0.889m$$

$$F = 0.15 (B_1 + B_2 + B_3) = 0.15 (19.28 + 61.7 + 0.889m)$$

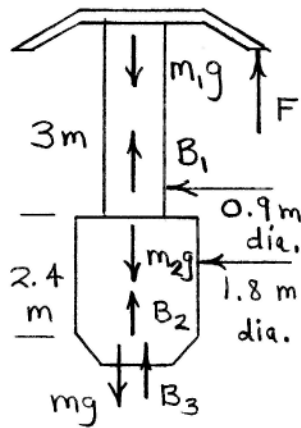
$$= 12.15 + 0.1333m$$

$$(m_1 + m_2)g = 5.7(9.81) = 55.9 \text{ kN}$$

$$\Sigma F = 0: F + B_1 + B_2 + B_3 - (m_1 + m_2)g - mg = 0$$

$$12.15 + 0.1333m + 19.28 + 61.7 + 0.889m - 55.9 - 9.81m = 0$$

$$\underline{m = 4.24 \text{ Mg}}$$



5/188 | The gage pressure 12 m below the surface

$$p = \rho gh = (1000)(9.81)(12) = 117\,700 \text{ N/m}^2$$

$$(a) \text{ Cover area } A_{\text{cov}} = \pi \left(\frac{0.75}{2}\right)\left(\frac{0.5}{2}\right) = 0.295 \text{ m}^2$$

$$\text{Force on cover} = p A_{\text{cov}} = 34\,700 \text{ N}$$

$$\text{Seal area } A_s = A_{\text{cov}} - \pi \left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) = 0.1325 \text{ m}^2$$

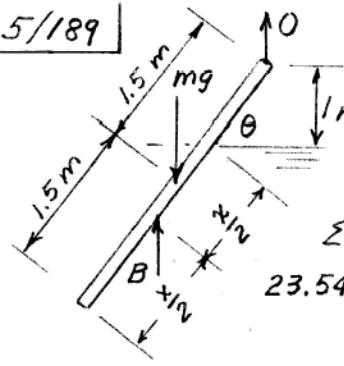
$$\sigma A_s = p A_{\text{cov}}, \quad \sigma = \frac{34\,700}{0.1325} = 262\,000 \frac{\text{N}}{\text{m}^2}$$

$$\text{or } \underline{\sigma = 262 \text{ kPa}}$$

$$(b) 16 \Delta T = p A_{\text{hole}} = 117\,700 \left[ \pi \left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) \right]$$

$$\underline{\Delta T = 1192 \text{ N}}$$

5/189

Let  $A$  = cross-sectional area of plank

$$mg = 800(3)A(9.81) = 23.54(10^3)A \text{ N}$$

$$B = \rho_w g A x$$

$$= 1000(9.81)A \left(3 - \frac{1}{\sin \theta}\right)$$

$$\sum M_O = 0;$$

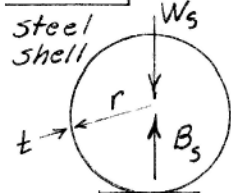
$$23.54(10^3)A(1.5 \cos \theta) - 9.81(10^3)A \left(3 - \frac{1}{\sin \theta}\right) \cdot \left[\frac{1}{2} \left(3 - \frac{1}{\sin \theta}\right) + \frac{1}{\sin \theta}\right] \cos \theta = 0$$

$$\left[\frac{1}{2} \left(3 - \frac{1}{\sin \theta}\right) + \frac{1}{\sin \theta}\right] \cos \theta = 0$$

Simplify & get  $23.54(1.5) = \frac{9.81}{2} \left(9 - \frac{1}{\sin^2 \theta}\right)$

&  $\sin^2 \theta = 0.5556$ ,  $\sin \theta = 0.7454$ ,  $\theta = 48.2^\circ$

5/190



Lead  
Ballast

For equilibrium  $W_s + W_L = B_s + B_L$

$\rho_s$  = density of steel =  $7.83 \text{ Mg/m}^3$

$\rho_w$  = " " salt water =  $1.03 \text{ Mg/m}^3$

$\rho_L$  = " " lead =  $11.37 \text{ Mg/m}^3$

$r = 1.00 \text{ m}$ ,  $t = 0.035 \text{ m}$

$V_L$  = volume of lead,  $\text{m}^3$

$m$  = mass of lead =  $\rho_L V_L$

$$\text{so } \rho_s g 4\pi r^2 t + \rho_L g V_L = \rho_w g \frac{4}{3}\pi \left(r + \frac{t}{2}\right)^3 + \rho_w g V_L$$

$$V_L g (\rho_L - \rho_w) = 4\pi r^2 g \left[ \frac{r}{3} \left(1 + \frac{t}{2r}\right)^3 \rho_w - \rho_s t \right]$$

$$m \left(1 - \frac{\rho_w}{\rho_L}\right) = 4\pi r^2 \left[ \frac{r}{3} \left(1 + \frac{t}{2r}\right)^3 \rho_w - \rho_s t \right]$$

$$m \left(1 - \frac{1.03}{11.37}\right) = 4\pi (1)^2 \left[ \frac{1}{3} \left(1 + \frac{0.035}{2}\right)^3 1.03 - 7.83(0.035) \right]$$

$$0.9094m = 1.1008, \quad m = \underline{1.210 \text{ Mg}} \text{ (metric tons)}$$



5/191 | The pressure at the bottom of the 3-m wall is  $p = \rho gh = 2400(9.81)(3) = 70\,600 \text{ N/m}^2$

Each tie controls an area  $A$  given by

$$pA = T, \quad A = \frac{T}{p} = \frac{6500}{70\,600} = 0.0920 \text{ m}^2$$

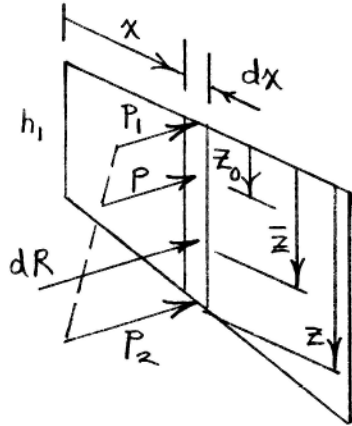
This square area has a side  $d$  given by

$$d^2 = A, \quad d = 0.303 \text{ m}$$

Using the pressure at the very bottom of the wall gives us a conservative design; a good figure for  $d$  would be  $d = 0.300 \text{ m}$ .

5/192

Method I: Direct integration



$\rho = \text{water density}, p = \rho g (b + z_0)$

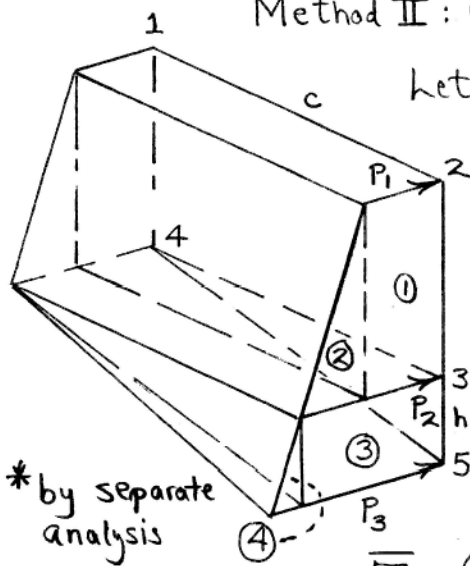
$$z = h_1 + kx = f(x)$$

(1) Calculate  $dR$  for elemental area  $z dx$  & find  $\bar{z}$

(2) Integrate & get  $R = \int dR$

$$(3) R\bar{Z} = \int \bar{z} dR$$

Method II: Geometry of pressure-area volumes



Let  $A_a = 1-2-3-4$ ,  $A_b = 3-4-5$

$$R_1 = \rho_1 A_a, \bar{z}_1 = h_1/2$$

$$R_2 = \frac{1}{2} (\rho_2 - \rho_1) A_a, \bar{z}_2 = \frac{2h_1}{3}$$

$$R_3 = \rho_2 A_b, \bar{z}_3 = h_1 + \frac{2}{3}(h_2 - h_1)$$

$$R_4 = \frac{1}{2} (\rho_3 - \rho_2) (h_2 - h_1) \frac{c}{3}$$

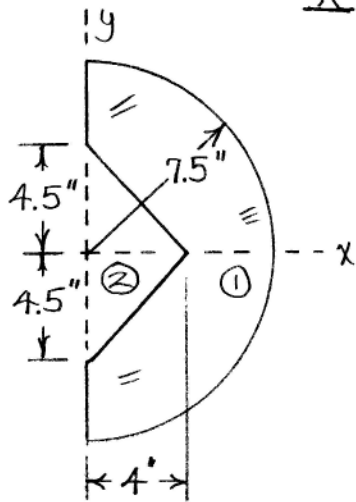
$$\bar{z}_4 = \frac{1}{2} (h_2 + h_1) *$$

$$R = R_1 + R_2 + R_3 + R_4$$

$$\bar{Z} = (R_1 \bar{z}_1 + R_2 \bar{z}_2 + R_3 \bar{z}_3 + R_4 \bar{z}_4) / R$$

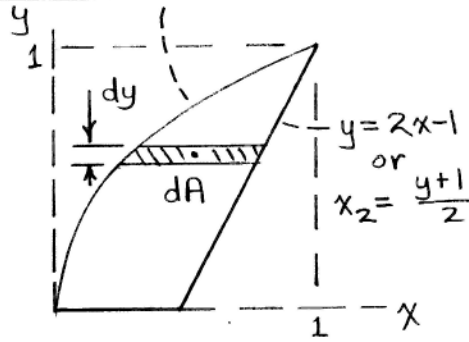
\* by separate analysis

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$$\begin{aligned} \bar{X} &= \frac{\sum A \bar{x}}{\sum A} \\ &= \frac{\pi \frac{7.5^2}{2} \left( \frac{4(7.5)}{3\pi} \right) - \frac{1}{2} (9)(4) \left( \frac{4}{3} \right)}{\pi \frac{7.5^2}{2} - \frac{1}{2} (9)(4)} \\ &= \underline{3.66 \text{ in.}} \end{aligned}$$

$$\frac{5}{194} \quad y = x^{1/3} \text{ or } x_1 = y^3$$



$$dA = (x_2 - x_1) dy$$

$$= \left( \frac{y}{2} + \frac{1}{2} - y^3 \right) dy$$

$$A = \int dA = \int_0^1 \left( \frac{y}{2} + \frac{1}{2} - y^3 \right) dy = \left( \frac{y^2}{4} + \frac{y}{2} - \frac{y^4}{4} \right) \Big|_0^1 = \frac{1}{2}$$

$$\int y_c dA = \int_0^1 \left( \frac{y^2}{2} + \frac{y}{2} - y^4 \right) dy = \left( \frac{y^3}{6} + \frac{y^2}{4} - \frac{y^5}{5} \right) \Big|_0^1 = \frac{13}{60}$$

$$\int x_c dA = \int_0^1 \left( \frac{x_1 + x_2}{2} \right) (x_2 - x_1) dy = \frac{1}{2} \int_0^1 (x_2^2 - x_1^2) dy$$

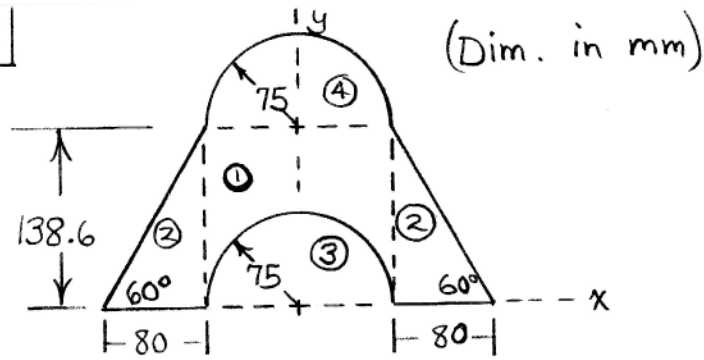
$$= \frac{1}{2} \int_0^1 \left( \frac{y^2}{4} + \frac{y}{2} + \frac{1}{4} - y^6 \right) dy = \frac{1}{2} \left( \frac{y^3}{12} + \frac{y^2}{4} + \frac{y}{4} - \frac{y^7}{7} \right) \Big|_0^1$$

$$= \frac{37}{168}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{13/60}{1/2} = \frac{13}{30}$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{37/168}{1/2} = \frac{37}{84}$$

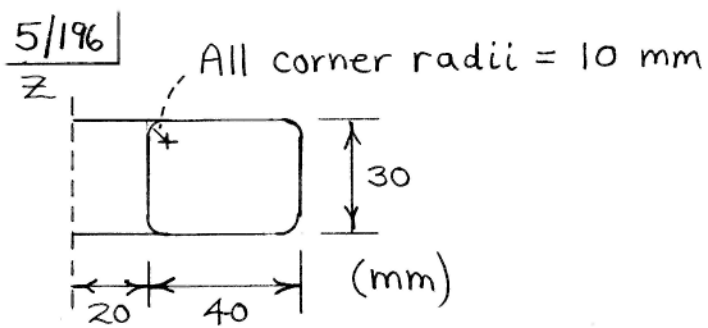
5/195



Comp.	$A$ ( $\text{mm}^2$ )	$\bar{y}$ (mm)	$A\bar{y}$ ( $\text{mm}^3$ )
①	$150(138.6)$	$138.6/2$	1 440 000
②	$2 \frac{1}{2}(80)(138.6)$	$\frac{1}{3}(138.6)$	512 000
③	$-\pi \frac{75^2}{2}$	$\frac{4(75)}{3\pi}$	- 281 250
④	$\pi \frac{75^2}{2}$	$(138.6 + \frac{4(75)}{3\pi})$	1 505 565
	$\Sigma A = 31 870$		$\Sigma A\bar{y} = 3.18(10^6)$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{99.7 \text{ mm}}$$

5/196  
z



$$V = 2\pi \bar{r} A = 2\pi \left(20 + \frac{40}{2}\right) \left[40(30) - 4(10^2) + \pi(10^2)\right] = \underline{280\,000 \text{ mm}^3}$$

$$A = 2\pi \bar{r} L = 2\pi \left(20 + \frac{40}{2}\right) \left[2(20) + 2(10) + 2\pi(10)\right] = \underline{30\,900 \text{ mm}^2}$$

5/197 |

For circular arc,  $\bar{y} = \frac{b \sin 30^\circ}{\pi/6} = 3b/\pi$

$\bar{z} = b$  (See Samp. Prob. 5/1)

$$\bar{Y} = \frac{\sum \bar{y} L}{\sum L} = \frac{b(0) + 2b\left(\frac{b}{2} \cos 30^\circ\right) + \frac{\pi}{3} b \left(\frac{3b}{\pi}\right)}{b + 2b + \frac{\pi}{3} b}$$

$$= \frac{0.461 b}{1}$$

$$\bar{Z} = \frac{\sum \bar{z} L}{\sum L} = \frac{b\left(\frac{b}{2}\right) + 2b(b) + \frac{\pi}{3} b(b)}{b + 2b + \frac{\pi}{3} b}$$

$$= \frac{0.876 b}{1}$$

5/198 | Same radial distribution as for semi-circular area, so  $\bar{z} = \frac{4r}{3\pi}$ , or by integration,

$dA = \pi z dL = \pi \frac{r}{h} x \frac{\sqrt{r^2 + h^2}}{h} dx$   
 $\int z_c dA = \int \frac{2z}{\pi} dA$   
 $= \int_0^h \frac{2r^2}{h^3} \sqrt{r^2 + h^2} x^2 dx$   
 $= \frac{2}{3} r^2 \sqrt{r^2 + h^2}$

$z_c = \frac{2z}{\pi}$  from Sample Prob. 5/1

$A = \int dA = \frac{1}{2} \pi r \sqrt{r^2 + h^2}$  so  $\bar{z} = \frac{\int z_c dA}{A} = \frac{4r}{3\pi}$

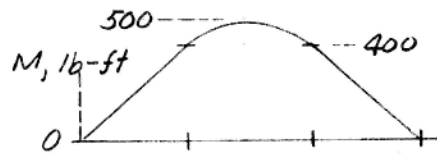
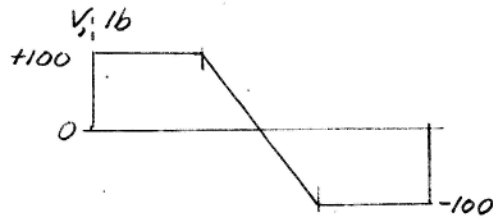
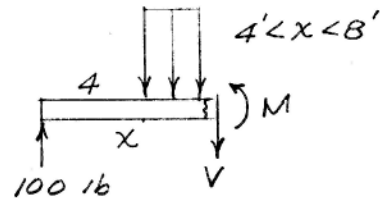
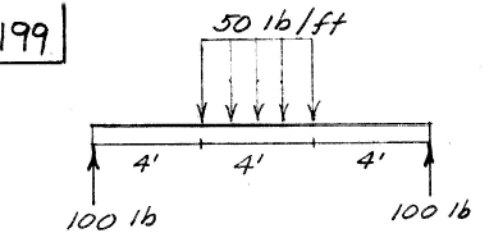
$\int x dA = \int_0^h \frac{\pi r \sqrt{r^2 + h^2}}{h^2} x^2 dx = \frac{\pi r \sqrt{r^2 + h^2}}{3} h$

$\bar{x} = \frac{\int x dA}{A} = \frac{2}{3} h$

This result can also be seen by inspection since the area is composed of elemental triangular areas each of which has the same  $x$ -centroidal coordinate of  $2h/3$ .



5/199



$$\sum F = 0; V + 50(x-4) - 100 = 0$$

$$V = 50(6-x)$$

$$\sum M_v = 0; M + 50 \frac{(x-4)^2}{2} - 100x = 0$$

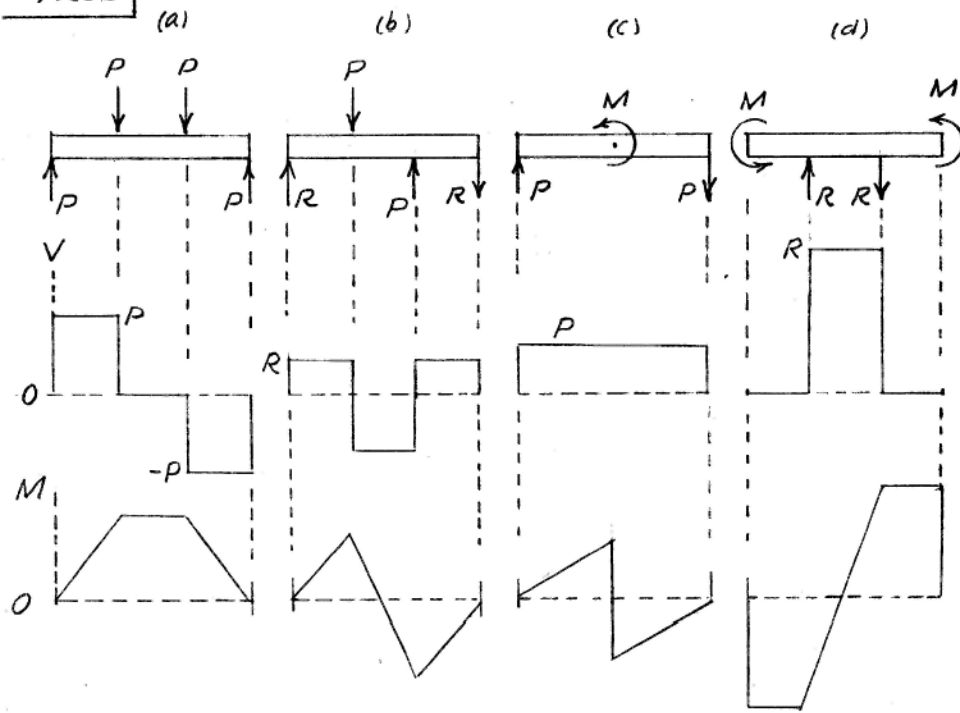
$$-100x = 0$$

$$M = 25(-x^2 + 12x - 16)$$

Set  $\frac{dM}{dx} = 0$  to get

$$M_{\max} = 500 \text{ lb-ft}$$

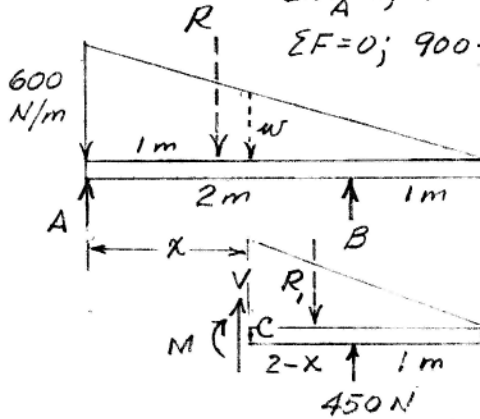
$$\text{@ } x = 6 \text{ ft}$$



$$\frac{5}{201} \quad R = \frac{600}{2}(3) = 900 \text{ N};$$

$$\sum M_A = 0; 900(1) - 2B = 0, B = 450 \text{ N}$$

$$\sum F = 0; 900 - 450 - A = 0, A = 450 \text{ N}$$



$$R_1 = \frac{w}{2}(2-x+1) = \frac{3-x}{2}w$$

$$\text{where } w = \frac{3-x}{3} 600 \text{ N/m}$$

$$\text{So } R_1 = 100(3-x)^2 \text{ N}$$

$$\sum M_C = 0;$$

$$450(2-x) = 100(3-x)^2 \frac{3-x}{3} + M$$

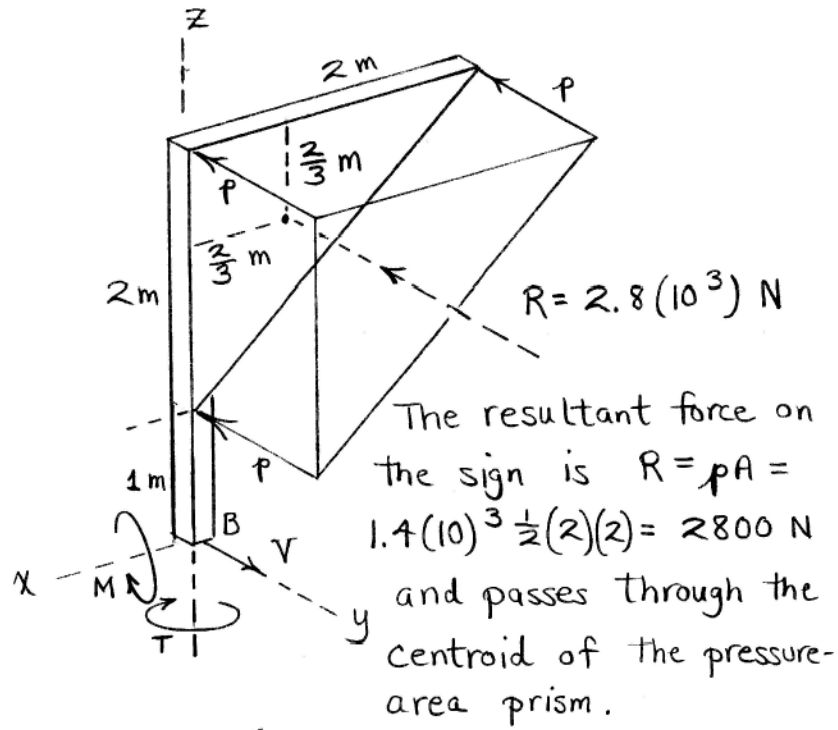
$$M = 450(2-x) - \frac{100}{3}(3-x)^3 \quad 0 \leq x \leq 2 \text{ m}$$

$$\frac{dM}{dx} = -450 + 100(3-x)^2 = 0 \text{ for max or min}$$

$$3-x = \pm \sqrt{4.5}, \quad x = 3(1 \pm \frac{1}{\sqrt{2}}) = \frac{0.879 \text{ m}}{\text{(or } 5.12)}$$

$$M_{\max} = 450(2-0.879) - \frac{100}{3}(3-0.879)^3 = \underline{186.4 \text{ N}\cdot\text{m}}$$

5/202



By inspection ,

$$\begin{cases} V = 2.8 \text{ kN} \\ M = 2.8 \left(3 - \frac{2}{3}\right) = 6.53 \text{ kN}\cdot\text{m} \\ T = 2.8 \left(\frac{2}{3}\right) = 1.867 \text{ kN}\cdot\text{m} \end{cases}$$

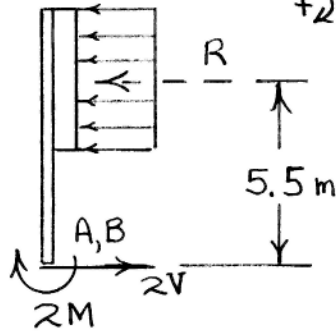
5/203

$$R = pA = 1.4(10^3)(6)(4) = 33.6(10^3) \text{ N}$$

$$\sum M_{AB} = 0 : 2M - 33.6(10^3)(5.5) = 0$$

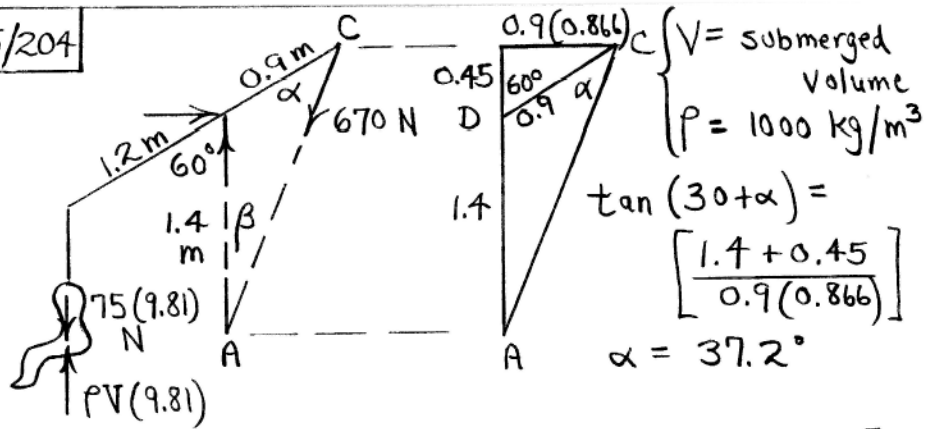
$$M = 92.4(10^3) \text{ N}\cdot\text{m}$$

$$\text{or } \underline{M = 92.4 \text{ kN}\cdot\text{m}}$$



(also forces at  
A & B)

5/204



$$\sum M_D = 0 : 670(0.9) \sin 37.2^\circ - [75(9.81) - \rho V(9.81)] \times 1.2 \sin 60^\circ = 0$$

$$\rho V = 39.3 \text{ kg}$$

$$V = \frac{39.3}{\rho} = \frac{39.3}{1000} = \underline{\underline{0.0393 \text{ m}^3}}$$

$$\begin{aligned}
 \underline{5/205} \quad S &= 2 \int_0^{L/2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2 \int_0^{L/2} \sqrt{1 + \left(\frac{wx}{T_0}\right)^2} dx \\
 &= 2 \left(\frac{1}{2}\right) \frac{T_0}{w} \left[ \frac{wx}{T_0} \sqrt{1 + \left(\frac{wx}{T_0}\right)^2} + \ln \left( \frac{wx}{T_0} + \sqrt{1 + \left(\frac{wx}{T_0}\right)^2} \right) \right]_0^{L/2} \\
 &= \frac{L}{2} \sqrt{1 + \left(\frac{wL}{2T_0}\right)^2} + \frac{T_0}{w} \ln \left( \frac{wL}{2T_0} + \sqrt{1 + \left(\frac{wL}{2T_0}\right)^2} \right)
 \end{aligned}$$

From Eq. 5/14  $\frac{w}{2T_0} = \frac{y}{x^2} = \frac{50}{(150)^2} = 0.02$ ,  $\frac{wL}{2T_0} = 2$

$$\begin{aligned}
 \text{So } S &= \frac{100}{2} \sqrt{1 + 2^2} + \frac{100}{4} \ln(2 + \sqrt{1 + 2^2}) \\
 &= 111.8 + 36.1 = \underline{147.9 \text{ ft}}
 \end{aligned}$$

$$\underline{5/206} \quad \text{Hole: } V = -9h \text{ (in.}^3\text{)}, \quad \bar{z} = h/2$$

$$\text{Cylinder: } V = \pi 6^2(10) = 1131 \text{ in.}^3, \quad \bar{z} = 5 \text{ in.}$$

$$\bar{z} = \frac{\sum \bar{z}V}{\sum V} = \frac{-9h\left(\frac{h}{2}\right) + 1131(5)}{-9h + 1131}$$

$$\text{For max. } \bar{z}, \quad \frac{d\bar{z}}{dh} = 0$$

$$\frac{d\bar{z}}{dh} = \frac{(-9h+1131)(-9h) - \left(-\frac{9h^2}{2} + 5655\right)(-9)}{(-9h+1131)^2} = 0$$

$$\Rightarrow 9h^2 - 1131h - 4.5h^2 + 5655 = 0$$

$$4.5h^2 - 1131h + 5655 = 0$$

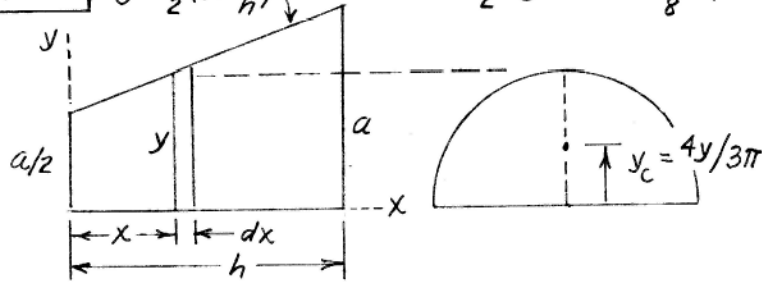
$$h = \frac{1131 \pm \sqrt{1131^2 - 4(4.5)(5655)}}{9} = \underline{5.10 \text{ in. or } 246 \text{ in.}}$$



5/207

$$y = \frac{a}{2} \left(1 + \frac{x}{h}\right)$$

$$dV = \frac{1}{2} \pi y^2 dx = \frac{\pi a^2}{8} \left(1 + \frac{x}{h}\right)^2 dx$$



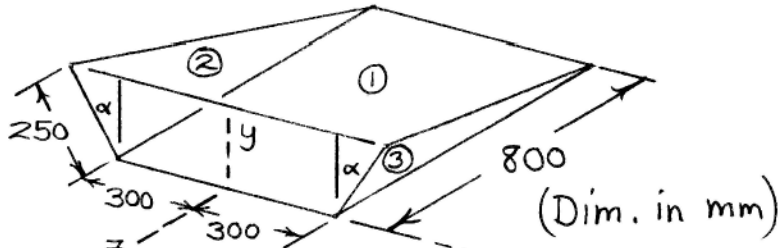
$$V = \frac{\pi a^2}{8} \int_0^h \left(1 + \frac{x}{h}\right)^2 dx = \frac{\pi a^2}{8} \left[ \frac{3}{3} \left(1 + \frac{x}{h}\right)^3 \right]_0^h = \frac{7\pi}{24} a^2 h$$

$$\begin{aligned} \int y_c dV &= \int_0^h \frac{4}{3\pi} \frac{a}{2} \left(1 + \frac{x}{h}\right) \cdot \frac{\pi a^2}{8} \left(1 + \frac{x}{h}\right)^2 dx = \frac{a^3}{12} \int_0^h \left(1 + \frac{x}{h}\right)^3 dx \\ &= \frac{a^3}{12} \left[ \frac{4}{4} \left(1 + \frac{x}{h}\right)^4 \right]_0^h = \frac{a^3}{48} 15h = \frac{5a^3 h}{16} \end{aligned}$$

$$\text{So } \bar{y} = \frac{\int y_c dV}{V} = \frac{5a^3 h}{16} \frac{24}{7\pi a^2 h} = \frac{15a}{14\pi}$$

5/208

( $\alpha = 30^\circ$ )



Comp.	(mm <sup>2</sup> ) A	(mm) $\bar{x}$	(mm) $\bar{y}$	(mm) $\bar{z}$	(mm <sup>3</sup> ) $A\bar{x}$	(mm <sup>3</sup> ) $A\bar{y}$	(mm <sup>3</sup> ) $A\bar{z}$
①	$600(800) = 4.8(10^5)$	0	0	-400	0	0	$-192(10^6)$
②	$\frac{250(800)}{2} = 10^5$	$-\frac{300}{3} = -342$	$\frac{250}{3} \csc 30^\circ = 72.2$	$-\frac{800}{3} = -267$	$-34.2(10^6)$	$7.22(10^6)$	$-26.7(10^6)$
③	$10^5$	342	72.2	-267	$+34.2(10^6)$	$7.22(10^6)$	$-26.7(10^6)$

$$\Sigma A = 6.8(10^5) \text{ mm}^2$$

$$\begin{cases} \Sigma A\bar{x} = 0 \\ \Sigma A\bar{y} = 14.43(10^6) \text{ mm}^3 \\ \Sigma A\bar{z} = -245(10^6) \text{ mm}^3 \end{cases}$$

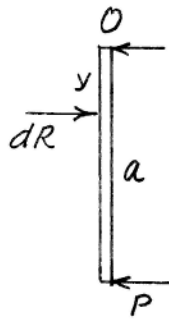
$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{14.43(10^6)}{6.8(10^5)} = 21.2 \text{ mm}$$

$$\bar{Z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{-245(10^6)}{6.8(10^5)} = -361 \text{ mm}$$

$$5/209 \quad dA = 2x dy = \frac{b}{a}(a-y) dy, \quad p = \rho g (h+y)$$

$$dR = \rho g (h+y) \frac{b}{a}(a-y) dy$$



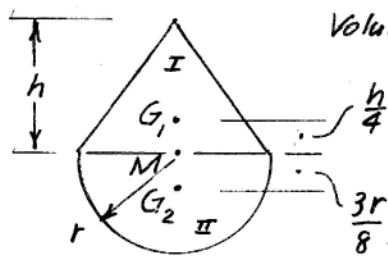
$$\sum M_O = 0; Pa - \int y dR = 0$$

$$Pa = \rho g \frac{b}{a} \int_0^a y(h+y)(a-y) dy$$

$$Pa = \rho g \frac{b}{a} \frac{a^3}{6} (h + \frac{a}{2})$$

$$P = \frac{\rho g a b}{6} (h + \frac{a}{2})$$

5/210 |  $M = \text{metacenter}$ , so combined center of mass cannot be above  $M$  for stability



Volume =  $V$ , density =  $\rho$

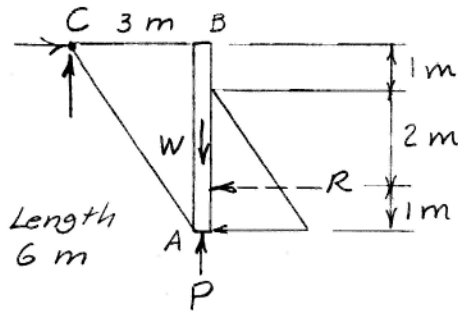
$$\rho V_I \frac{h}{4} = \rho V_{II} \frac{3r}{8}$$

$$\frac{1}{3} \pi r^2 h \frac{h}{4} = \frac{2}{3} \pi r^3 \frac{3r}{8}$$

$$h^2 = 3r^2, \quad \underline{h = r\sqrt{3}}$$

5/211 | Av. pressure =  $\frac{1}{2} \rho g h = \frac{1}{2} 1.0 (9.81) 3$

= 14.72 kN/m<sup>2</sup>



$R = pA = 14.72 (3)(6)$

= 264.9 kN

$W = 8.5(9.81) = 83.4$  kN

$\sum M_C = 0; 264.9(3) + 83.4(3) - P(3) = 0$

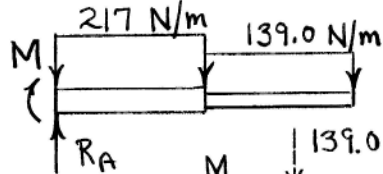
$P = 348$  kN

$$5/212 \quad \rho_{\text{Steel}} g = 7830 (9.81) = 76.8 (10^3) \text{ N/m}^3$$

$$w = \text{weight/meter} = \frac{\pi d^2}{4} (1) (76.8) 10^3 = 60.3 d^2 (10^3) \frac{\text{N}}{\text{m}}$$

$$\text{For } d = 60 (10^{-3}) \text{ m, } w = 217 \text{ N/m} \quad \left. \vphantom{\text{For}} \right\} \text{ (d in meters)}$$

$$\text{For } d = 48 (10^{-3}) \text{ m, } w = 139.0 \text{ N/m}$$



$$0.4 < x < 0.8 \text{ m}$$

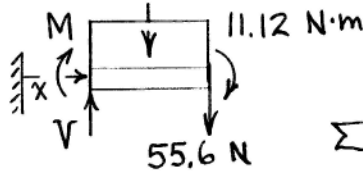
$$\sum F = 0: V = 139.0 (0.8 - x)$$

$$\sum M = 0: M + 139.0 (0.8 - x) \frac{0.8 - x}{2} = 0$$

$$M = -69.5x^2 + 111.2x - 44.5$$

$$217(0.4 - x)$$

$$\text{At } x = 0.4 \text{ m, } \begin{cases} M = -11.12 \text{ N}\cdot\text{m} \\ V = 55.6 \text{ N} \end{cases}$$



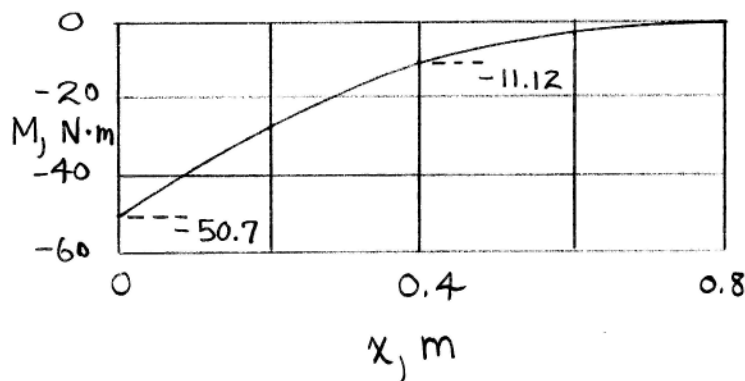
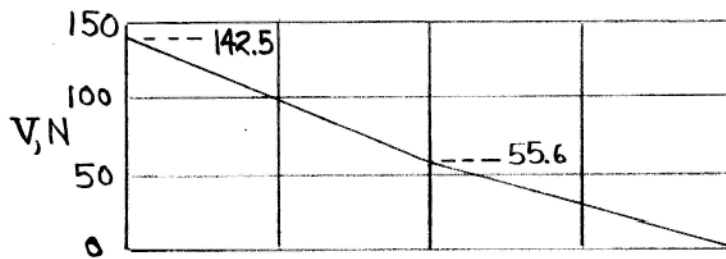
$$0 < x < 0.4 \text{ m}$$

$$\sum F = 0: V = 142.5 - 217x$$

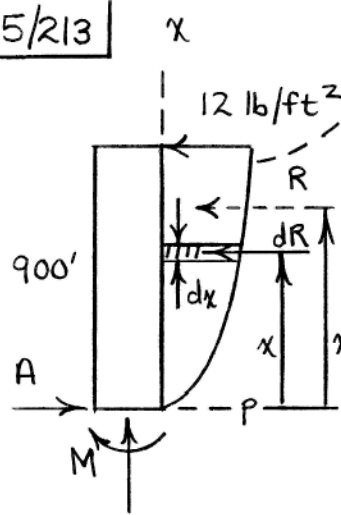
$$\sum M = 0: M + 11.12 + 55.6(0.4 - x) + 217 \frac{(0.4 - x)^2}{2} = 0$$

$$M = -108.6x^2 + 142.5x - 50.7$$

$$\text{At } x = 0, \begin{cases} M = -50.7 \text{ N}\cdot\text{m} \\ V = 142.5 \text{ N} \end{cases}$$



5/213



$$p = k\sqrt{x} : 12 = k\sqrt{900}$$

$$k = 0.4 \frac{\text{lb}}{\text{ft}^{5/2}}, \quad p = 0.4\sqrt{x}$$

$$dR = 200 p dx = 200(0.4\sqrt{x})$$

$$= 80\sqrt{x} dx$$

$$R = \int dR = \int_0^{900'} 80\sqrt{x} dx$$

$$= 80 \frac{2}{3} x^{3/2} \Big|_0^{900'} = 1.440(10^6) \text{ lb}$$

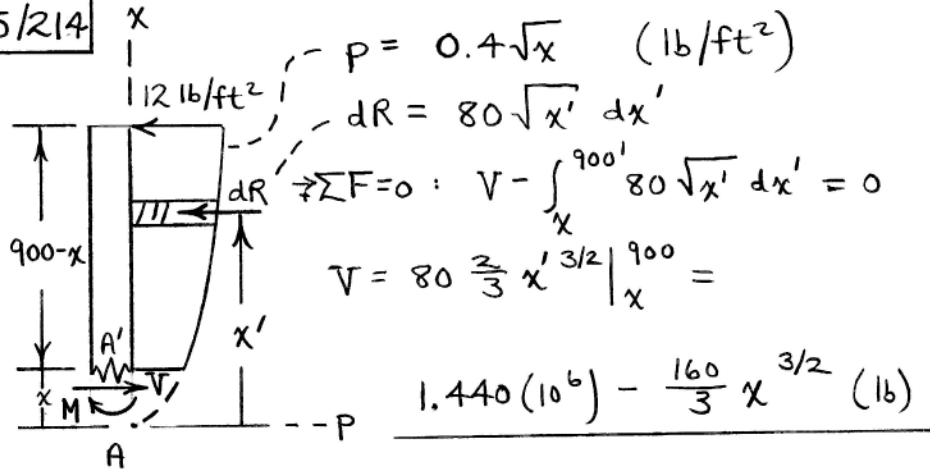
$$\int x dR = \int_0^{900} 80 x^{3/2} dx = 80 \frac{2}{5} x^{5/2} \Big|_0^{900'}$$

$$= 7.78(10^8) \text{ lb-ft}$$

Thus ,

$$\frac{A = 1.440(10^6) \text{ lb}}{M = 7.78(10^8) \text{ lb-ft}}$$

5/214



$$V \Big|_{x=450'} = 0.931(10^6) \text{ lb}$$

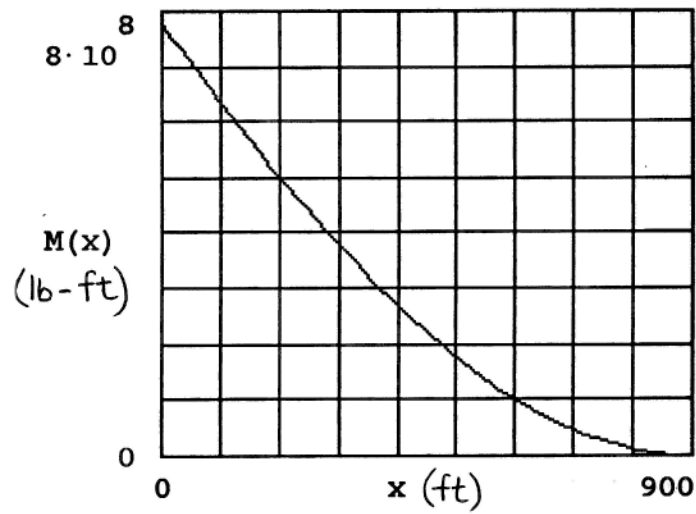
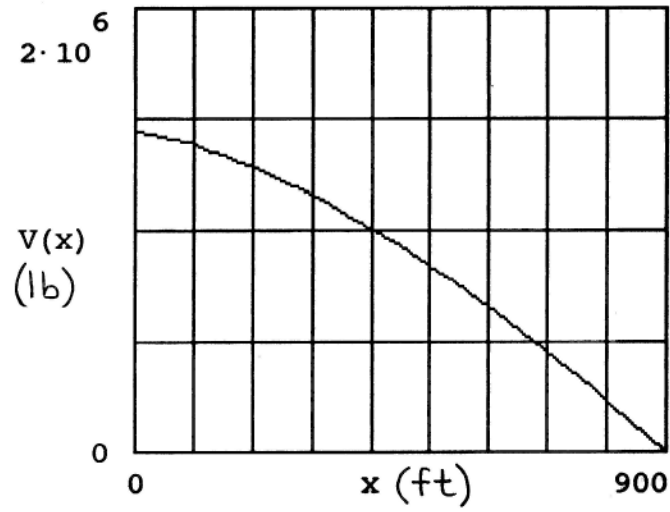
$$\sum M_{A'} = 0: M - \int_x^{900} 80\sqrt{x'} (x' - x) dx' = 0$$

$$M = 80 \left\{ \frac{2}{5} x'^{5/2} - x \frac{2}{3} x'^{3/2} \right\} \Big|_x^{900}$$

$$= 7.78(10^8) - 1.440(10^6)x + \frac{64}{3} x^{5/2}$$

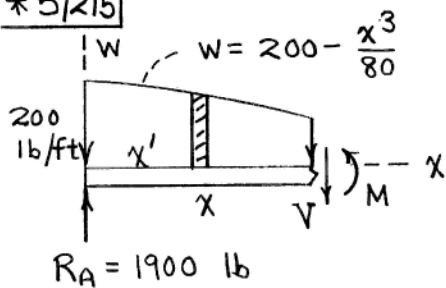
$$M \Big|_{x=450'} = 2.21(10^8) \text{ lb-ft}$$





\*5/215

( $w$  &  $R_A$  from Prob. 5/107)

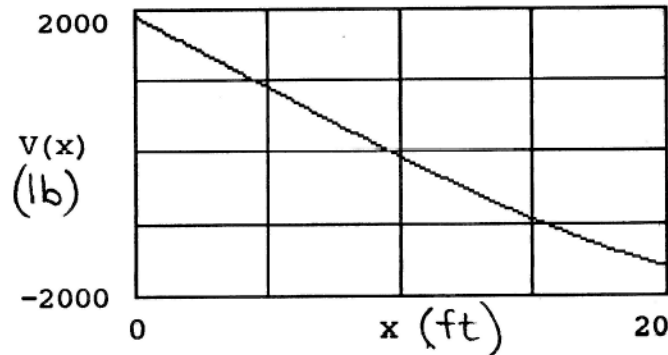


$$+\uparrow \Sigma F = 0 : 1900 - V - \int_0^x \left( 200 - \frac{x'^3}{80} \right) dx'$$

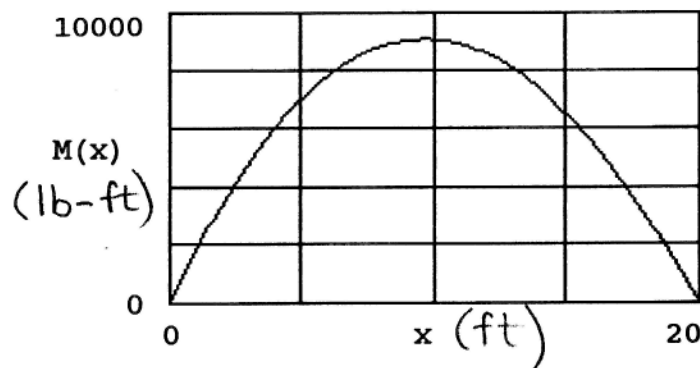
$$V = 1900 - 200x + \frac{x^4}{320} \quad (1b)$$

$$+\curvearrowright \Sigma M = 0 : M + \int_0^x \left( 200 - \frac{x'^3}{80} \right) (x - x') dx' - 1900x = 0$$

$$M = 1900x - 100x^2 + \frac{x^5}{1600} \quad (1b-ft)$$

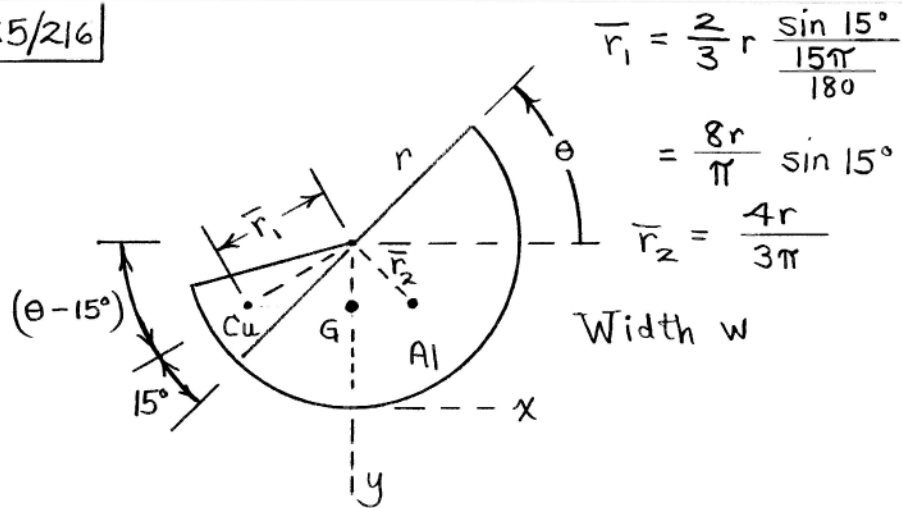


$$V_{max} = 1900 \text{ lb @ } x = 0$$



$$M_{max} = 9080 \text{ lb-ft @ } x = 9.63'$$

\*5/216



$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} = 0 \text{ for equilibrium}$$

$$\text{So } m_{Cu} |\bar{x}_1| = m_{Al} |\bar{x}_2|$$

$$\left(\frac{30}{360} \pi r^2 w \rho_{Cu}\right) \left(\frac{8r}{\pi} \sin 15^\circ\right) \cos(\theta - 15^\circ) = \left(\frac{\pi r^2}{2} w \rho_{Al}\right) \frac{4r}{3\pi} \sin \theta$$


$$\text{Reduces to } \frac{\rho_{Cu}}{\rho_{Al}} \sin 15^\circ \cos(\theta - 15^\circ) = \sin \theta$$

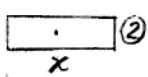
With  $\rho_{Cu} = 8910 \text{ kg/m}^3$  and  $\rho_{Al} = 2690 \text{ kg/m}^3$ ,  
a numerical solution yields  $\theta = 46.8^\circ$ .

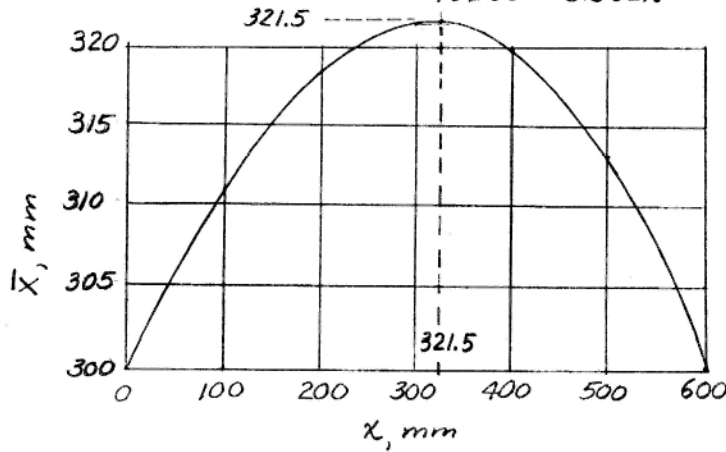
\*5/217

$$V_1 = \frac{\pi(180)^2}{4} 600 = 15268 (10^3) \text{ mm}^3$$

Dia. 600 mm  $\bar{x}_1 = 300 \text{ mm}$

180 mm   $V_2 = -\frac{\pi(90)^2}{4} x = -6.362(10^3)x$ ,  $\bar{x}_2 = -x/2$

90 mm   $\bar{x} = \frac{\sum V\bar{x}}{\sum V} = \frac{15268(300)10^3 - 3.181(10^3)x^2}{15268(10^3) - 6.362(10^3)x}$   
 $= \frac{4580(10^3) - 3.181x^2}{15268 - 6.362x}$

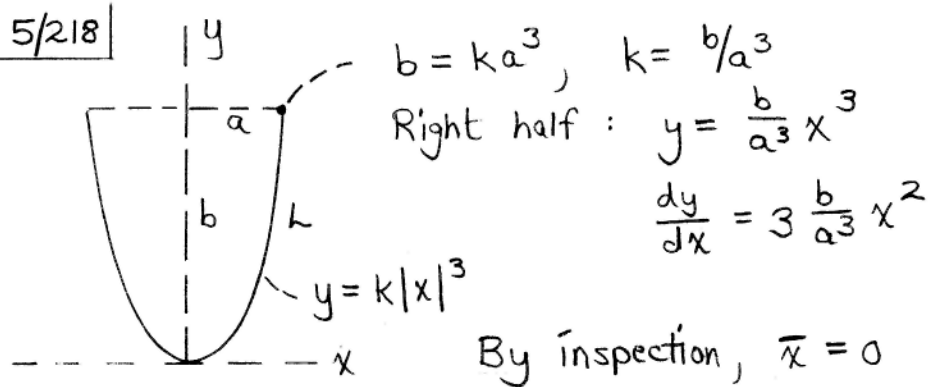


$$\bar{x}_{\max} = 322 \text{ mm}$$

at

$$x = 322 \text{ mm}$$

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$$L = \int dL = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_0^a \sqrt{1 + 9 \frac{b^2}{a^6} x^4} dx$$

$$\int y dL = \int_0^a \frac{b}{a^3} x^3 \sqrt{1 + 9 \frac{b^2}{a^6} x^4} dx$$

$$= \frac{b}{a^3} \int_0^a x^3 \sqrt{1 + 9 \frac{b^2}{a^6} x^4} dx$$

$$\bar{y} = \frac{\int y dL}{\int dL} = \frac{\frac{b}{a^3} \int_0^a x^3 \sqrt{1 + 9 \frac{b^2}{a^6} x^4} dx}{\int_0^a \sqrt{1 + 9 \frac{b^2}{a^6} x^4} dx}$$

(Units of length ✓; integrals not in most tables)

For the given values,

$$k = b/a^3 = 8/2^3 = 1, y = x^3, \text{ and}$$

$$\bar{y} = \frac{\int_0^2 x^3 \sqrt{1 + 9x^4} dx}{\int_0^2 \sqrt{1 + 9x^4} dx} = \frac{N}{D}$$

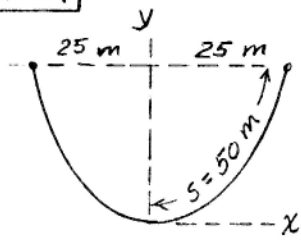
Use a numerical approach such as the rectangular, trapezoidal, or Simpson technique for area determination and find

$$N = 32.3, D = 8.63, \bar{y} = \frac{N}{D} = \underline{3.74}$$



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\*5/219



$$\text{Eq. 5/20 } S = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$$

$$50 = \frac{T_0}{\mu} \sinh \frac{25\mu}{T_0}$$

$$\frac{50\mu}{T_0} - \sinh \frac{25\mu}{T_0} = R = 0$$

Write and run program for  $R = f\left(\frac{\mu}{T_0}\right)$  & find  $\mu/T_0$  for  $R=0$ . Result is  $\mu/T_0 = 0.0871$

$$\text{From Eq. 5/19, } y = \frac{T_0}{\mu} \left( \cosh \frac{\mu x}{T_0} - 1 \right)$$

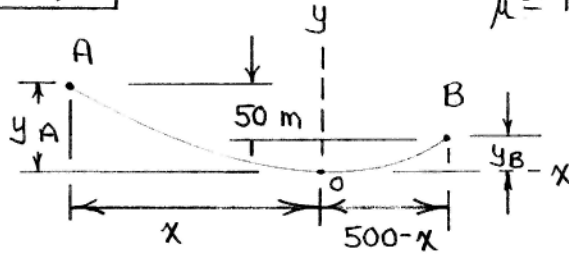
$$= \frac{1}{0.0871} \left( \cosh 0.0871[25] - 1 \right)$$

$$h = y = 3.468/0.0871 = \underline{39.8 \text{ m}}$$

Result depends only on the geometry of the catenary.

\* 5/220

$$\mu = 12(9.81) = 117.7 \text{ N/m}$$



From  $s = \frac{T_0}{\mu} \sinh \frac{\mu}{T_0} x$ :

$$s_{0A} = \frac{T_0}{\mu} \sinh \frac{\mu}{T_0} x \quad (1)$$

$$s_{0B} = \frac{T_0}{\mu} \sinh \left[ \frac{\mu}{T_0} (500-x) \right] \quad (2)$$

Also:  $s_{0A} + s_{0B} = 505 \quad (3)$

From  $y = \frac{T_0}{\mu} \left[ \cosh \frac{\mu}{T_0} x - 1 \right]$ :

$$y_A = \frac{T_0}{\mu} \left[ \cosh \frac{\mu}{T_0} x - 1 \right] \quad (4)$$

$$y_B = \frac{T_0}{\mu} \left[ \cosh \frac{\mu}{T_0} (500-x) - 1 \right] \quad (5)$$

Also:  $y_A - y_B = 50 \quad (6)$

Numerical Sol.  
of (1) - (6):

$$\frac{T_0}{\mu} = 1439 \text{ m}$$

$$s_A = 398 \text{ m}$$

$$s_B = 107.1 \text{ m}$$

$$x = 393 \text{ m}$$

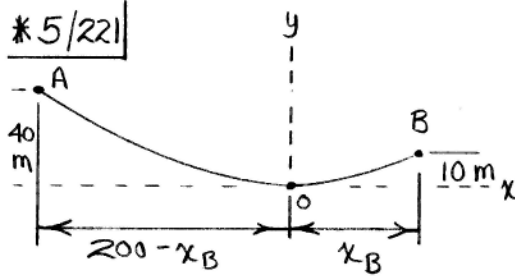
$$y_A = 54.0 \text{ m}$$

$$y_B = 3.98 \text{ m}$$

$$\frac{T_0}{\mu} = \frac{T_0}{117.7} = 1439 \Rightarrow T_0 = 169\,400 \text{ N}$$

$$T = T_0 + \mu y : \begin{cases} T_A = T_0 + 117.7(54) = 175\,800 \text{ N} \\ T_B = T_0 + 117.7(3.98) = 169\,900 \text{ N} \end{cases}$$

\*5/221



$$\mu_{\text{cable}} = 20(9.81) = 196.2 \frac{\text{N}}{\text{m}}$$

$$\text{Eq. 5/19: } y = \frac{T_0}{\mu} \left[ \cosh \frac{x}{T_0/\mu} - 1 \right]$$

$$\text{At B: } 10 = \frac{T_0}{\mu} \left[ \cosh \frac{x_B}{T_0/\mu} - 1 \right]$$

$$\text{At A: } 40 = \frac{T_0}{\mu} \left[ \cosh \frac{200 - x_B}{T_0/\mu} - 1 \right]$$

$$\text{Simultaneous numerical solution: } \begin{cases} x_B = 67.1 \text{ m} \\ T_0/\mu = 227 \text{ m} \end{cases}$$

$$T_A = T_0 + \mu y_A : 75\,000 = 227\mu + \mu(40)$$

$$\mu = 281 \text{ N/m}$$

$$\mu = \mu_{\text{cable}} + \mu_{\text{ice}} : 281 = 196.2 + \mu_{\text{ice}}$$

$$\mu_{\text{ice}} = 84.7 \text{ N/m}$$

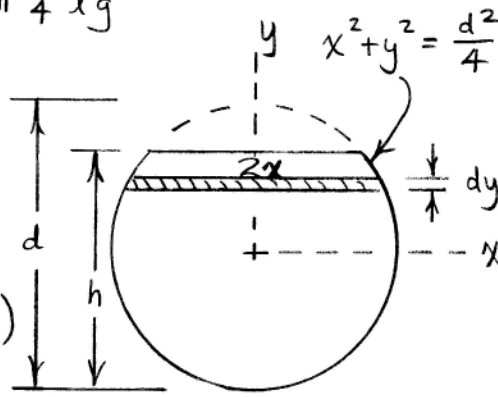
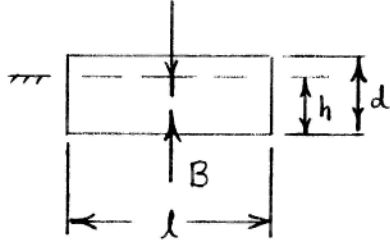
$$p = \frac{\mu}{g} = \frac{84.7}{9.81} = 8.63 \text{ kg/m}$$

The Configuration does not depend on  $\mu$ .



\*5/222

$$W = \rho_1 V g = \rho_1 \pi \frac{d^2}{4} l g$$



$$B = \rho_2 V_{sub} g \quad (\text{Need } V_{sub})$$

$$A = \int dA = \int_{-d/2}^{h-d/2} 2x dy$$

$$= 2 \int_{-d/2}^{h-d/2} \sqrt{\frac{d^2}{4} - y^2} dy = 2 \cdot \frac{1}{2} \left[ y \sqrt{\frac{d^2}{4} - y^2} + \frac{d^2}{4} \sin^{-1} \frac{2y}{d} \right]_{-d/2}^{h-d/2}$$

$$= \left( h - \frac{d}{2} \right) \sqrt{hd - h^2} + \frac{d^2}{4} \sin^{-1} \left( \frac{2h-d}{d} \right) + \frac{\pi d^2}{8}$$

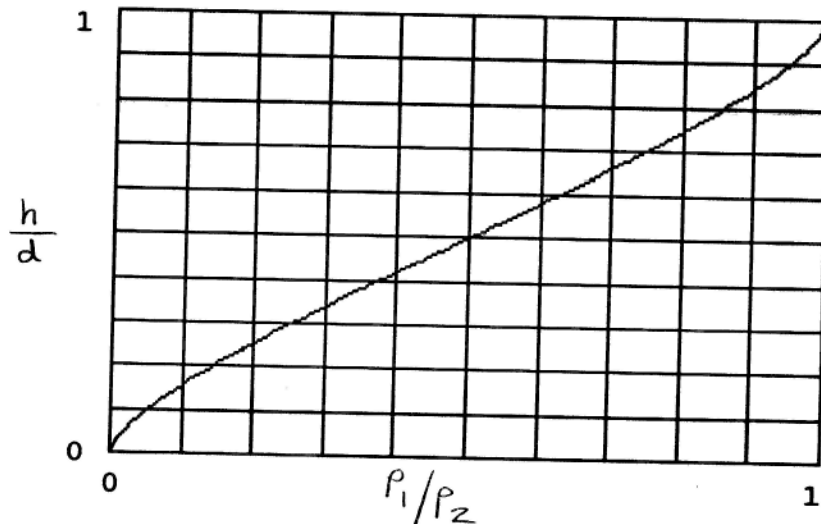
So  $B = \rho_2 g A l$ ,  $A$  given just above

$$\uparrow \Sigma F = 0 : \rho_2 g A l - \rho_1 \pi \frac{d^2}{4} l g = 0$$

$$\text{or } \left( h - \frac{d}{2} \right) \sqrt{hd - h^2} + \frac{d^2}{4} \sin^{-1} \left( \frac{2h-d}{d} \right) + \frac{\pi d^2}{8} = \frac{\rho_1}{\rho_2} \pi \frac{d^2}{4}$$

Strategy : Set  $d=1$  and numerically solve the above equation for  $h$  for values of

$\frac{\rho_1}{\rho_2}$  between 0 and 1:



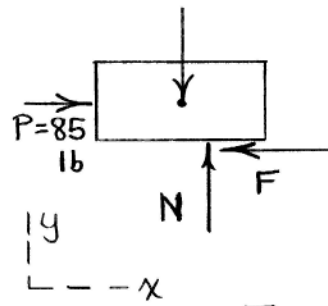
For pine wood and salt water,  $\rho_1 = 480 \frac{\text{kg}}{\text{m}^3}$  and  $\rho_2 = 1030 \frac{\text{kg}}{\text{m}^3}$ . So  $\frac{\rho_1}{\rho_2} = \frac{480}{1030} = 0.466$

Numerical



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6/1



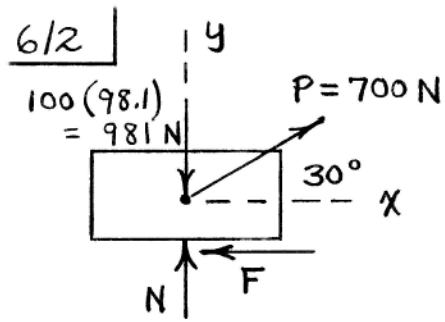
Assume equilibrium.

$$\sum F_x = 0 \Rightarrow F = 85 \text{ lb}$$

$$\sum F_y = 0 \Rightarrow N = 200 \text{ lb}$$

$$F_{\max} = \mu_s N = 0.5(200) = 100 \text{ lb}$$

$F < F_{\max}$ , assumption valid,  $F = 85 \text{ lb}$



Assume equilibrium.

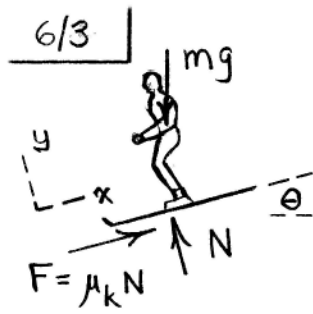
$$\sum F_x = 0 : 700 \cos 30^\circ - F = 0, \quad F = 606 \text{ N}$$

$$\sum F_y = 0 : N - 981 + 700 \sin 30^\circ = 0, \quad N = 631 \text{ N}$$

$$F_{\max} = \mu_s N = 0.8 (631) = 505 \text{ N} < F = 606 \text{ N}$$

Assumption invalid, motion occurs.

$$F = \mu_k N = 0.6 (631) = \underline{379 \text{ N}}$$



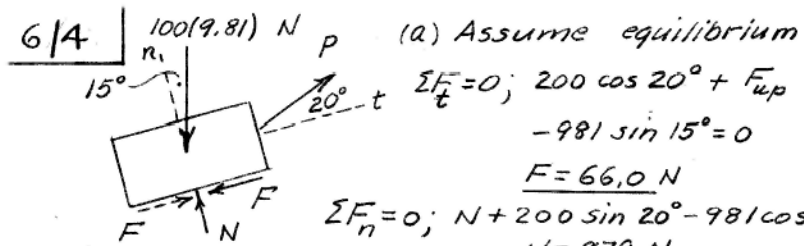
$$\Sigma F_x = 0: 0.08N - mg \sin \theta = 0$$

$$N = \frac{mg \sin \theta}{0.08}$$

$$\Sigma F_y = 0: N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$\text{So } \frac{mg \sin \theta}{0.08} = mg \cos \theta, \tan \theta = 0.08, \underline{\theta = 4.57^\circ}$$



$$\Sigma F_t = 0; 200 \cos 20^\circ + F_{up} - 981 \sin 15^\circ = 0$$

$$F = 66.0 \text{ N}$$

$$\Sigma F_n = 0; N + 200 \sin 20^\circ - 981 \cos 15^\circ = 0$$

$$N = 879 \text{ N}$$

&  $F_{s \max} = 0.3(879) = 264 \text{ N} > 66.0 \text{ N}$  so equil. assumpt. OK

(b)  $F_{down} = \mu_s N; \Sigma F_t = 0; P \cos 20^\circ - 981 \sin 15^\circ - 0.3N = 0$

$$\Sigma F_n = 0; N + P \sin 20^\circ - 981 \cos 15^\circ = 0$$

Solve & get  $P = 576 \text{ N}$

(c) Assume slipping up plane with  $F_{down} = \mu_k N$

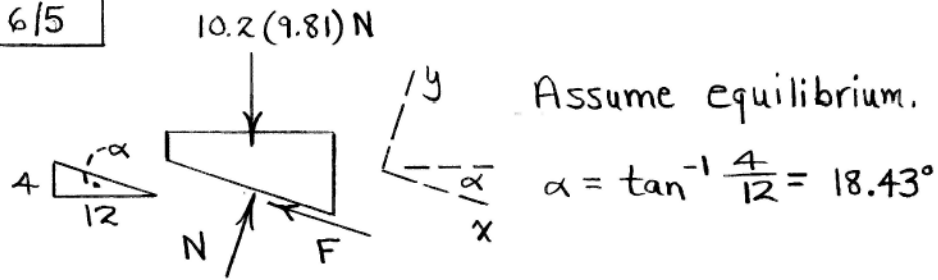
$$\Sigma F_n = 0; 600 \sin 20^\circ + N - 981 \cos 15^\circ = 0, N = 742 \text{ N}$$

$$F = 0.2(742) = 148.5 \text{ N}$$

$\Sigma F_t = 600 \cos 20^\circ - 981 \sin 15^\circ - 148.5 = 161.4 \text{ N} > 0$   
so block moves up plane as assumed



6/5



$$\alpha = \tan^{-1} \frac{4}{12} = 18.43^\circ$$

$$\sum F_x = 0: -F + 10.2(9.81) \sin \alpha = 0, \quad F = 31.6 \text{ N}$$

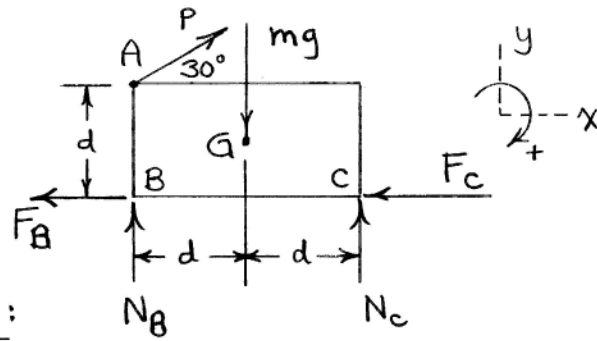
$$\sum F_y = 0: N - 10.2(9.81) \cos \alpha = 0, \quad N = 94.9 \text{ N}$$

$$F_{\max} = \mu_s N = 0.9(94.9) = 85.4 \text{ N} > F = 31.6 \text{ N}$$

Assumption valid;  $F = 31.6 \text{ N}$  ←

Total force  $P = 10.2(9.81) = \underline{100.1 \text{ N}}$  ↑

6/6

Slips:

$$\sum F_x = 0 : -F_B - F_C + P \cos 30^\circ = 0 \quad (1)$$

$$\sum F_y = 0 : N_B + N_C + P \sin 30^\circ - mg = 0 \quad (2)$$

With  $F_B = \mu_s N_B$  &  $F_C = \mu_s N_C$ , combine (1)

$$\& (2) \text{ to obtain } P = \frac{\mu_s mg}{\mu_s \sin 30^\circ + \cos 30^\circ}$$

$$\text{With } \mu_s = 0.5, \quad P = P_{\text{slip}} = \underline{0.448 mg}$$

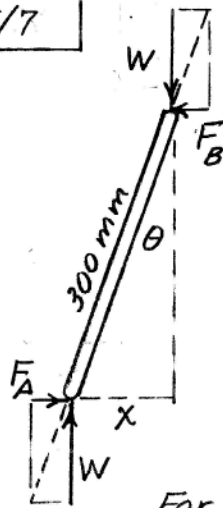
Tips ( $N_B, F_B \rightarrow 0$ ):

$$\sum M_G = 0 : (P \cos 30^\circ)d + (P \sin 30^\circ)(2d) - mg(d) = 0$$

$$\Rightarrow P = \frac{mg}{\cos 30^\circ + 2 \sin 30^\circ} = \underline{0.536 mg = P_{\text{tip}}}$$

For these conditions, slipping would occur first.

6/7



$$\text{For } x = 75 \text{ mm, } \theta = \sin^{-1} \frac{75}{300} = 14.5^\circ$$

Friction angle  $\phi = \tan^{-1} \mu_s$

$$\text{for A is } \phi_A = \tan^{-1} 0.40 = 21.8^\circ$$

$$\text{" B " } \phi_B = \tan^{-1} 0.30 = 16.7^\circ$$

Since  $\theta < \phi_A$  &  $\phi_B$ , bar does not

slip &  $F_A = F_B = W \tan \theta$

$$= 50(9.81) \tan 14.5^\circ$$

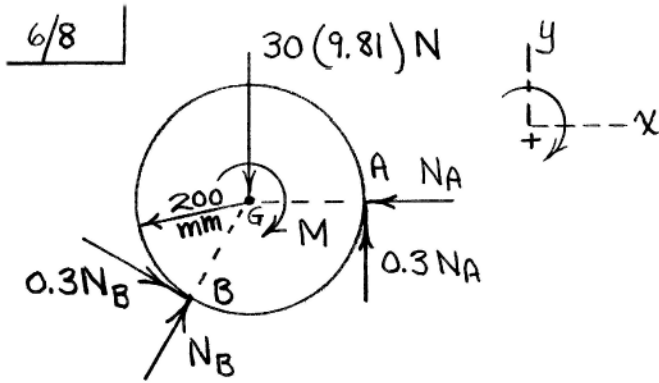
$$= \underline{126.6 \text{ N}}$$

For increased  $x$  bar slips first at B

with  $\theta = \phi_B = 16.7^\circ$ . Thus  $x_{\max} = 300 \sin 16.7^\circ$

$$= \underline{86.2 \text{ mm}}$$





$$\left\{ \begin{array}{l} \sum M_G = 0: M - 0.3(N_A + N_B) \cdot 0.2 = 0 \end{array} \right. \quad (1)$$

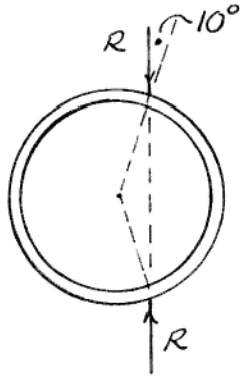
$$\left\{ \begin{array}{l} \sum F_x = 0: N_B \sin 30^\circ + 0.3 N_B \cos 30^\circ - N_A = 0 \end{array} \right. \quad (2)$$

$$\left\{ \begin{array}{l} \sum F_y = 0: N_B \cos 30^\circ - 0.3 N_B \sin 30^\circ - 30(9.81) \\ \quad + 0.3 N_A = 0 \end{array} \right. \quad (3)$$

Solution of Eqs. (1)-(3):

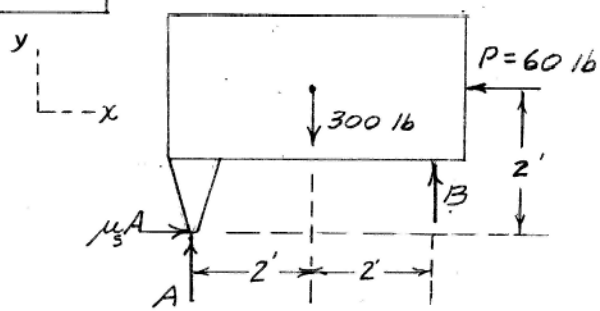
$$\begin{cases} N_B = 312 \text{ N} \\ N_A = 237 \text{ N} \\ M = 32.9 \text{ N}\cdot\text{m} \end{cases}$$

6/9



$$\begin{aligned}\mu_{s \min} &= \tan \phi = \tan 10^\circ \\ &= \underline{0.176}\end{aligned}$$

6/10



$$\begin{aligned} \Sigma M_A = 0; & 60(2) + 4B - 300(2) = 0, & B = 120 \text{ lb} \\ \Sigma F_y = 0; & A + 120 - 300 = 0, & A = 180 \text{ lb} \\ \Sigma F_x = 0; & \mu_s(180) - 60 = 0, & \underline{\mu_s = 0.33} \end{aligned}$$

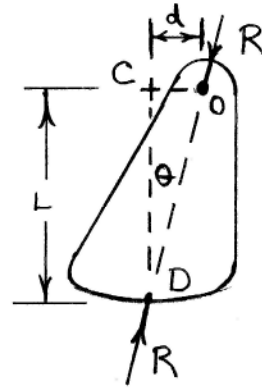
6/11

$$\theta_{\max} = \phi = \tan^{-1} \mu_s = \tan^{-1} \frac{d}{L}$$

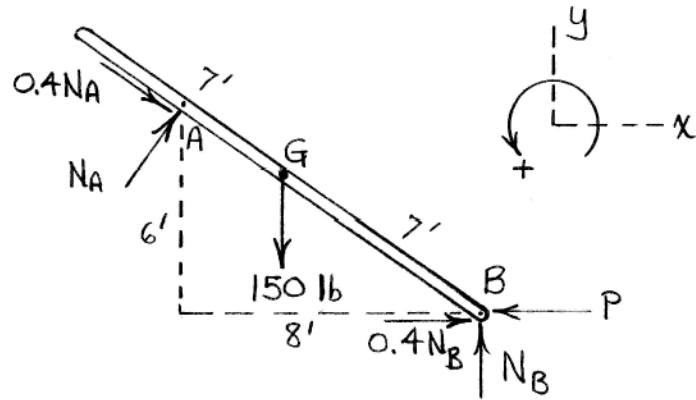
$$\text{So } \mu_s = \frac{d}{L}, \quad d = \mu_s L = 0.2L$$

Device will work for all

$$\underline{d \leq 0.2L}$$



6/12



$$\sum M_B = 0 : 150 \left( \frac{4}{5} \cdot 7 \right) - 10 N_A = 0 \quad N_A = 84 \text{ lb}$$

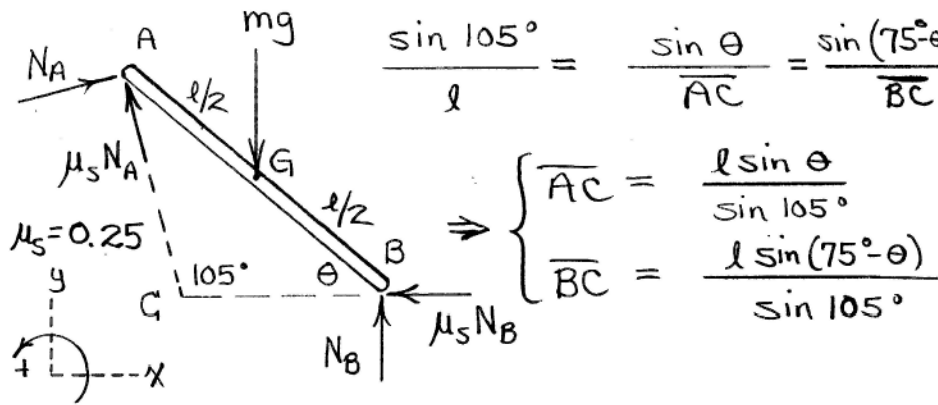
$$\sum F_y = 0 : N_B - 150 + \frac{4}{5} (84) - 0.4 (84) \frac{3}{5} = 0,$$
$$N_B = 103.0 \text{ lb}$$

$$\sum F_x = 0 : -P + 0.4 (103.0) + 84 \left( \frac{3}{5} \right) + 0.4 (84) \frac{4}{5} = 0$$
$$P = 118.5 \text{ lb}$$

6/13

From law of sines :

$$\frac{\sin 105^\circ}{l} = \frac{\sin \theta}{AC} = \frac{\sin(75^\circ - \theta)}{BC}$$



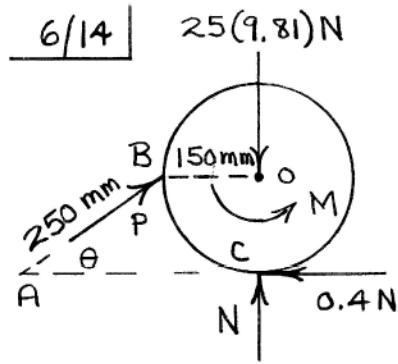
$$\Rightarrow \begin{cases} AC = \frac{l \sin \theta}{\sin 105^\circ} \\ BC = \frac{l \sin(75^\circ - \theta)}{\sin 105^\circ} \end{cases}$$

$$\sum F_x = 0 : N_A \cos 15^\circ - 0.25 N_A \sin 15^\circ - 0.25 N_B = 0$$

$$\sum F_y = 0 : N_A \sin 15^\circ + 0.25 N_A \cos 15^\circ + N_B - mg = 0$$

$$\sum M_c = 0 : -N_A \left( \frac{l \sin \theta}{\sin 105^\circ} \right) + N_B \left( \frac{l \sin(75^\circ - \theta)}{\sin 105^\circ} \right) - mg \left[ \frac{l \sin(75^\circ - \theta)}{\sin 105^\circ} - \frac{l}{2} \cos \theta \right] = 0$$

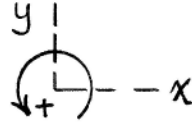
$$\text{Solution : } \begin{cases} N_A = 0.244 mg \\ N_B = 0.878 mg \\ \theta = 59.9^\circ \end{cases}$$



$$\sin \theta = \frac{150}{250} = 0.6$$

$$\cos \theta = 0.8$$

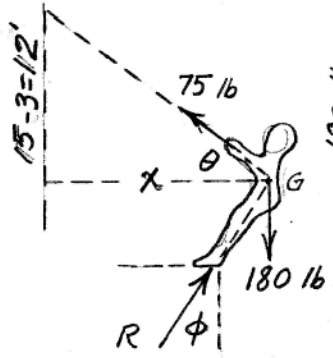
$$\overline{AC} = 0.8(0.25) + 0.15 = 0.35 \text{ m}$$



$$\begin{cases} \sum F_x = 0: 0.8P - 0.4N = 0 \\ \sum F_y = 0: N + 0.6P - 25(9.81) = 0 \\ \sum M_A = 0: M + (N - 25(9.81))(0.35) = 0 \end{cases}$$

Solution:  $N = 188.7 \text{ N}$ ,  $P = 94.3 \text{ N}$ ,  $M = \underline{19.81 \text{ N}\cdot\text{m}}$

6/15



$$\phi = \tan^{-1} 0.40 = 21.8^\circ$$

$$\beta = 90 - 21.8 = 68.2^\circ$$

Law of sines

$$\frac{180}{\sin(\theta + \beta)} = \frac{75}{\sin 21.8^\circ}$$
$$\theta + \beta = \sin^{-1} \frac{180 \sin 21.8}{75}$$

$$= \sin^{-1} 0.891 = 63.0^\circ$$

or:  $117.0^\circ$

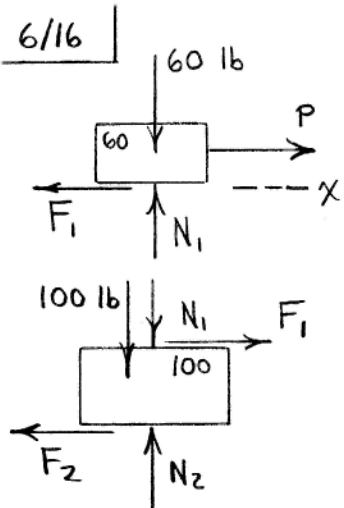
$63.0^\circ$  sol. not possible,

$$\text{so } \theta = 117.0 - 68.2 = 48.8^\circ$$

$$\frac{12}{x} = \tan 48.8^\circ, \quad x = 12 / 1.14 = \underline{10.52 \text{ ft}}$$



6/16



(a)  $P$  applied to 60-lb block  
 (Note:  $N_1 = 60$  lb  $\neq$   $N_2 = 160$  lb throughout)

Assume 60-lb block slips by itself ( $F_1 = \mu_{s1} N_1$ )

$$\sum F_x = 0: P - \mu_{s1} N = 0$$

$$P = \mu_{s1} N_1 = 0.4(60) = 24 \text{ lb}$$

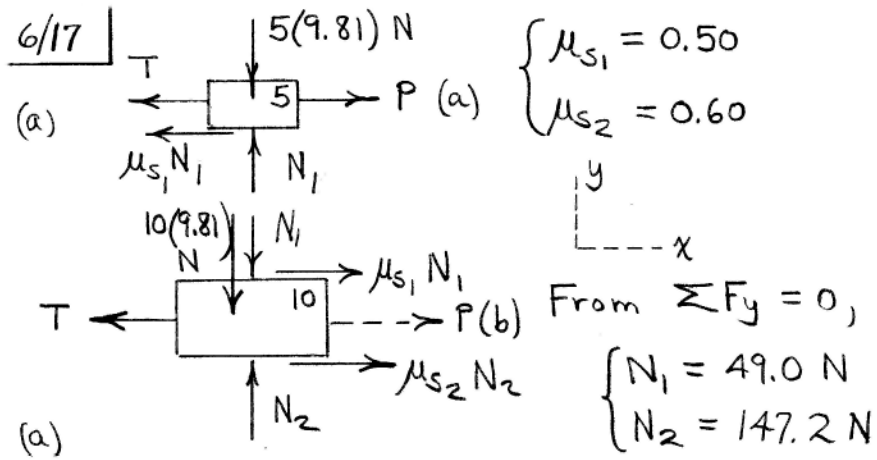
Check on 100-lb block:

$$\sum F_x = 0: 24 - F_2 = 0, \quad F_2 = 24 \text{ lb}$$

$$\text{But } F_{2\text{max}} = \mu_{s2} N_2 = 0.12(160) = 19.2 \text{ lb}$$

So the 60-lb does not slip by itself; rather, the two blocks move as a unit. In both cases (a)  $\neq$  (b),

$$P = \mu_{s2} N_2 = 0.12(160) = \underline{19.2 \text{ lb}}$$



$$\sum F_x = 0: \begin{cases} P - T - 0.50(49.0) = 0 \\ -T + 0.50(49.0) + 0.60(147.2) = 0 \end{cases}$$

$$\underline{T = 112.8 \text{ N}}, \quad \underline{P = 137.3 \text{ N}}$$

(b) Now  $P$  is applied to 10-kg block & we reverse all friction forces above:

$$\sum F_x = 0: \begin{cases} -T + 0.50(49.0) = 0 \\ -T - 0.50(49.0) - 0.60(147.2) + P = 0 \end{cases}$$

$$\underline{T = 24.5 \text{ N}}, \quad \underline{P = 137.3 \text{ N}}$$

$$\underline{6/18} \quad \Sigma F_y = 0 : T - W \cos 10^\circ = 0$$

$$T = W \cos 10^\circ$$

100-lb block:

$$\Sigma F_y = 0 : N - 100 \cos 20^\circ = 0$$

$$N = 94.0 \text{ lb (throughout)}$$

(a) Motion impends down incline: y

$$\Sigma F_x = 0 : 2T - 100 \sin 20^\circ + F_{\max} = 0$$

$$\text{With } F_{\max} = \mu_s N = 0.3(94.0)$$

$$= 28.2 \text{ lb and } T = W \cos 10^\circ,$$

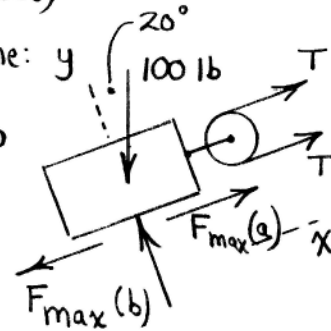
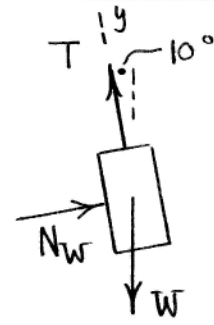
$$W = 3.05 \text{ lb}$$

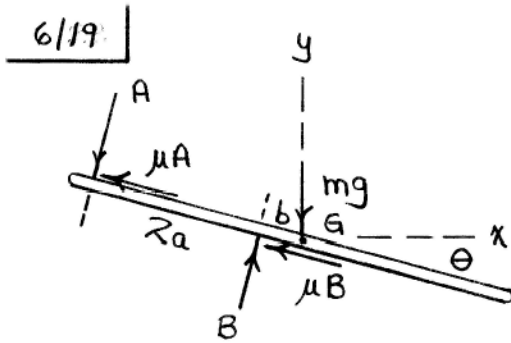
(b) Motion impends up incline :

$$\Sigma F_x = 0 : 2T - 100 \sin 20^\circ - F_{\max} = 0$$

$$\text{Similarly, } W = 31.7 \text{ lb}$$

Hence the allowable range is  $\underline{3.05 \leq W \leq 31.7 \text{ lb}}$





$$\sum M_G = 0 : A(2a+b) - Bb = 0, \quad \frac{B}{A} = \frac{2a+b}{b}$$

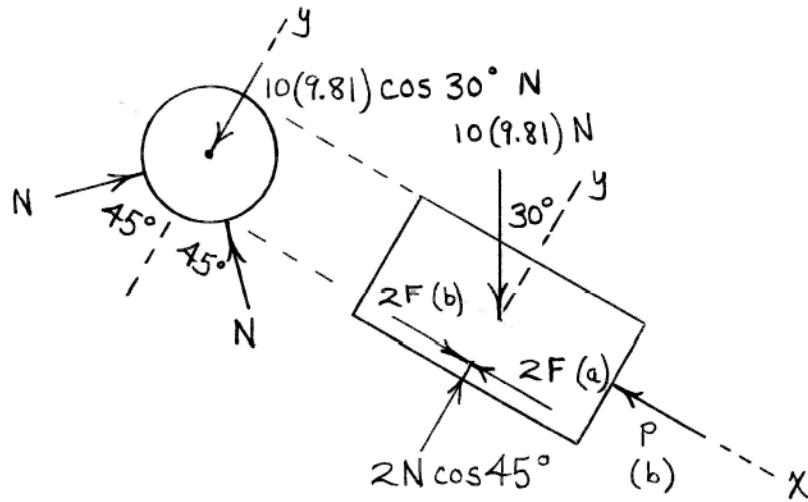
$$\sum F_x = 0 : (B-A)\sin\theta - \mu(A+B)\cos\theta = 0$$

$$\tan\theta = \mu \frac{A+B}{B-A} = \mu \frac{1 + B/A}{\frac{B}{A} - 1}$$

$$\text{Substitute } \frac{B}{A} : \tan\theta = \mu \frac{1 + (2a+b)/b}{(2a+b)/b - 1}$$

$$\text{or } \underline{\theta = \tan^{-1}\left(\mu \frac{a+b}{a}\right)}$$

6/20



$$\Sigma F_y = 0: 2N \cos 45^\circ - 10(9.81) \cos 30^\circ = 0, N = 60.1 \text{ N}$$

(a)  $P = 0$

$$\Sigma F_x = 0: -2F + 10(9.81) \sin 30^\circ = 0, \underline{F = 24.5 \text{ N}}$$

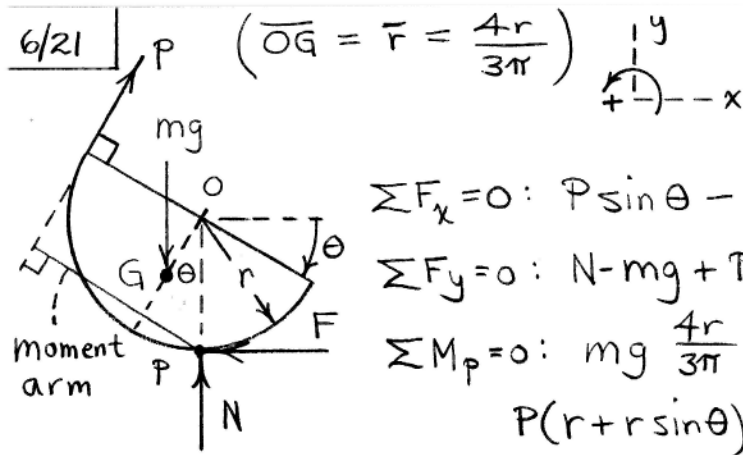
Check:  $F_{\max} = \mu_s N = 0.5(60.1) = 30.0 \text{ N} > F = 24.5 \text{ N}$

So we indeed have static equilibrium.

(b)  $P \neq 0$

$$\Sigma F_x = 0: -P + 10(9.81) \sin 30^\circ + 2(0.5 \cdot 60.1) = 0$$

$$\underline{P = 109.1 \text{ N}}$$



When slipping impends,  $F = \mu_s N \quad (4)$

$(4) \rightarrow (1): P \sin \theta - \mu_s N = 0 \quad (5)$

$(2): N = mg - P \cos \theta \quad (6)$

$(6) \rightarrow (5): P \sin \theta - \mu_s (mg - P \cos \theta) = 0$

$\Rightarrow P = \frac{\mu_s mg}{\sin \theta + \mu_s \cos \theta} \quad (7)$

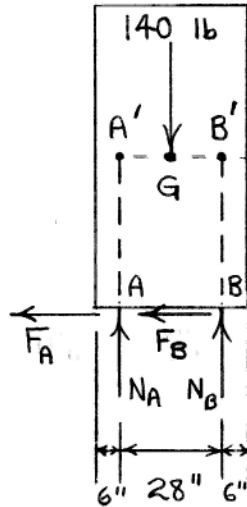
$(7) \rightarrow (3)$  & simplification yields

$$\mu_s = \frac{4 \sin^2 \theta}{3\pi (1 + \sin \theta) - 4 \sin \theta \cos \theta}$$

For  $\theta = 40^\circ$ ,  $\mu_s = 0.1223$

$(7)$  then gives  $P = 0.1661 mg$

6/22



(a) Set  $F_B = 0$ ,  $F_A = \mu N_A$ .

$$\sum M_{B'} = 0 : 140(14) - N_A(28)$$

$$- 0.3 N_A(40) = 0, N_A = 49 \text{ lb}$$

$$\sum F = 0 : P - \mu N_A = 0, \underline{P = 14.7 \text{ lb}}$$

(b) Set  $F_A = 0$ ,  $F_B = \mu N_B$ .

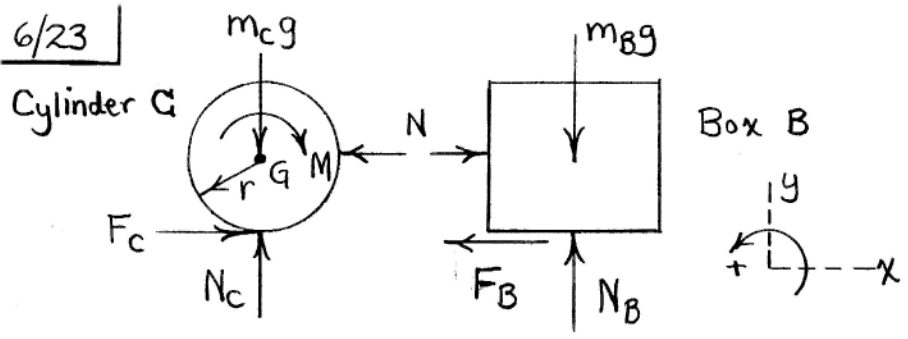
$$\sum M_{A'} = 0 : -140(14) + N_B(28)$$

$$- 0.3 N_B(40) = 0, N_B = 122.5 \text{ lb}$$

$$\sum F = 0 : P - \mu N_B = 0, \underline{P = 36.8 \text{ lb}}$$

(Note : Above  $\mu = \mu_k = 0.3$ )

6/23



Assume that box slips but cylinder does not.

$$F_B = (\mu_s)_B N_B$$

$$B \left\{ \begin{array}{l} \sum F_x = 0 : N - F_B = 0 \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} \sum F_y = 0 : N_B - m_B g = 0 \end{array} \right. \quad (2)$$

$$\text{So } N_B = m_B g, \quad N = F_B = (\mu_s)_B m_B g$$

$$C \left\{ \begin{array}{l} \sum F_x = 0 : F_c - N = 0 \end{array} \right. \quad (3)$$

$$\left\{ \begin{array}{l} \sum F_y = 0 : N_c - m_c g = 0 \end{array} \right. \quad (4)$$

$$\left\{ \begin{array}{l} \sum M_G = 0 : F_c r - M = 0 \end{array} \right. \quad (5)$$

$$M = F_c r = N r = (\mu_s)_B m_B g r$$

$$= 0.5(3)(9.81)(0.2) = \underline{2.94 \text{ N}\cdot\text{m}}$$

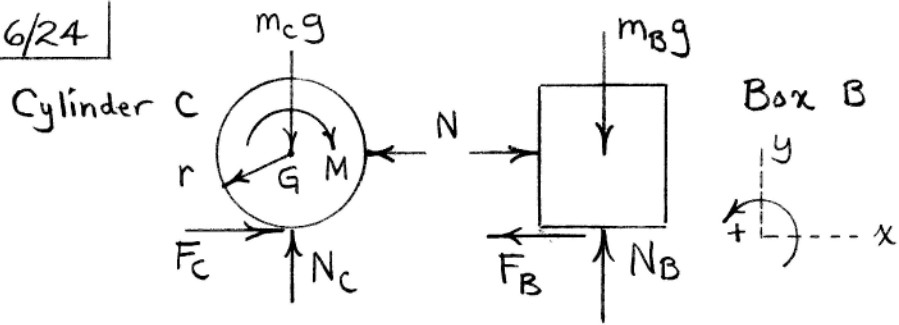
$$F_c = N = (\mu_s)_B m_B g = (0.5)(3)(9.81) = 14.72 \text{ N}$$

$$< (F_c)_{\text{max}} = (\mu_s)_c m_c g = (0.4)(6)(9.81) = 23.5 \text{ N}$$





6/24



Assume that cylinder slips but box does not.

$$F_c = (\mu_s)_c N_c$$

$$B \begin{cases} \sum F_x = 0 : N - F_B = 0 & (1) \\ \sum F_y = 0 : N_B - m_B g = 0 & (2) \end{cases}$$

$$\text{So } N_B = m_B g, \quad N = F_B$$

$$C \begin{cases} \sum F_x = 0 : (\mu_s)_c N_c - N = 0 & (3) \\ \sum F_y = 0 : N_c - m_c g = 0 & (4) \\ \sum M_G = 0 : (\mu_s)_c N_c r - M = 0 & (5) \end{cases}$$

$$M = (\mu_s)_c m_c g r = 0.2(6)(9.81)(0.2) = \underline{2.35 \text{ N}\cdot\text{m}}$$

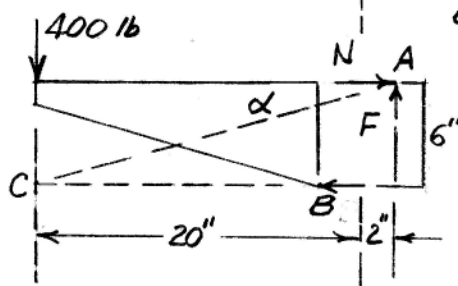
$$N = (\mu_s)_c m_c g = 0.2(6)(9.81) = 11.77 \text{ N}$$

$$F_B = N = 11.77 \text{ N}$$

$$(F_B)_{\max} = (\mu_s)_B N_B = 0.5(3)(9.81) = 14.72 \text{ N} > F_B$$

Assumption OK

6/25



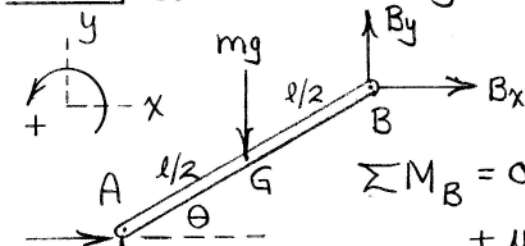
For equilibrium forces must be concurrent at C so that

$$\alpha = \tan^{-1} \frac{6}{22} = \tan^{-1} 0.27$$

Since  $0.27 < 0.40$ , collar will not slip &

$$\Sigma F = 0; \quad \underline{F = 400 \text{ lb}}$$

6/26 | (a) P to the right.



$$\sum M_B = 0: mg \frac{l}{2} \cos \theta - N_A l \cos \theta + \mu_s N_A l \sin \theta = 0 \quad (1)$$

$$(Box) \sum F_x = 0: P - \mu_s N_A - \mu_s N = 0 \quad (2)$$

$$\sum F_y = 0: N - m_o g - N_A = 0 \quad (3)$$

Solve for P as

$$P = \mu_s g \left[ \frac{m \cos \theta}{\cos \theta - \mu_s \sin \theta} + m_o \right]$$

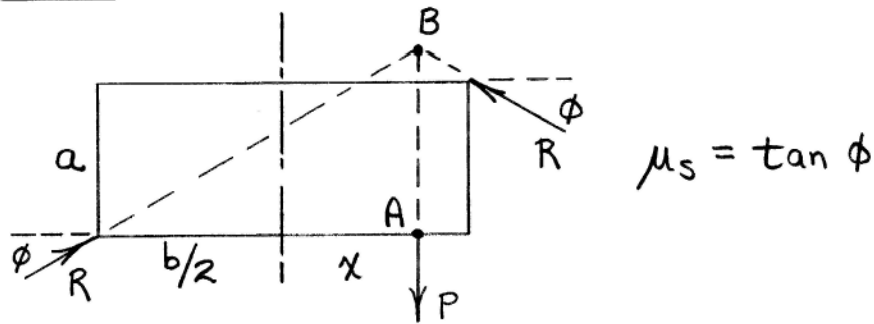
(b) P to the left. Reverse P and all friction forces in the above FBD's & obtain

$$P = \mu_s g \left[ \frac{m \cos \theta}{\cos \theta + \mu_s \sin \theta} + m_o \right]$$

With  $\theta = 30^\circ$ ,  $m = m_o = 3 \text{ kg}$ , and  $\mu_s = 0.60$ ,

$$\text{we obtain } \begin{cases} (a) P = 44.7 \text{ N} \\ (b) P = 30.8 \text{ N} \end{cases}$$

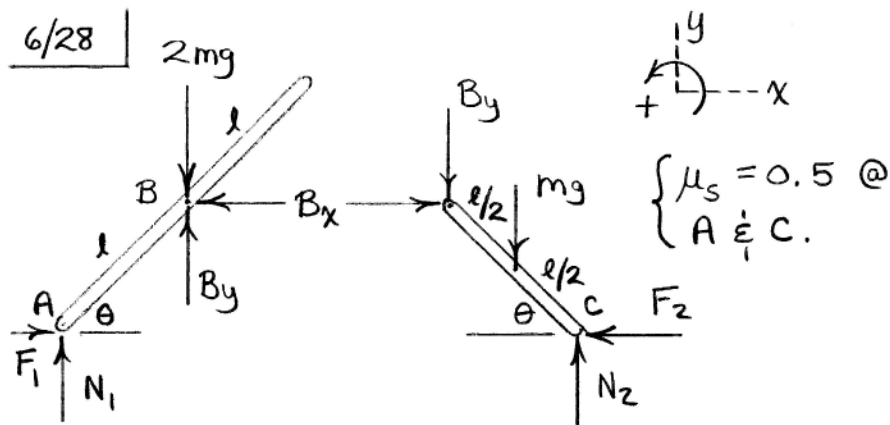
6/27



Binding occurs for max.  $x$  when  $\phi$  becomes the friction angle or less. For equilibrium, forces are concurrent at B. Thus

$$\overline{AB} = \left(\frac{b}{2} + x\right) \tan \phi = a + \left(\frac{b}{2} - x\right) \tan \phi$$

$$\Rightarrow \underline{x = \frac{a}{2\mu_s}}$$



$$\begin{cases}
 \Sigma F_x = 0 : F_1 - B_x = 0 & (1) \\
 \Sigma F_y = 0 : N_1 + B_y - 2mg = 0 & (2) \\
 \Sigma M_A = 0 : B_x(l \sin \theta) + B_y(l \cos \theta) - 2mg(l \cos \theta) = 0 & (3)
 \end{cases}$$

$$\begin{cases}
 \Sigma F_x = 0 : B_x - F_2 = 0 & (4) \\
 \Sigma F_y = 0 : -B_y - mg + N_2 = 0 & (5) \\
 \Sigma M_C = 0 : mg\left(\frac{l}{2} \cos \theta\right) + B_y(l \cos \theta) - B_x(l \sin \theta) = 0 & (6)
 \end{cases}$$

Assume first slippage at A:  $F_1 = 0.5N_1$ , Solve seven equations to obtain  $\theta = 63.4^\circ$ ,  $F_2 = 0.625mg$ ,  $N_2 = 1.75mg$ . Note  $F_2 < F_{2max} = 0.875mg$ .

Then assume first slippage at B:  $F_2 = 0.5N_2$ . Obtain  $\theta = 55.0^\circ$ ,  $F_1 = 0.875mg$  &  $N_1 = 1.25mg$ . Note  $F_1 > F_{1max} = 0.625mg$ . So A slips first.

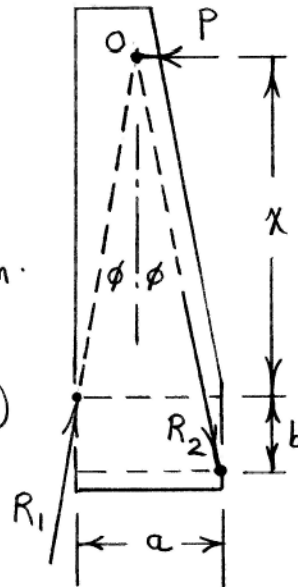
6/29

For equilibrium, the forces must be concurrent at  $O$  and  $\phi = \tan^{-1} \mu_s$  for impending motion with  $x = x_{\min}$ .

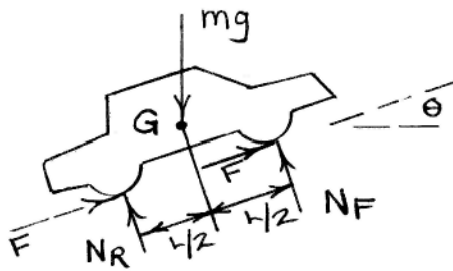
$$a = x \tan \phi + (x+b) \tan \phi$$

$$= x \mu_s + (x+b) \mu_s = \mu_s (2x+b)$$

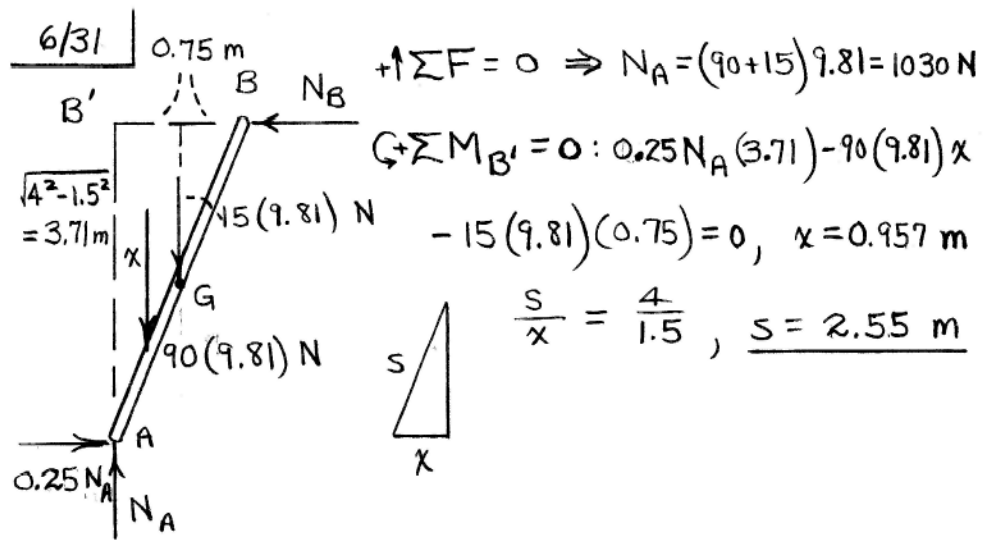
$$x = \frac{a - b \mu_s}{2 \mu_s}$$



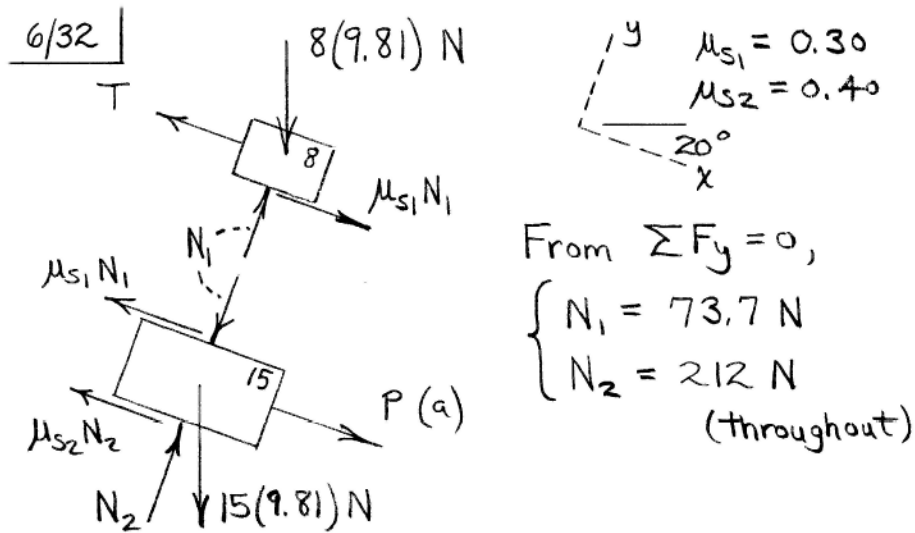
6/30 | Consider the FBD below and the equilibrium equation  $\sum M_G = 0$ . The presence of the propulsive friction forces  $F$ , whether applied at the front or at the rear, increases the rear normal forces and decreases the front ones. Increased normals mean increased available propulsive friction forces.



Thus the rear-wheel drive car would climb the steeper grade.





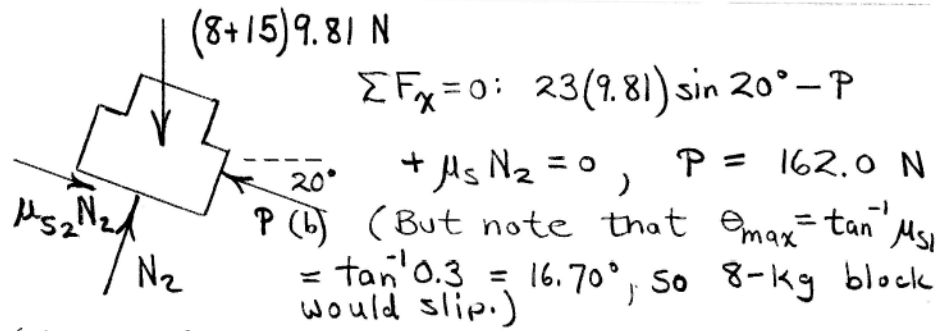


(a)  $\Sigma F_x = 0$ :

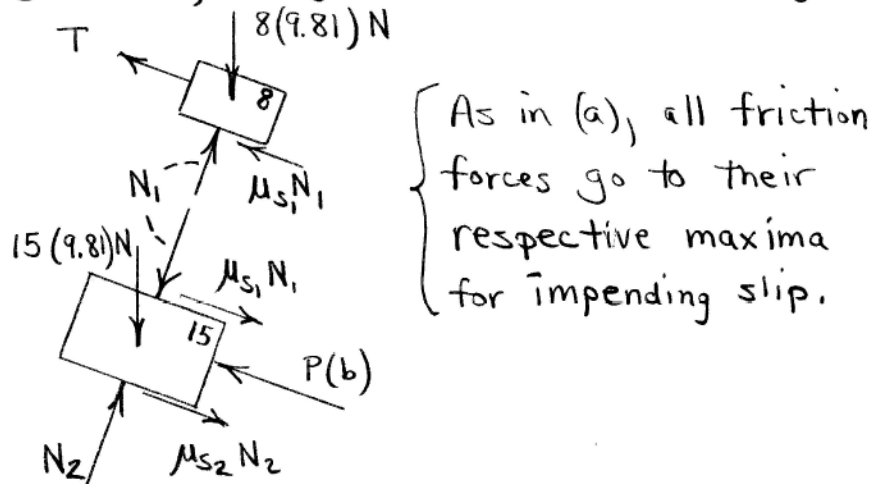
$$\begin{cases} -T + 8(9.81) \sin 20^\circ + \mu_{s1} N_1 = 0 \\ -\mu_{s1} N_1 - \mu_{s2} N_2 + 15(9.81) \sin 20^\circ + P = 0 \end{cases}$$

Solution:  $\underline{P = 56.6 \text{ N}}$ ,  $T = 49.0 \text{ N}$

(b) Assume T goes slack and both masses move as one unit.



(b), continued. Assume that T does not go slack; 8-kg block remains stationary.



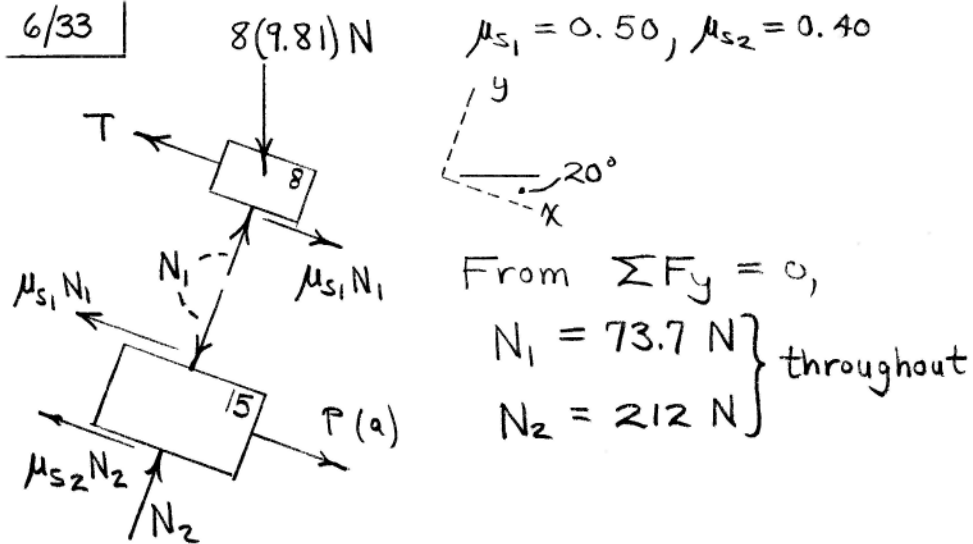
$$\Sigma F_x = 0:$$

$$\left. \begin{aligned} -T + 8(9.81) \sin 20^\circ - \mu_{s1} N_1 &= 0 \\ \mu_{s1} N_1 + \mu_{s2} N_2 + 15(9.81) \sin 20^\circ - P &= 0 \end{aligned} \right\}$$

$$\text{Solution: } \underline{P = 157.3 \text{ N}}, T = 4.72 \text{ N}$$

The 15-kg block slips up the incline while the 8-kg block remains stationary.

6/33

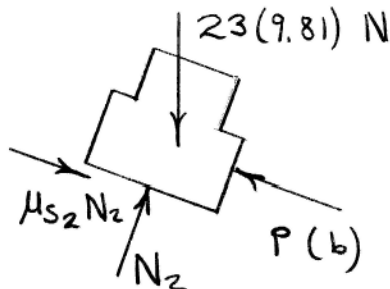


(a)  $\sum F_x = 0$ :

$$\left. \begin{aligned} -T + 8(9.81) \sin 20^\circ + \mu_{s1} N_1 &= 0 \\ -\mu_{s1} N_1 - \mu_{s2} N_2 + 15(9.81) \sin 20^\circ + P &= 0 \end{aligned} \right\}$$

Solution,  $\underline{P = 71.4 \text{ N}} \quad T = 63.7 \text{ N}$

(b) Assume that T goes slack and motion impends for A and B as a unit.



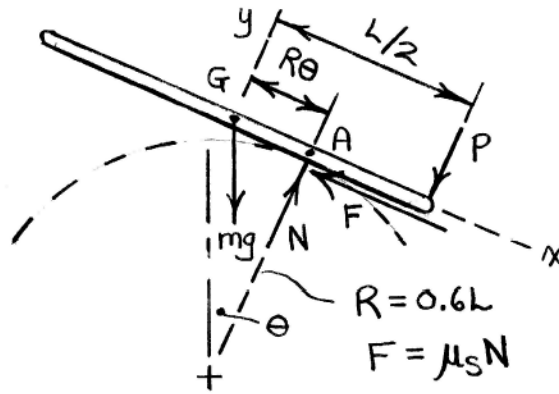
$$\sum F_x = 0: 23(9.81) \sin 20^\circ - P + \mu_{s2} N_2 = 0$$

$$\underline{P = 162.0 \text{ N}}$$

Note that  $\theta_{\max} = \tan^{-1} \mu_{s1} = \tan^{-1} 0.5$   
 $= 26.6^\circ > 20^\circ$

Hence, the possibility that the 8-kg block remains stationary as the 15-kg block slips beneath it is ruled out.

6/34



$$\sum F_x = 0: mg \sin \theta - \mu_s N = 0 \quad (1)$$

$$\sum F_y = 0: N - P - mg \cos \theta = 0 \quad (2)$$

$$\sum M_A = 0: mg R \cos \theta - P \left( \frac{L}{2} - R \right) = 0$$

$$\text{With } R = 0.6L: 0.6 mg \cos \theta - P \left( \frac{1}{2} - 0.6 \theta \right) = 0 \quad (3)$$

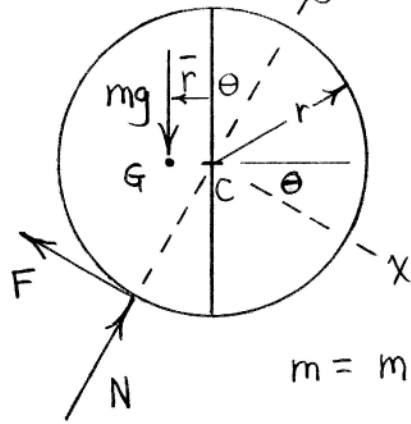
$$(1) \text{ \& } (2): mg \sin \theta = \mu_s (P + mg \cos \theta) \quad (4)$$

$$(3) \text{ \& } (4): mg \sin \theta = \mu_s \left[ \frac{0.6 mg \cos \theta}{0.5 - 0.6 \theta} + mg \cos \theta \right]$$

$$= \mu_s mg \cos \theta \left( \frac{0.5}{0.5 - 0.6 \theta} \right)$$

$$\mu_s = \left( 1 - \frac{6}{5} \theta \right) \tan \theta = \left( 1 - \frac{6}{5} \cdot 20^\circ \cdot \frac{\pi}{180^\circ} \right) \tan 20^\circ = \underline{0.212}$$

6/35



$$m_{Al} = \rho_{Al} \pi r^2 t_{Al} \quad (t = \text{depth})$$

$$= 2690 \pi (0.080)^2 (0.040)$$

$$= 2.16 \text{ kg}$$

$$m_{st} = \rho_{st} \pi r^2 t_{st} \cdot \frac{1}{2}$$

$$= 7830 \pi (0.080)^2 (0.016) \frac{1}{2}$$

$$= 1.259 \text{ kg}$$

$$m = m_{Al} + m_{st} = 3.42 \text{ kg}$$

$$\bar{r}_{st} = \frac{4r}{3\pi} = \frac{4(80)}{3\pi} = 34.0 \text{ mm}$$

$$\bar{r} = \frac{\sum m \bar{r}}{\sum m} = \frac{2.16(0) + 1.259(34.0)}{3.42} = 12.49 \text{ mm}$$

$$\sum M_C = 0 : mg \bar{r} - Fr = 0, \quad F = mg \frac{\bar{r}}{r}$$

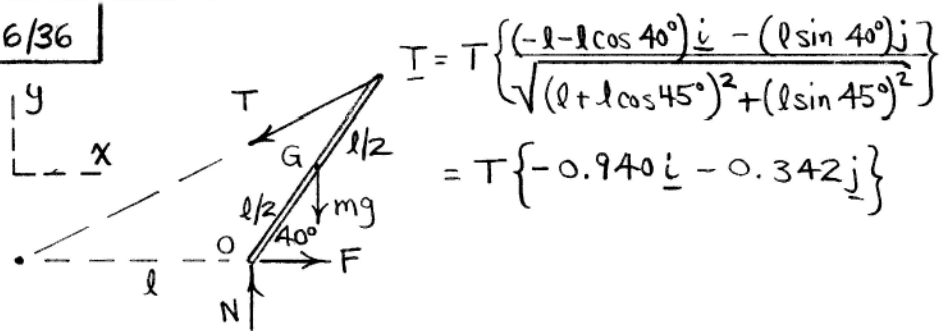
$$F = 3.42(9.81) \frac{12.49}{80} = 5.24 \text{ N}$$

$$\sum F_x = 0 : -5.24 + 3.42(9.81) \sin \theta = 0, \quad \theta = 8.98^\circ$$

$$\sum F_y = 0 : N - 3.42(9.81) \cos 8.98^\circ = 0, \quad N = 33.2 \text{ N}$$

$$\mu_s = \frac{F}{N} = \frac{5.24}{33.2} = 0.158$$

6/36



$$\underline{T} = T \left\{ \frac{(-l - l \cos 40^\circ) \underline{i} - (l \sin 40^\circ) \underline{j}}{\sqrt{(l + l \cos 40^\circ)^2 + (l \sin 40^\circ)^2}} \right\}$$

$$= T \{-0.940 \underline{i} - 0.342 \underline{j}\}$$

$$\circlearrowleft + \sum M_o = 0: 0.342T(l) - mg \frac{l}{2} \cos 40^\circ = 0$$

$$T = 1.120 mg$$

$$\sum F_x = 0: F - 1.120 mg (0.940) = 0, F = 1.052 mg$$

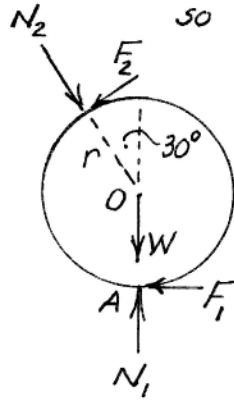
$$\sum F_y = 0: N - 1.120 mg (0.342) - mg = 0$$

$$N = 1.383 mg$$

$$\mu_s = \frac{F}{N} = \frac{1.052 mg}{1.383 mg} = \underline{0.761}$$

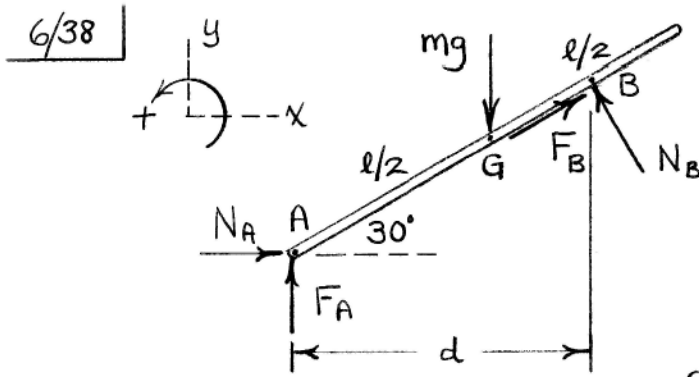
6/37

Lower roller;  $\Sigma M_O = 0$ ;  $F_1 = F_2$  But  $N_1 > N_2$   
 so  $F_2$  reaches limiting value  $\mu_s N_2$  before  $F_1$ .



$$\Sigma M_A = 0; \mu_s N_2 \left( r + \frac{\sqrt{3}}{2} r \right) - N_2 \frac{r}{2} = 0$$

$$\text{so } \mu_s = \frac{r/2}{r + \frac{\sqrt{3}r}{2}} = \frac{0.5}{1 + 0.866} = \underline{0.268}$$



FBD assumes slipping CCW  $\left\{ \begin{array}{l} \text{We expect} \\ \frac{l}{2} > \frac{d}{\cos 30^\circ} ! \end{array} \right.$

$$\begin{cases} \sum F_x = 0 : N_A + F_B \cos 30^\circ - N_B \sin 30^\circ = 0 \\ \sum F_y = 0 : F_A + F_B \sin 30^\circ + N_B \cos 30^\circ - mg = 0 \\ \sum M_A = 0 : -mg \frac{l}{2} \cos 30^\circ + N_B \frac{d}{\cos 30^\circ} = 0 \end{cases}$$

Limiting friction:  $F_A = \mu_s N_A$ ,  $F_B = \mu_s N_B$

Solve 5 equations to obtain

$$\begin{cases} N_A = 0.1362mg & F_A = 0.0545mg \\ N_B = 0.887mg & F_B = 0.355mg \end{cases} \quad l = 2.37d$$

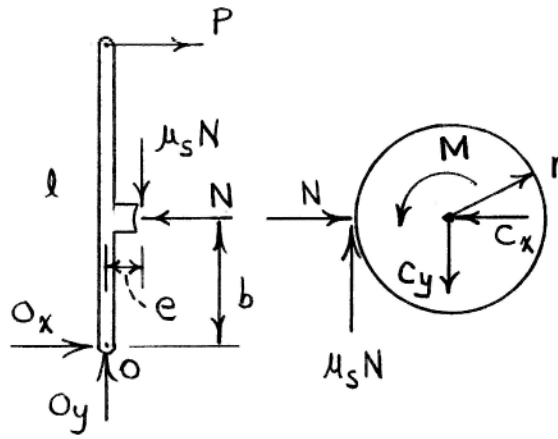
For slipping CW, reverse  $F_A$  &  $F_B$  on FBD and first two equations & obtain

$$\begin{cases} N_A = 2.58mg & F_A = 1.034mg \\ N_B = 3.05mg & F_B = 1.222mg \end{cases} \quad l = 8.14d$$

For equilibrium:  $\underline{2.37 \leq \frac{l}{d} \leq 8.14}$



6/39



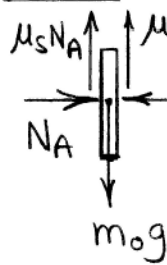
Wheel:  $\curvearrowright \sum M_c = 0: M - \mu_s N r = 0, \mu_s N = M/r$

Lever:  $\curvearrowright \sum M_o = 0: Nb - Pl - \mu_s Ne = 0$

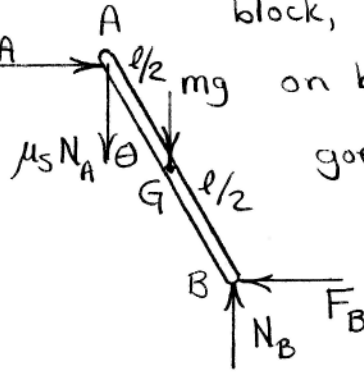
$$P = \frac{M}{rl} \left( \frac{b}{\mu_s} - e \right)$$

If  $b = \mu_s e$ ,  $P = 0$   $\hat{=}$  brake would be self-locking.

6/40



For impending slip of the block, the friction force on both of its faces goes to the maximum.



$$\text{(Block)} \uparrow \sum F = 0: 2\mu_s N_A - m_0 g = 0, N_A = \frac{m_0 g}{2\mu_s} \quad (1)$$

$$\text{(Bar)} \curvearrowright \sum M_B = 0: mg \left(\frac{l}{2} \sin \theta\right) + \mu_s N_A (l \sin \theta) - N_A (l \cos \theta) = 0 \quad (2)$$

With Eq. (1), Eq. (2) becomes

$$mg \left(\frac{l}{2} \sin \theta\right) + \mu_s \frac{m_0 g}{2\mu_s} l \sin \theta - \frac{m_0 g}{2\mu_s} l \cos \theta = 0$$

$$\text{Solving for } \theta: \theta = \tan^{-1} \left[ \frac{1}{\mu_s \left(1 + \frac{m}{m_0}\right)} \right] = \theta_{\min}$$

$$\text{(Bar)} \rightarrow \sum F = 0: N_A - F_B = 0$$

$$F_B = N_A = \frac{m_0 g}{2\mu_s}$$

$$\uparrow \sum F = 0: N_B - mg - \mu_s N_A = 0$$

$$N_B = mg + \mu_s \frac{m_0 g}{2\mu_s} = \left(m + \frac{m_0}{2}\right) g$$

$$(\mu_s)_B = \frac{F_B}{N_B} = \frac{m_0 g / (2\mu_s)}{\left(m + \frac{m_0}{2}\right) g} = \frac{1}{\mu_s \left(1 + 2 \frac{m}{m_0}\right)}$$

Numbers ( $\mu_s = 0.5$  throughout)

$$(a) \frac{m}{m_0} = 0.1: \theta_{\min} = 61.2^\circ, (\mu_s)_B = 1.667$$

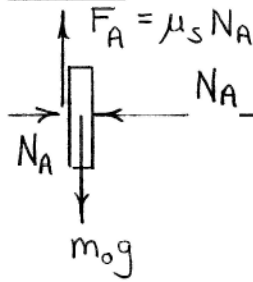
(not possible)

$$(b) \frac{m}{m_0} = 1: \theta_{\min} = 45^\circ, (\mu_s)_B = 0.667$$

$$(c) \frac{m}{m_0} = 10: \theta_{\min} = 10.30^\circ, (\mu_s)_B = 0.0952$$



6/41



The friction force  $F_A$  between the block  $m_0$  and the vertical wall is set to the maximum.

Block:  $\uparrow \Sigma F = 0$ :

$$\begin{aligned} \mu_s N_A &= m_0 g \\ N_A &= \frac{m_0 g}{\mu_s} \quad (1) \end{aligned}$$

Bar:  $\curvearrowright \Sigma M_B = 0$ :  $mg \left( \frac{l}{2} \sin \theta \right) - N_A (l \cos \theta) = 0$

$$N_A = \frac{mg}{2} \tan \theta \quad (2)$$

Combine (1) & (2):  $\frac{m_0 g}{\mu_s} = \frac{mg}{2} \tan \theta$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{2}{\mu_s} \frac{m_0}{m} \right) = \theta_{\min}$$

$$\text{Numbers: } \theta_{\min} = \tan^{-1} \left( \frac{2}{0.5} \cdot \frac{1}{10} \right) = \underline{21.8^\circ}$$

Check to see that no slippage occurs at B.

(Bar)  $\rightarrow \Sigma F = 0$ :  $N_A - F_B = 0$

So  $F_B = N_A = \frac{mg}{2} \tan \theta$  (from Eq. (2))

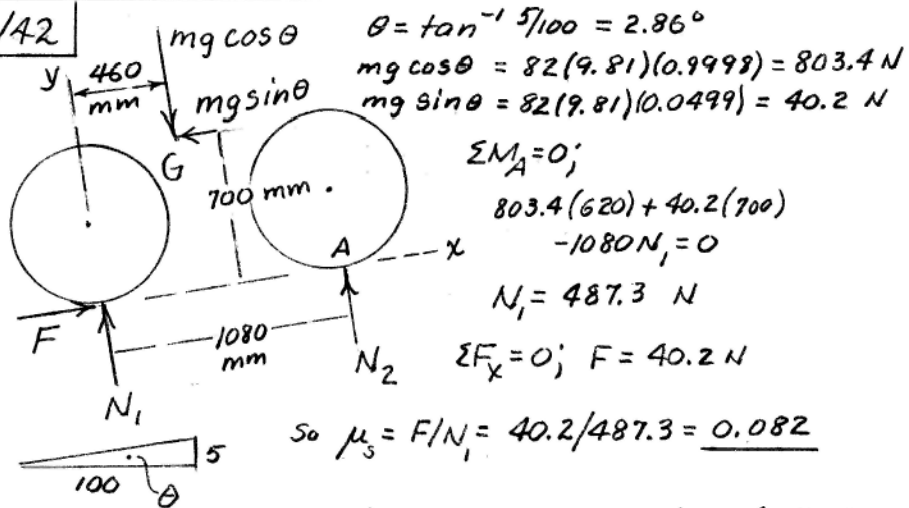
$\uparrow \Sigma F = 0$ :  $N_B - mg = 0$ ,  $N_B = mg$

$$(F_B)_{\max} = \mu_s N_B = \mu_s mg$$

$$\text{Numbers: } \left\{ \begin{aligned} F_B &= \frac{mg}{2} \tan 21.8^\circ = 0.2mg \\ (F_B)_{\max} &= 0.5mg \end{aligned} \right.$$

$F_B < (F_B)_{\max}$ , so no slippage at B.

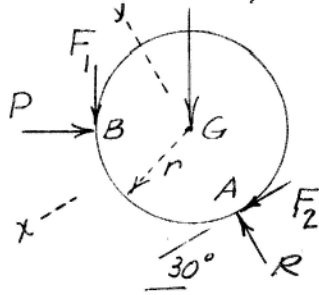
6/42



If  $\mu_s$  were doubled, friction force remains 40.2 N

6/43

$$mg = 1200(9.81) \text{ N}$$



Assume roll slips at B  
but not at A. Then

$$F_1 = 0.4P$$

$$\sum M_A = 0;$$

$$mg r \sin 30^\circ + 0.4P(r + r \sin 30^\circ)$$

$$- P(r \cos 30^\circ) = 0$$

$$P(0.866 - 0.4[1 + 0.5]) = \frac{1200(9.81)}{2}$$

$$P = 22\,126 \text{ N or } \underline{P = 22.1 \text{ kN}}$$

Check on assumption:

$$\sum F_y = 0; R - 1200(9.81) \cos 30^\circ - 0.4(22\,126) \cos 30^\circ - 22\,126 \sin 30^\circ = 0$$

$$R = 28\,922 \text{ N}$$

$$\sum M_G = 0; F_2 r - F_1 r = 0, F_2 = F_1 = 0.4(22\,126) = 8850 \text{ N}$$

But  $(F_2 = 8850 \text{ N}) < (0.4R = 11\,569 \text{ N})$  so assumption OK

6/44

For equilibrium, the three forces must be concurrent.

(a) Slipping at A is not possible as long as

$$\phi_A > (\theta_A = \tan^{-1} \frac{1/8}{1/2} = 14.04^\circ)$$

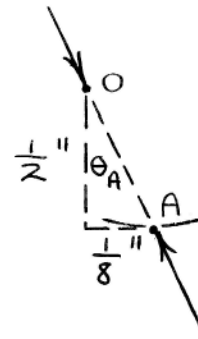
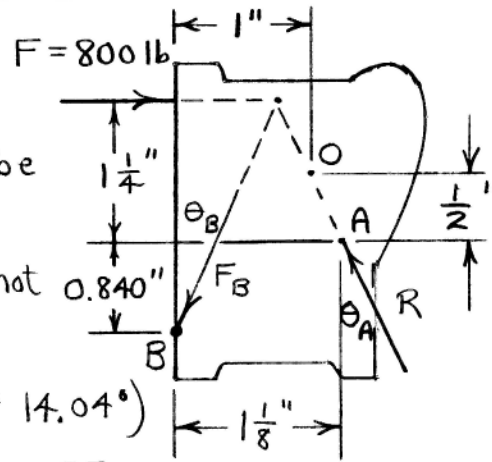
$$\text{So } (\mu_s)_{\min} = \tan \theta_A = \underline{0.25 @ A}$$

Slipping at B is prevented as long as  $\phi_B > \theta_B$ .

$$\tan \theta_B = \frac{1.125 - 1.25(0.25)}{1.25 + 0.840}$$

$$= 0.389$$

$$(\mu_s)_{\min} = \underline{0.389 @ B}$$

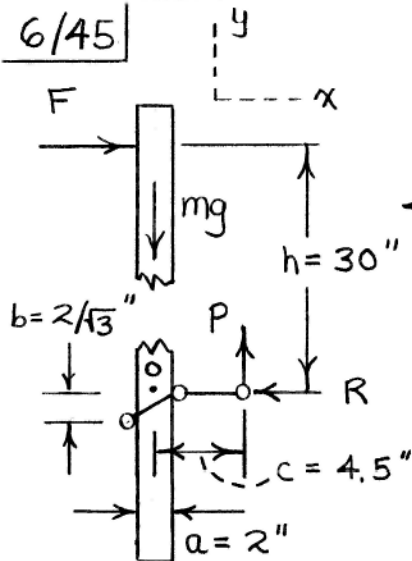


$$(b) \sum M_B = 0 : 800(1.25 + 0.840) - R \cos 14.04^\circ (1.125)$$

$$- R \sin 14.04^\circ (0.840) = 0$$

$$\underline{R = 1291 \text{ lb}}$$

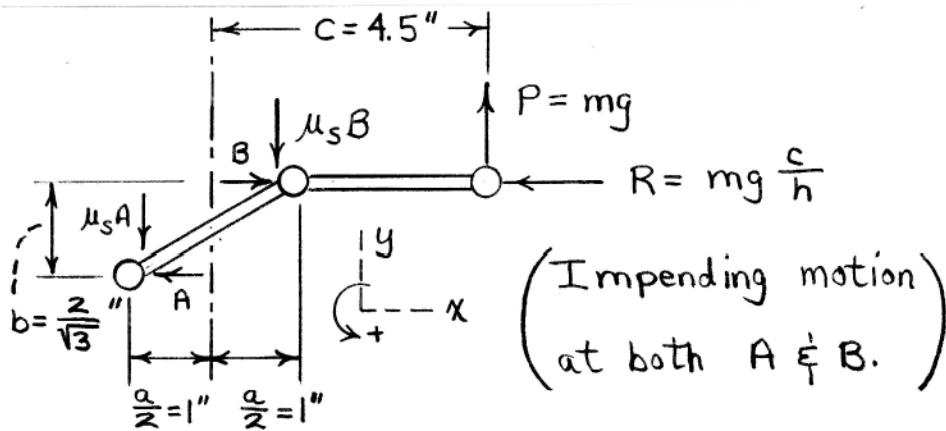
6/45



System of panel &amp; carrier:

$$\begin{cases} \sum F_x = 0 : F = R \\ \sum F_y = 0 : P = mg \\ \sum M_o = 0 : Fh = Pc \end{cases}$$

$$\Rightarrow R = mg \frac{c}{h}$$

(m is  $\frac{1}{2}$  panel mass)

$$\begin{cases} \sum F_x = 0 : B - A - mg \frac{c}{h} = 0 & (1) \\ \sum F_y = 0 : mg - \mu_s A - \mu_s B = 0 & (2) \\ \sum M_B = 0 : mg(c - \frac{a}{2}) + \mu_s A a - Ab = 0 & (3) \end{cases}$$

Eliminate B from (1) &amp; (2):

$$mg - \mu_s A - \mu_s (A + mg \frac{c}{h}) = 0, \quad A = \frac{mg}{2} \left( \frac{1}{\mu_s} - \frac{c}{h} \right)$$

$$(3): mg(c - \frac{a}{2}) + \frac{mg}{2} \left( \frac{1}{\mu_s} - \frac{c}{h} \right) (\mu_s a - b) = 0$$

$$\text{Simplify to get } \mu_s^2 - \mu_s \frac{b+2h}{a} + \frac{bh}{ac} = 0$$

$$\mu_s = \frac{b+2h}{2a} \pm \frac{1}{2} \sqrt{\frac{b^2 + 4bh + 4h^2}{a^2} - \frac{4bh}{ac}}$$

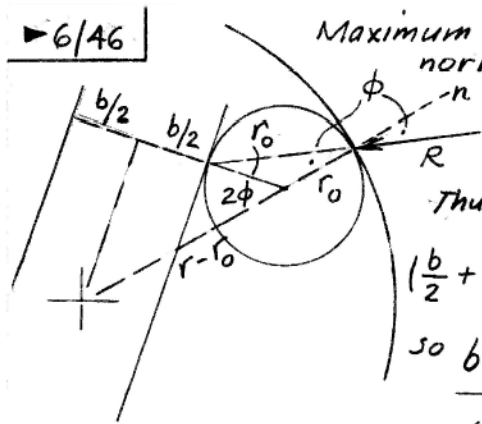
$$\text{With } a = 2", \quad b = \frac{2}{\sqrt{3}}", \quad c = 4.5" \text{ and } h = 30",$$

$$" = 0.1264$$



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► 6/46



Maximum angle between R and normal is the friction angle

$$\phi = \tan^{-1} \mu$$

so  $\phi$  increases as b decreases

Thus min. value of b is given by

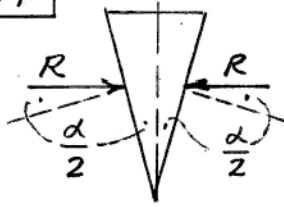
$$\left(\frac{b}{2} + r_0\right) / \cos 2\phi + r_0 = r$$

$$\text{so } b = \frac{2[(r - r_0) \cos 2\phi - r_0]}{\cos 2\phi}$$

where  $\phi = \tan^{-1} \mu$



6/47



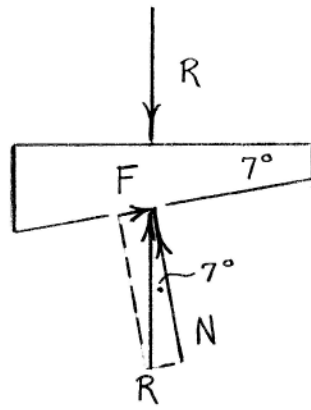
Critical  $\alpha$  occurs when

$$\frac{\alpha}{2} = \phi = \tan^{-1} \mu$$

$$\text{so } \alpha = 2 \tan^{-1} 0.20$$

$$= 2(11.31^\circ) = \underline{22.6^\circ}$$

6/48 |



$$\frac{F}{N} = \mu_s = \tan 7^\circ$$

$$\mu_s = 0.1228$$

$$\underline{6/49} \quad \text{Helix angle } \alpha = \tan^{-1} \frac{24}{40\pi} = 10.81^\circ$$

$$\text{Friction angle } \phi = \tan^{-1} \mu = \tan^{-1} 0.15 = 8.53^\circ$$

$\alpha > \phi$  so screw is not self-locking.

$$\alpha + \phi = 19.34^\circ ; \quad \alpha - \phi = 2.28^\circ$$

$$(a) \quad M = P r \tan(\alpha - \phi) : 60 = P (0.020) \tan 2.28^\circ$$

$$P = 75\,300 \text{ N or } \underline{75.3 \text{ kN}}$$

$$(b) \quad M = P r \tan(\alpha + \phi) : 60 = P (0.020) \tan 19.34^\circ$$

$$P = 8550 \text{ N or } \underline{8.55 \text{ kN}}$$

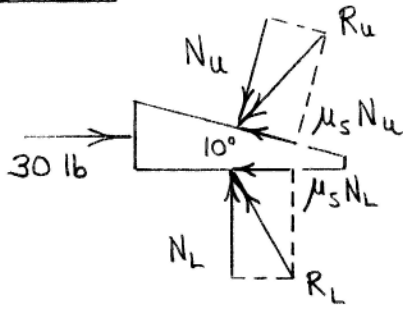
6/50 Friction angle  $\phi = \tan^{-1}\mu = \tan^{-1}0.15 = 8.53^\circ$

$\tan \alpha = \frac{L}{2\pi r}$  ; Critical when  $\alpha = \phi$

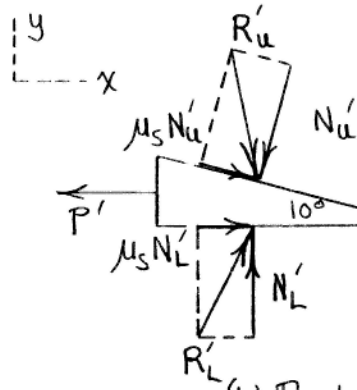
$\therefore$  Lead  $L = 2\pi r \tan \phi = 2\pi \frac{3/8}{2} \tan 8.53^\circ$   
 $= 0.1767$  in. per revolution

$N = 1/L = 1/0.1767 = \underline{5.66 \text{ threads per inch}}$

6/51



(a) Insertion



(b) Retraction

$$(a) \sum F_x = 0 : 30 - N_u \sin 10^\circ - \mu_s N_u \cos 10^\circ - \mu_s N_L = 0$$

$$\sum F_y = 0 : -N_u \cos 10^\circ + \mu_s N_u \sin 10^\circ + N_L = 0$$

$$\text{Solution: } \underline{N_u = 53.5 \text{ lb}}, \quad \underline{N_L = 50.8 \text{ lb}}$$

(b) Primes denote new values of  $N_u$  and  $N_L$ .

There are now 3 unknowns ( $P'$ ,  $N_u'$ , and  $N_L'$ ), so we cannot solve for  $P'$  without more information.

6/52

$$\text{Helix angle } \alpha = \tan^{-1} \frac{l}{\pi d} = \tan^{-1} \frac{1/12}{\pi (3/8)} = 4.05^\circ$$

$$\text{Friction angle } \phi = \tan^{-1} \mu_s = \tan^{-1} (0.20) = 11.31^\circ$$

$$\text{Tighten: } M = Pr \tan (\alpha + \phi) = 80 \frac{3/8}{2} \tan (4.05^\circ + 11.31^\circ)$$

$$= \underline{4.12 \text{ lb-in.}}$$

$$\text{Loosen: } M' = Pr \tan (\phi - \alpha) = 80 \frac{3/8}{2} \tan (11.31^\circ - 4.05^\circ)$$

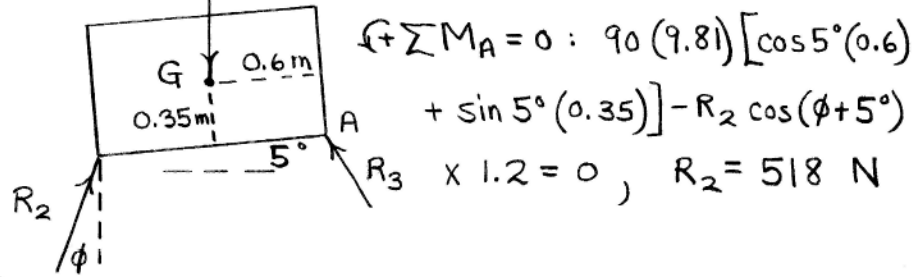
$$= \underline{1.912 \text{ lb-in.}} \quad \left( \begin{array}{l} \text{in direction opposite} \\ \text{to that of } M \end{array} \right)$$

Note:  $\alpha < \phi$ , so screw is self-locking (a good feature for a clamp!)

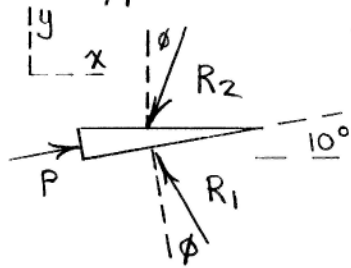
6/53

90(9.81) N

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.4 = 21.8^\circ$$



$$\begin{aligned} \sum M_A = 0: & 90(9.81) [\cos 5^\circ (0.6) \\ & + \sin 5^\circ (0.35)] - R_2 \cos(\phi + 5^\circ) \\ & \times 1.2 = 0, \quad R_2 = 518 \text{ N} \end{aligned}$$



$$\begin{aligned} \sum F_x = 0: & P \cos 10^\circ - 518 \sin \phi \\ & - R_1 \sin(\phi + 10^\circ) = 0 \end{aligned}$$

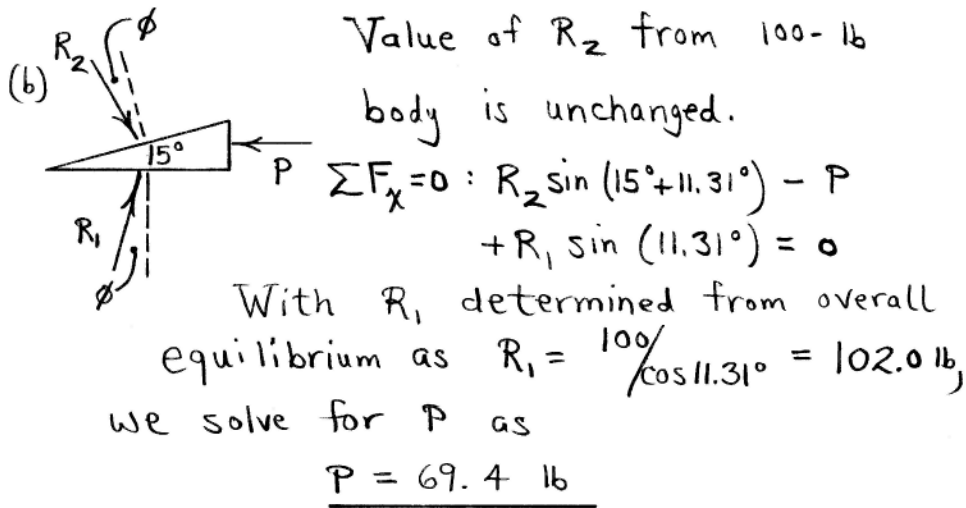
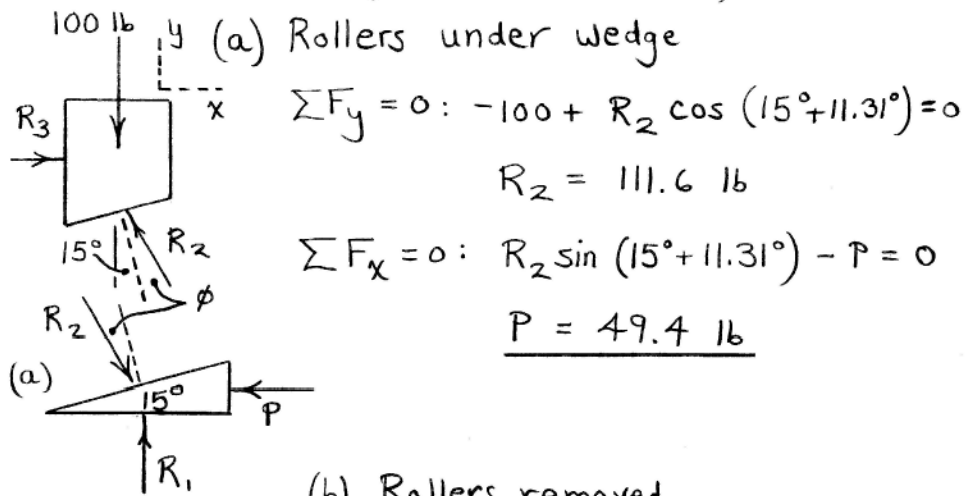
$$\begin{aligned} \sum F_y = 0: & P \sin 10^\circ - 518 \cos \phi \\ & + R_1 \cos(\phi + 10^\circ) = 0 \end{aligned}$$

Solve simultaneously to obtain

$$R_1 = 471 \text{ N}$$

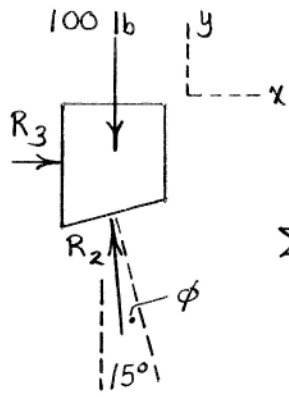
$$P = \underline{449 \text{ N}}$$

6/54 | Friction angle  $\phi = \tan^{-1}(0.2) = 11.31^\circ$





6/55 | Friction angle  $\phi = \tan^{-1}(0.2) = 11.31^\circ$



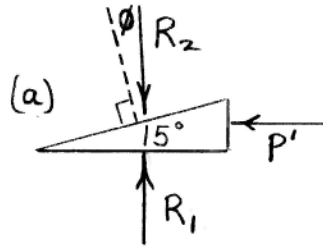
(a) Rollers under wedge

$$\sum F_y = 0: -100 + R_2 \cos(15^\circ - 11.31^\circ) = 0$$

$$R_2 = 100.2 \text{ lb}$$

$$\sum F_x = 0: R_2 \sin(15^\circ - 11.31^\circ) - P' = 0$$

$$P' = 6.45 \text{ lb (to the left)}$$



(b) Rollers removed

$$R_2 = 100.2 \text{ lb, from (a).}$$

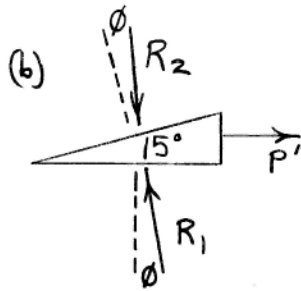
From overall equilibrium,

$$R_1 = \frac{100}{\cos 11.31^\circ} = 102.0 \text{ lb}$$

$$\sum F_x = 0: R_2 \sin(15^\circ - 11.31^\circ) + P' - R_1 \sin(11.31^\circ) = 0$$

$$-R_1 \sin(11.31^\circ) = 0$$

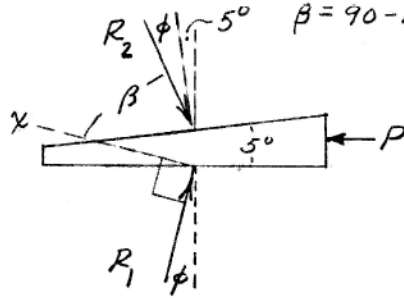
$$P' = 13.55 \text{ lb (to the right)}$$



6/56

$$\phi = \tan^{-1} 0.40 = 21.80^\circ$$

$$\beta = 90 - 2\phi - 5^\circ = 41.40^\circ$$



Column & wedge

$$\Sigma F_{\text{vert.}} = 0$$

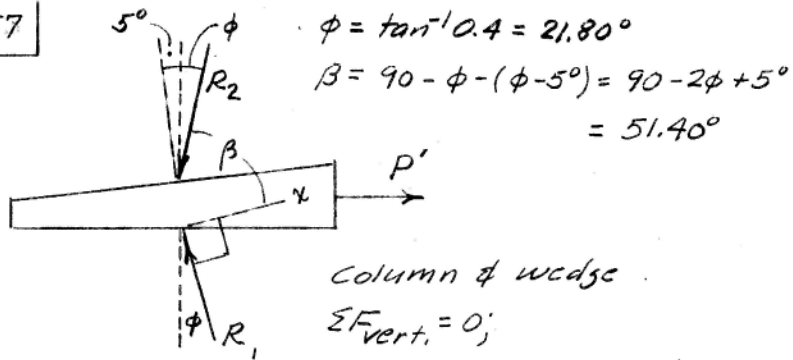
$$R_2 \cos(21.80 + 5)^\circ = 5 \text{ kN}$$

$$R_2 = \frac{5}{\cos 26.80^\circ} = 5.60 \text{ kN}$$

$$\text{Wedge: } \Sigma F_x = 0; P \cos 21.80^\circ - 5.60 \cos 41.40^\circ = 0$$

$$P = 5.60 (0.7501) / 0.9285 = \underline{4.53 \text{ kN}}$$

6/57



Column & wedge

$$\Sigma F_{\text{vert.}} = 0;$$

$$R_2 \cos(21.80 - 5)^\circ = 5 \text{ kN}$$

$$R_2 = \frac{5}{\cos 16.80^\circ} = 5.22 \text{ kN}$$

$$\text{Wedge; } \Sigma F_x = 0; P' \cos 21.80 - 5.22 \cos 51.40 = 0$$

$$P' = 5.22(0.6239) / 0.9285 = \underline{3.51 \text{ kN}}$$

6/58 | Helix angle:  $\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{1/5}{2\pi \left(\frac{1.150}{2}\right)} = 3.17^\circ$

Friction angle  $\phi = \tan^{-1} \mu = \tan^{-1}(0.25) = 14.04^\circ$

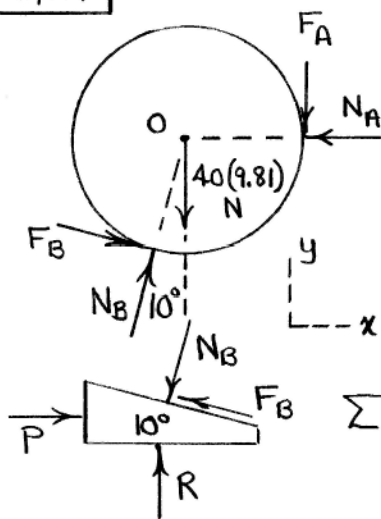
(a) To tighten,  $M_a = 2Tr \tan(\alpha + \phi)$

$M_a = 2(10,000) \frac{1.150}{2} \tan(3.17^\circ + 14.04^\circ) = \underline{3560 \text{ lb-in.}}$

(b) To loosen,  $M_b = 2Tr \tan(\phi - \alpha)$

$M_b = 2(10,000) \frac{1.150}{2} \tan(14.04^\circ - 3.17^\circ) = \underline{2210 \text{ lb-in.}}$

6/59



By inspection  $N_A < N_B$ ,  
so slipping occurs first at  
A. Thus  $F_A = 0.25 N_A$

Cylinder:  $\sum M_o = 0$ :

$$F_B r - F_A r = 0, \quad F_B = F_A = 0.25 N_A$$

$$\sum F_y = 0: \quad N_B \cos 10^\circ - F_A \sin 10^\circ - 40(9.81) - F_A = 0$$

$$\text{or } N_B \cos 10^\circ - (0.25 N_A) \sin 10^\circ - 40(9.81) - 0.25 N_A = 0 \quad (a)$$

$$\sum F_x = 0: \quad N_B \sin 10^\circ + F_B \cos 10^\circ - N_A = 0$$

$$\text{or } N_B \sin 10^\circ + (0.25 N_A) \cos 10^\circ - N_A = 0 \quad (b)$$

$$\text{Solve (a) \& (b): } N_A = 98.6 \text{ N}, \quad N_B = 428 \text{ N}; \quad \underline{F_B = F_A = 24.6 \text{ N}}$$

$$\text{Wedge: } \sum F_x = 0: \quad P - 24.6 \cos 10^\circ - 428 \sin 10^\circ = 0, \quad \underline{P = 98.6 \text{ N}}$$

$$\frac{6}{60} \quad M = Tr \tan(\alpha + \phi) + Tr \tan(\alpha - \phi)$$

$$\text{where } \alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{4}{2\pi(16/2)} = 4.55^\circ$$

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.24 = 13.50^\circ$$

$$\alpha + \phi = 18.05^\circ, \quad \phi - \alpha = 13.50 - 4.55 = 8.95^\circ$$

$$\text{So } M = 8000 \left(\frac{16}{2}\right) [\tan 18.05^\circ + \tan 8.95^\circ]$$

$$= 8000(8)(0.3258 + 0.1574) = 30\,926 \text{ N}\cdot\text{mm}$$

$$\text{or } \underline{M = 30.9 \text{ N}\cdot\text{m}} \quad (\text{Same for both directions})$$

6/61 | For the screw,

$$\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{6}{20\pi} = 5.45^\circ$$
$$\phi = \tan^{-1}(0.25) = 14.04^\circ$$

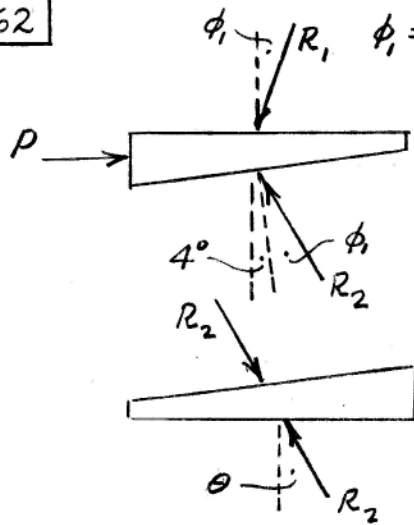
Eq. 6/3:  $M = Wr \tan(\alpha + \phi)$

$$24 = P \frac{20/2}{1000} \tan(5.45^\circ + 14.04^\circ)$$
$$P = 6780 \text{ N (to remove collar)}$$

Collar:  $\mu p A = P : 0.30p (0.050\pi \cdot 0.060) = 6780$

$$p = 2.40 (10^6) \text{ Pa or } \underline{2400 \text{ kPa}}$$

6/62



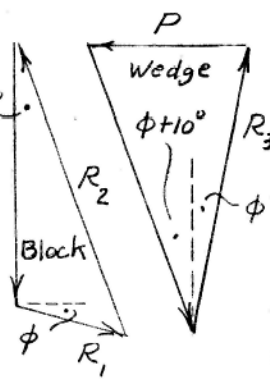
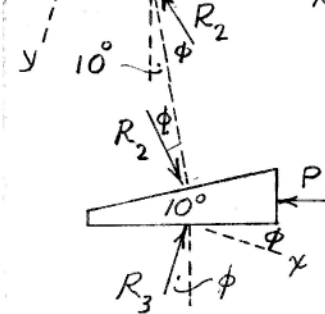
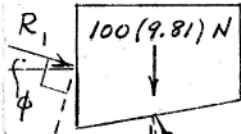
$$\phi_1 = \tan^{-1} 0.3 = 16.70^\circ$$

In order for column to be raised, bottom wedge must not move. Therefore  $\mu_2$  must be greater than  $\tan \theta$  or

$$(\mu_2)_{\min} = \tan (4 + 16.70)$$
$$= \underline{0.378}$$



6/63  $\phi = \tan^{-1} 0.40 = 21.80^\circ$



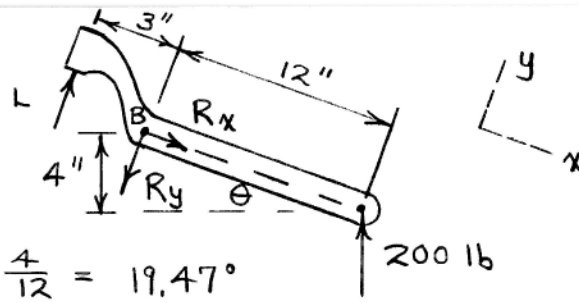
$2\phi + 10^\circ = 53.60^\circ$   
 $90^\circ - (2\phi + 10^\circ) = 36.40^\circ$

Block:  $\Sigma F_y = 0;$   
 $981 \cos 21.80^\circ = R_2 \cos 53.60^\circ$   
 $R_2 = 1535 \text{ N}$   
 Wedge:  $\Sigma F_x = 0;$   
 $1535 \cos 36.40^\circ = P \cos 21.80^\circ$   
 $P = 1331 \text{ N}$

Screw:  $\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{10}{2\pi(15)} = 6.06^\circ; \phi = \tan^{-1} 0.25 = 14.04^\circ$   
 $\phi + \alpha = 20.09^\circ$

$M = Pr \tan(\phi + \alpha) = 1331(0.015) \tan 20.09^\circ = \underline{7.30 \text{ N}\cdot\text{m}}$

6/64



$$\theta = \sin^{-1} \frac{4}{12} = 19.47^\circ$$

$$\sum M_B = 0: 200 \cos 19.47^\circ (12) - 3L = 0, \quad L = 754 \text{ lb}$$

$$\text{Screw: } \begin{cases} \text{helix angle } \alpha = \tan^{-1} \frac{1/12}{\pi(1/2)} = 3.04^\circ \\ \text{friction angle } \phi = \tan^{-1}(0.20) = 11.31^\circ \end{cases}$$

$$\begin{aligned} \text{Tighten screw: } M &= Lr \tan(\phi + \alpha) \\ &= 754 (0.25) \tan(11.31^\circ + 3.04^\circ) = 48.2 \text{ lb-in.} \end{aligned}$$

$$\begin{aligned} \text{Loosen screw: } M' &= Lr \tan(\phi - \alpha) \\ &= 754 (0.25) \tan(11.31^\circ - 3.04^\circ) = 27.4 \text{ lb-in.} \end{aligned}$$

► 6/65 | For equil. of screw (refer to prob. illust.)

$$\Sigma F = 0; W = \Sigma R, \cos(\alpha + \gamma) = \cos(\alpha + \gamma) \Sigma R,$$

$$\Sigma M = 0; M = \Sigma R, r \sin(\alpha + \gamma) = r \sin(\alpha + \gamma) \Sigma R,$$

$$\text{Combine \& set } M = Wr \tan(\alpha + \gamma) = Wr \frac{\tan \alpha + \tan \gamma}{1 - \tan \alpha \tan \gamma}$$

$$\text{But } \tan \gamma = \frac{R \sin \phi}{R \cos \phi \cos \beta/2} = \mu / \cos \beta/2$$

$$\& \tan \beta/2 = \frac{L}{2h} \cos \alpha, \tan \frac{\theta}{2} = \frac{L}{2h}, \text{ so } \tan \frac{\beta}{2} = \tan \frac{\theta}{2} \cos \alpha$$

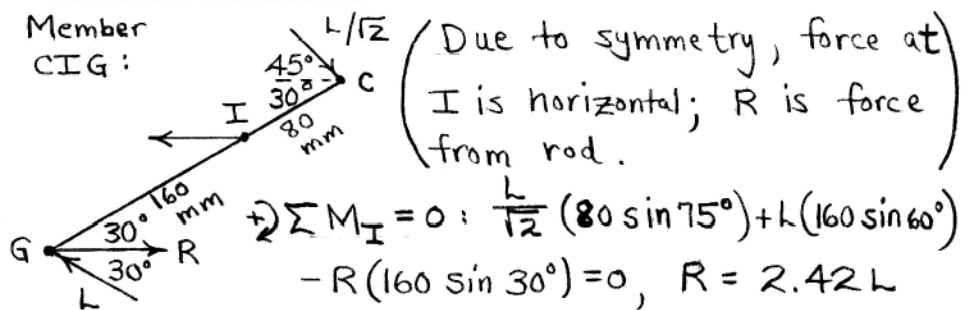
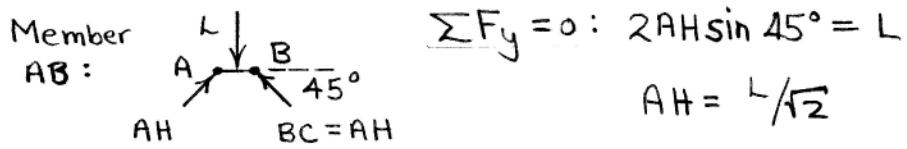
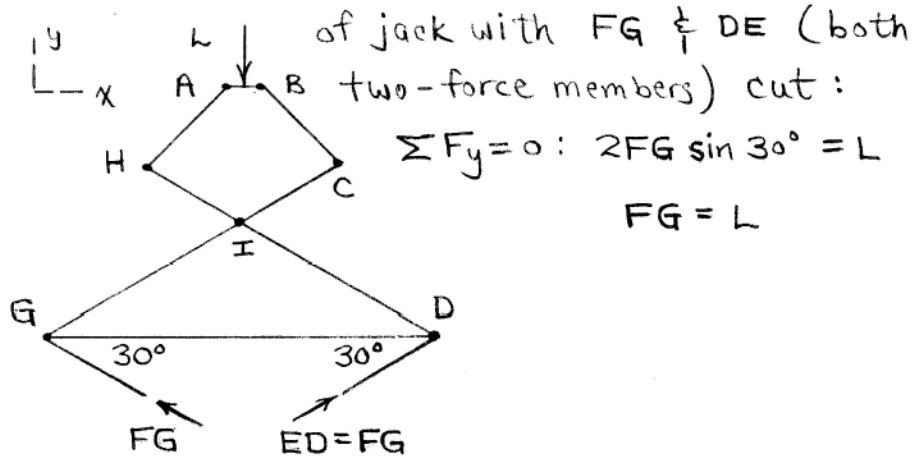
$$\& \cos \frac{\theta}{2} = 1 / \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}. \text{ Thus } \tan \gamma = \mu \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}$$

Hence

$$M = Wr \frac{\tan \alpha + \mu \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}}{1 - \mu \tan \alpha \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}}$$

$$\text{where } \tan \alpha = \frac{L}{2\pi r}$$

6/66 | Calculate force in member FG: FBD of jack with FG & DE (both two-force members) cut:



Friction angle  $\phi = \tan^{-1} \mu = \tan^{-1} (0.20) = 11.31^\circ$

$\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{0.003}{2\pi (0.010/2)} = 5.45^\circ$

To raise load:  $M = Rr \tan(\alpha + \phi)$

$= [2.42(1000)(9.81)] \left[ \frac{0.010}{2} \right] \tan(11.31^\circ + 5.45^\circ)$

$= \underline{35.7 \text{ N}\cdot\text{m}}$

To lower load:  $M' = Rr \tan(\phi - \alpha)$

$= [2.42(1000)(9.81)] \left[ \frac{0.010}{2} \right] \tan(11.31^\circ - 5.45^\circ)$

$= \underline{12.15 \text{ N}\cdot\text{m}}$

$$\underline{6/67} \quad M = \frac{2}{3} \mu PR$$

$$A \text{ on } B: M = \frac{2}{3} (0.40)(80)\left(\frac{9}{2}\right) = \underline{96.0 \text{ lb-in.}}$$

$$B \text{ on } C: 96.0 = \frac{2}{3} \mu (80)\left(\frac{12}{2}\right), \quad \underline{\mu = 0.300}$$

$$\frac{6}{68} \mid M = Rr \sin \phi, \sin \phi = \frac{M}{Rr} = \frac{3}{2(40)(9.81)(0.040/2)}$$
$$\phi = 11.02^\circ$$

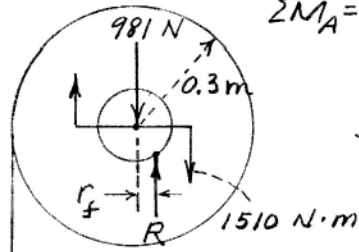
$$\mu = \tan \phi = \underline{0.1947}$$

$$r_f = r \sin \phi = \frac{0.040}{2} \sin 11.02^\circ = 0.00382 \text{ m}$$

$$\text{or } \underline{r_f = 3.82 \text{ mm}}$$

6/69

$$r_f = r \sin \phi = 0.025 \sin \phi$$



$$\sum M_A = 0; 1510 - 981(0.025 \sin \phi)$$

$$-500(9.81)(0.3 + 0.025 \sin \phi) = 0$$

$$\sin \phi = 0.2616, \quad \phi = 15.17^\circ$$

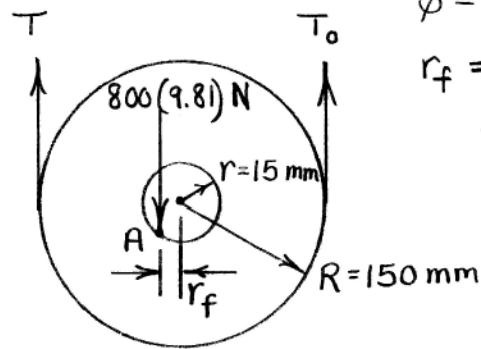
$$\mu = \tan \phi = \underline{0.271}$$

$$500(9.81) \text{ N}$$

$$(\text{using } r_f \approx r \phi, \quad \phi = 15.0^\circ)$$

$$\mu = \tan \phi = \underline{0.268}$$

6/70



$$\phi = \tan^{-1} 0.25 = 14.04^\circ$$

$$r_f = r \sin \phi = 15 \sin 14.04 \\ = 3.64 \text{ mm}$$

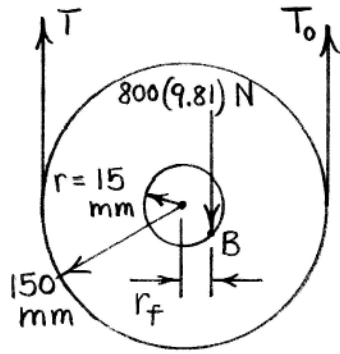
$$+\uparrow \sum F = 0 : T + T_0 - 800(9.81) = 0 \quad (1)$$

$$\curvearrowright \sum M_A = 0 : T(150 - 3.64) - T_0(150 + 3.64) = 0 \quad (2)$$

$$\text{Solve (1) \& (2) : } \begin{cases} T = 4020 \text{ N} \\ T_0 = \underline{3830 \text{ N}} \end{cases}$$



6/71



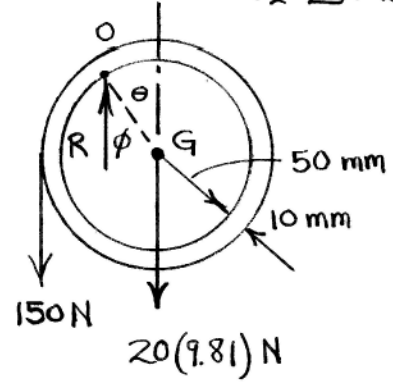
From the solution to  
Prob. 6/70,  $r_f = 3.64 \text{ mm}$

$$\uparrow \sum F = 0: T + T_0 - 800(9.81) = 0 \quad (1)$$

$$\curvearrowright \sum M_B = 0: T(150 + 3.64) - T_0(150 - 3.64) = 0 \quad (2)$$

$$\text{Solve (1) \& (2): } \begin{cases} T = 3830 \text{ N} \\ T_0 = 4020 \text{ N} \end{cases}$$

6/72



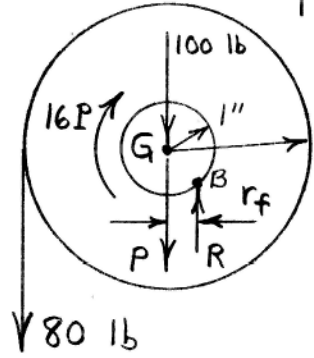
$$\sum M_o = 0: 20(9.81)(50 \sin \phi)$$

$$- 150(60 - 50 \sin \phi) = 0$$

$$\phi = \theta = \underline{31.3^\circ}$$

$$\mu = \tan \phi = \underline{0.609}$$

6/73 | FBD of shaft and attached drum, with force P replaced by a force-couple system at G:



(a)  $r_f = 0$

$$\sum M_G = 0: 80(10) - 16P = 0$$

$$P = \underline{50 \text{ lb}}$$

(b)  $\phi = \tan^{-1}(0.2) = 11.31^\circ$

$$r_f = r \sin \phi = 1 \sin 11.31^\circ = 0.1961 \text{ in.}$$

$$\sum M_B = 0: 80(10 + 0.1961) + 100(0.1961)$$

$$+ P(0.1961) - 16P = 0$$

$$P = \underline{52.9 \text{ lb}}$$

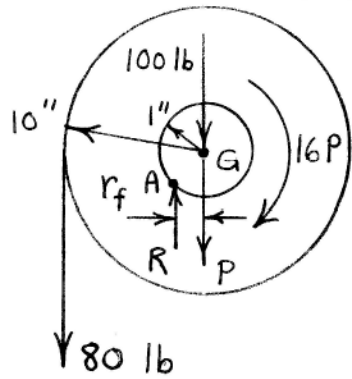
(This solution assumes that the bearing reaction can be represented by a single force R as shown above.)

6/74 |  $r_f = 0.1961$  in. (from Prob. 6/73)

$$\sum M_A = 0: 80(10 - 0.1961) - 16P$$

$$-(100 + P)(0.1961) = 0$$

$$\underline{P = 47.2 \text{ lb}}$$



The values are NOT

$$\text{Symmetric: } \begin{cases} P_{\text{down}} = 47.215 \text{ lb} \\ P_{\text{n.f.}} = 50 \text{ lb} \\ P_{\text{up}} = 52.854 \text{ lb} \end{cases}$$

$$\underline{6/75} \quad dM = (\mu p dA) r \quad \text{where } p = k/r^2$$

$$M = \int_0^{2\pi} \int_{r_i}^{r_o} \mu p r (r dr d\theta) = 2\pi \mu k \int_{r_i}^{r_o} dr = 2\pi \mu k (r_o - r_i)$$

$$\text{Also } L = \int p dA = \int_0^{2\pi} \int_{r_i}^{r_o} \frac{k}{r^2} r dr d\theta = 2\pi k \ln r \Big|_{r_i}^{r_o}$$

$$\text{or } L = 2\pi k \ln \frac{r_o}{r_i}, \quad 2\pi k = \frac{L}{\ln r_o/r_i}$$

$$\text{Thus } M = \mu L \frac{r_o - r_i}{\ln(r_o/r_i)}$$

---

6/76 For constant pressure Eq. 6/5a gives

$$M = \frac{2}{3} \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{2}{3} (0.35)(1) \frac{(150)^3 - (75)^3}{(150)^2 - (75)^2} = 40.8 \text{ N}\cdot\text{m}$$

For wheel  $\Sigma M = 0$ ;  $F(0.3) - 40.8 = 0$ ,  $F = \underline{136.1 \text{ N}}$

6/77 | Force of shaft on bearing is

$$R = \sqrt{W^2 + T^2}$$

$$\sum M_o = 0: Tr_o - Rr_f = 0$$

$$T = \frac{r_f}{r_o} \sqrt{T^2 + W^2}$$

where  $r_f = r \sin \phi$  and  $\phi = \tan^{-1} \mu$

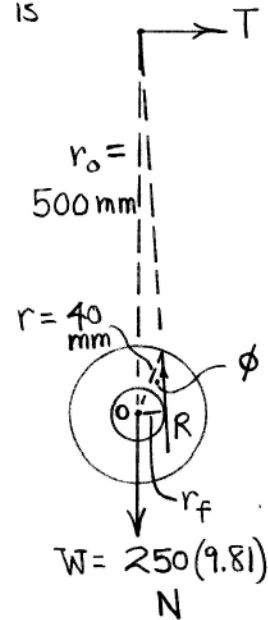
$$\text{Thus } T^2 = \frac{r^2 \sin^2 \phi}{r_o^2} (T^2 + W^2)$$

$$T = \frac{Wr \sin \phi}{\sqrt{r_o^2 - r^2 \sin^2 \phi}}$$

$$\phi = \tan^{-1}(0.30) = 16.70^\circ$$

$$\sin \phi = 0.287$$

$$\text{So } T = \frac{250(9.81)(0.040)(0.287)}{\sqrt{0.5^2 - 0.040^2 (0.287)^2}} = \underline{56.4 \text{ N}}$$

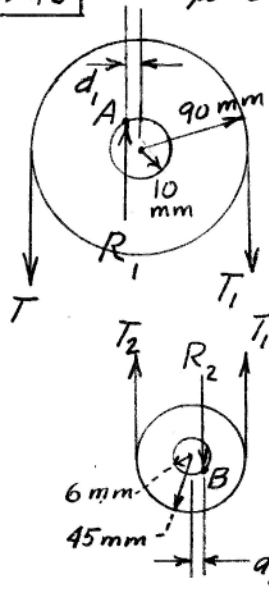


6/78

$$\mu = 0.25; \phi = \tan^{-1} 0.25 = 14.04^\circ$$

$$d_1 = r_0 \sin \phi = 10 \sin 14.04^\circ = 2.43 \text{ mm}$$

$$d_2 = 6 \sin 14.04^\circ = 1.455 \text{ mm}$$



$$\sum M_A = 0; T(90 - 2.43) - T_1(90 + 2.43) = 0$$

$$T = \frac{92.43}{87.57} T_1 = 1.055 T_1$$

$$\sum M_B = 0; T_1(45 - 1.456) - T_2(45 + 1.456) = 0$$

$$T_1 = \frac{46.455}{43.54} T_2 = 1.067 T_2$$

$$\sum F = 0; T_1 + T_2 - R_2 = 0, T_1 + T_2 = 200(9.81)$$

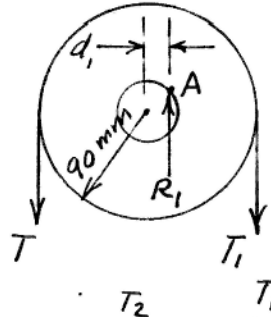
$$\text{Combine \& get } 2.067 T_2 = 1962$$

$$\text{so } T_2 = \underline{949 \text{ N}}, T_1 = \underline{1013 \text{ N}}$$

$$T = 1.055(1013) = \underline{1069 \text{ N}}$$



6/79 | From Prob. 6/78,  $d_1 = 2.43 \text{ mm}$ ,  $d_2 = 1.455 \text{ mm}$   
 $R_2 = 200(9.81) = 1962 \text{ N}$



$$\sum M_A = 0; T_1(90 - 2.43) - T(90 + 2.43) = 0$$

$$T = \frac{87.57}{92.43} T_1 = 0.948 T_1$$

$$\sum M_B = 0; T_2(45 - 1.455) - T_1(45 + 1.455) = 0$$

$$T_1 = \frac{43.54}{46.455} T_2 = 0.937 T_2$$

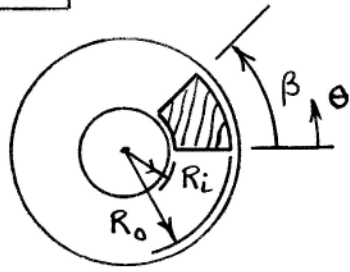
$$\sum F = 0; T_1 + T_2 - 1962 = 0$$

combine & get

$$1.937 T_2 = 1962, T_2 = \frac{1013 \text{ N}}{1}$$

$$T_1 = \frac{949 \text{ N}}{1}$$

$$T = 0.948(949) = \underline{899 \text{ N}}$$



$$P = pA = p \int_0^{\beta} \int_{R_i}^{R_o} r dr d\theta$$

$$= \frac{p}{2} \int_0^{\beta} (R_o^2 - R_i^2) d\theta$$

$$= \frac{p}{2} (R_o^2 - R_i^2) \beta$$

$$M = 2 \int \mu p r dA = 2 \mu p \int_0^{\beta} \int_{R_i}^{R_o} r^2 dr d\theta$$

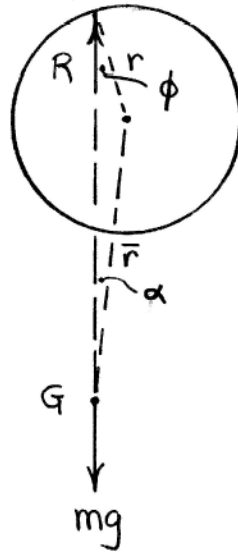
$$= \frac{2 \mu p}{3} (R_o^3 - R_i^3) \beta$$

$$= \frac{2 \mu}{3} \frac{2 p}{(R_o^2 - R_i^2) \beta} (R_o^3 - R_i^3) \beta$$

$$= \frac{4 \mu p}{3} \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

Same form as Eq. 6/5a except for factor of 2 for 2 pads. No  $\beta$  dependence. Pressure variation with  $\theta$  would not change the moment  $M$ .

6/81



$$r \sin \phi = \bar{r} \sin \alpha$$

$$\mu = \tan \phi = \frac{\bar{r} \sin \alpha}{\sqrt{r^2 - \bar{r}^2 \sin^2 \alpha}}$$

or

$$\mu = \frac{1}{\sqrt{\left(\frac{d/2}{\bar{r} \sin \alpha}\right)^2 - 1}}$$

$$\underline{6/82} \quad p = p_0 \left(1 - \frac{r}{2a}\right); \quad dA = 2\pi r dr$$

$$L = \int p dA = \int_0^a p_0 \left(1 - \frac{r}{2a}\right) 2\pi r dr = 2\pi p_0 \left[ \frac{r^2}{2} - \frac{r^3}{6a} \right]_0^a$$

$$= \frac{2}{3} \pi p_0 a^2 \quad \text{so} \quad p_0 = \frac{3L}{2\pi a^2}$$

$$M = \int \mu p r dA = \int_0^a \mu p_0 \left(r - \frac{r^2}{2a}\right) 2\pi r dr = 2\pi \mu p_0 \left[ \frac{r^3}{3} - \frac{r^4}{8a} \right]_0^a$$

$$= \frac{5}{12} \pi \mu p_0 a^3 = \underline{\underline{\frac{5}{8} \mu L a}}$$

$$6/83 \quad L = \frac{1}{4} (960 - 4 \times 40) = 200 \text{ lb}$$

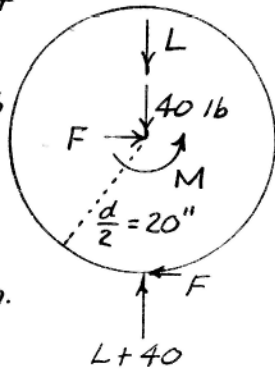
$$P = 4F$$

$$F = \frac{1}{4} 16 = 4 \text{ lb}$$

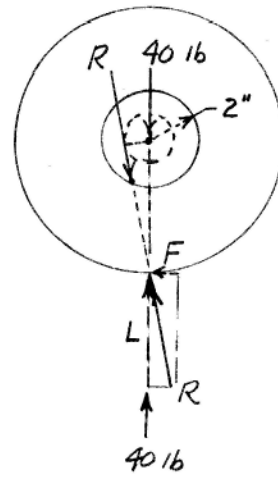
$$M = F \frac{d}{2}$$

$$= 4(20)$$

$$= 80 \text{ lb-in.}$$



≡



$$M = Rr \sin \phi \approx Lr \sin \phi$$

$$80 \approx 200(2) \sin \phi$$

$$\phi = \sin^{-1} 0.2 = 11.54^\circ, \quad \mu = \tan \phi = \underline{0.204}$$

$$\frac{6}{84} \quad \alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{11}{2\pi \frac{120}{2}} = 1.671^\circ$$

$$\phi = \tan^{-1} 0.15 = 8.53^\circ$$

Screw: (a) Raise :  $M_s = Wr \tan(\alpha + \phi)$   
 $= \frac{1}{2} (10+3)(9.81) \frac{120}{2} \tan(1.671^\circ + 8.53^\circ) = 689 \text{ N}\cdot\text{m}$

(b) Lower :  $M_s = Wr \tan(\phi - \alpha)$   
 $= \frac{1}{2} (10+3)(9.81) \frac{120}{2} \tan(8.53^\circ - 1.671^\circ) = 460 \text{ N}\cdot\text{m}$

Collar bearing :  $M_c = \frac{2}{3} \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$   
 $= \frac{2}{3} (0.15) \left( \frac{10+3}{2} + 0.9 \right) (9.81) \frac{(250/2)^3 - (125/2)^3}{(250/2)^2 - (125/2)^2}$   
 $= 1059 \text{ N}\cdot\text{m}$

Total moment per screw  $\left\{ \begin{array}{l} \text{(a)} \quad M = 689 + 1059 = \underline{1747 \text{ N}\cdot\text{m}} \\ \text{(b)} \quad M = 460 + 1059 = \underline{1519 \text{ N}\cdot\text{m}} \end{array} \right.$

6/85

$$dM = \mu p dA \times r \cos \alpha$$
$$= \mu (k \sin \alpha) (r d\alpha \cdot 2\pi r \cos \alpha) (r \cos \alpha)$$

$$M = 2\pi \mu k r^3 \int_0^{\pi/2} \sin \alpha \cos^2 \alpha d\alpha$$

$$= 2\pi \mu k r^3 \left[ -\frac{\cos^3 \alpha}{3} \right]_0^{\pi/2} = \frac{2}{3} \pi \mu k r^3$$

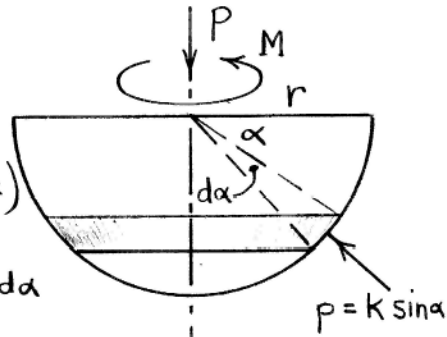
$$\Sigma F = 0 \Rightarrow P = \int p \sin \alpha dA$$

$$P = \int_0^{\pi/2} (k \sin^2 \alpha) (r d\alpha \cdot 2\pi r \cos \alpha)$$

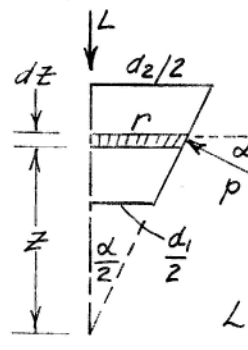
$$= 2\pi k r^2 \int_0^{\pi/2} \sin^2 \alpha \cos \alpha d\alpha = 2\pi k r^2 \left[ \frac{\sin^3 \alpha}{3} \right]_0^{\pi/2}$$

$$= \frac{2}{3} \pi k r^2$$

$$\text{So } \underline{M = \mu P r}$$



6/86 |  $dL = p(2\pi r) ds \sin \frac{\alpha}{2}$  where  $ds = dz / \cos \frac{\alpha}{2}$



$$dL = 2\pi p \sin \frac{\alpha}{2} \left( z \tan \frac{\alpha}{2} \right) \frac{dz}{\cos \frac{\alpha}{2}} \left[ dz \sqrt{\frac{dz}{ds}} \right]^{\frac{\alpha}{2}}$$

$$= 2\pi p \tan^2 \frac{\alpha}{2} z dz$$

$$L = 2\pi p \tan^2 \frac{\alpha}{2} \int_{z_1}^{z_2} z dz \quad \text{where} \quad z_1 = \frac{d_1/2}{\tan \alpha/2}$$

$$z_2 = \frac{d_2/2}{\tan \alpha/2}$$

$$L = \frac{\pi p (d_2^2 - d_1^2)}{4}$$

$$M = \int r \mu p dA = \mu p \int_{z_1}^{z_2} \left( z \tan \frac{\alpha}{2} \right) \left( 2\pi z \tan \frac{\alpha}{2} \right) \frac{dz}{\cos \frac{\alpha}{2}}$$

$$= 2\pi \mu p \frac{\tan^2 \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \int_{z_1}^{z_2} z^2 dz = 2\pi \mu p \frac{1}{24 \sin \frac{\alpha}{2}} (d_2^3 - d_1^3)$$

$$M = \frac{\mu L}{3 \sin \frac{\alpha}{2}} \frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}$$



$$\frac{6187}{T_1} = e^{\mu\beta}, \quad \frac{100}{50} = e^{\pi\mu}, \quad \pi\mu = \ln 2$$
$$\mu = \frac{0.6931}{\pi} = \underline{0.221}$$

6/88 | Use  $T_2 = T_1 e^{\mu\beta}$

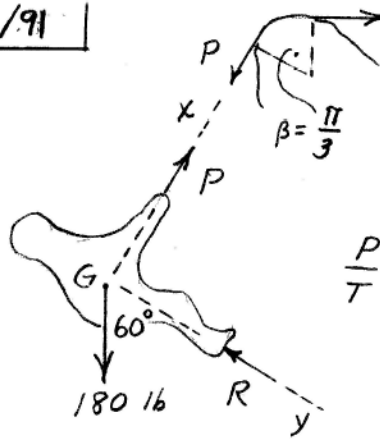
$$(a) \quad P = 40(9.81)e^{0.3(\pi)} = \underline{1007 \text{ N}}$$

$$(b) \quad 40(9.81) = P e^{0.3(\pi)}, \quad \underline{P = 152.9 \text{ N}}$$

$$\frac{6/89}{\frac{T_2}{T_1}} = e^{\mu\beta} : \frac{mg}{mg/10} = e^{\mu(3\pi)}, \quad \underline{\mu = 0.244}$$

$$\frac{6/90}{\rho} = e^{\mu\beta}, \quad \frac{mg}{mg/6} = e^{\mu(\frac{5}{4}2\pi)}$$
$$\frac{5}{2}\pi\mu = \ln 6 = 1.792, \quad \mu = \frac{2(1.792)}{5\pi} = \underline{0.228}$$

6/91



Equil. of climber,  $\Sigma F_x = 0$   
gives  $P = 180 \sin 60^\circ$

$$= 155.9 \text{ lb}$$

$$\frac{P}{T} = e^{\mu\beta}, \quad \frac{155.9}{75} = e^{\pi\mu/3}$$

$$\frac{\pi}{3}\mu = \ln 2.078 = 0.732$$

$$\mu = 3(0.732)/\pi = \underline{0.699}$$

$$\frac{6}{92} \quad T_2 = T_1 e^{\mu\beta}, \quad \beta = 2 \text{ turns} + 60^\circ$$

$$= 2(360^\circ) + 60^\circ = 780^\circ$$

$$\text{or } 13.61 \text{ rad}$$

$$T = \left(\frac{2}{16}\right) e^{0.7(13.61)} = \underline{1720 \text{ lb}}$$

$$\frac{6}{93} \quad \frac{4}{mg} = e^{\mu\beta}, \quad \frac{mg}{1.6} = e^{\mu\beta}$$

$$\text{Thus } \frac{4}{mg} = \frac{mg}{1.6}, \quad m^2 g^2 = 4(1.6)$$

$$m = \frac{\sqrt{4(1.6)}}{9.81} = 0.258 \text{ Mg} \quad \text{or} \quad \underline{m = 258 \text{ kg}}$$

6/94 | Use  $T_2 = T_1 e^{\mu\beta}$

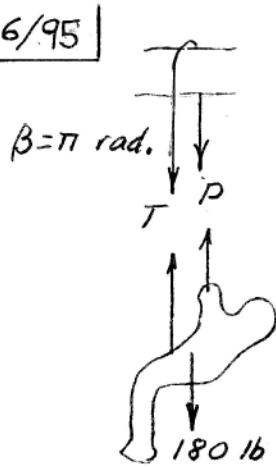
(a) Lower:  $50(9.81) = 70 e^{\mu\pi}$  ,  $\mu = 0.620$

(b) Raise:  $P' = 50(9.81) e^{0.620\pi}$

$P' = 3440 \text{ N}$  or  $\underline{P' = 3.44 \text{ kN}}$



6/95



$$T = P e^{\mu \beta}$$

$$= P e^{0.6\pi} = 6.59 P$$

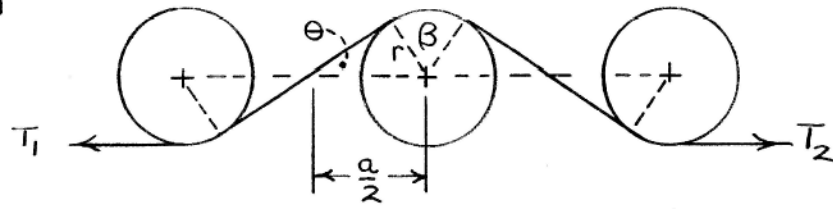
$$\text{Man; } \Sigma F = 0; T + P = 180$$

Combine & set

$$P(1 + 6.59) = 180 \text{ lb}$$

$$P = \frac{180}{7.59} = \underline{23.71 \text{ lb}}$$

6/96



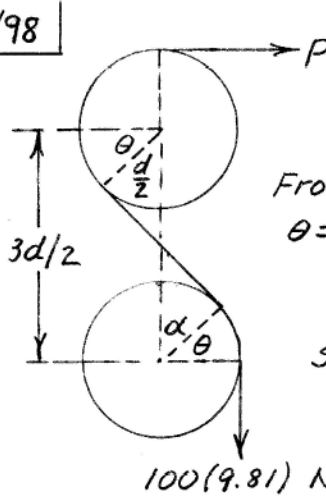
$$\theta = \sin^{-1} \frac{r}{a/2}, \quad \beta = 2\theta = 2 \sin^{-1} \frac{2r}{a}$$

$$\frac{T_2}{T_1} = e^{\mu\beta} : 1.15 = e^{0.1\beta}, \quad \beta = 1.398 \text{ rad or } 80.1^\circ$$

$$\text{Thus } 80.1^\circ = 2 \sin^{-1} \frac{2(20)}{a}, \quad \underline{a = 62.2 \text{ mm}}$$

6/97 | Normal force under section BC is  
 $N = 5(1.2)(9.81) = 58.9 \text{ N}$  and friction  
there is  $F = T_1 = \mu N = 0.5(58.9) = 29.4 \text{ N}$   
 $T_2 = T_1 e^{\mu\beta} = 29.4 e^{0.4(\pi/2)} = 55.2 \text{ N}$   
Neglecting effects of the hose mass from  
A to B,  $P_x = 55.2 \text{ N}$

6/98



$$\frac{P}{981} = e^{0.4\beta}$$

$$\text{where } \beta = \pi/2 + 2\theta$$

From geometry

$$\theta = \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \cos^{-1} \frac{d/2}{3d/4}$$

$$= 90^\circ - 48.19^\circ = 41.81^\circ$$

$$\text{So } \beta = 90 + 2(41.81) = 173.6^\circ$$

$$\text{or } \beta = \frac{173.6}{180} \pi = 3.03 \text{ rad}$$

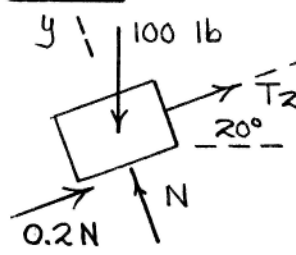
$$100(9.81) \text{ N}$$

$$\text{So } P = 981 e^{0.4(3.03)}$$

$$= 981 (3.36) = 3297 \text{ N}$$

$$\text{or } \underline{P = 3.30 \text{ kN}}$$

6/99 | Motion impending down plane:



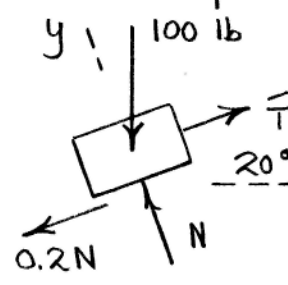
$$\sum F_y = 0 \Rightarrow N = 100 \cos 20^\circ = 94.0 \text{ N (throughout)}$$

$$\sum F_x = 0: T_2 - 100 \sin 20^\circ + 0.2(94.0) = 0$$

$$T_2 = 15.41 \text{ lb}$$

$$T_2 = T_1 e^{\mu\beta}: 15.41 = W e^{0.3(1.920)}, W = 8.66 \text{ lb}$$

Motion impending up plane:



$$\sum F_x = 0: T_1 - 100 \sin 20^\circ - 0.2(94.0) = 0$$

$$T_1 = 53.0 \text{ lb}$$

$$T_2 = T_1 e^{\mu\beta}: W = 53.0 e^{0.3(1.920)}$$

$$W = 94.3 \text{ lb}$$

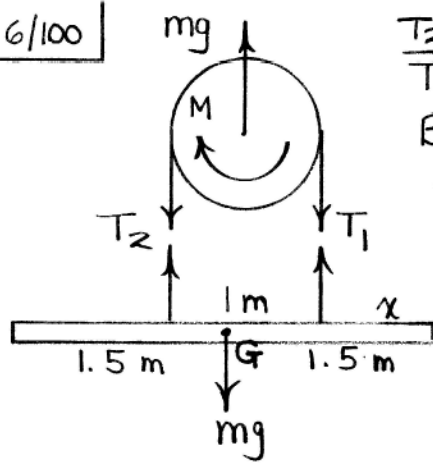
So range is  $8.66 \leq W \leq 94.3 \text{ lb}$

(Note that  $\beta = 90^\circ + 20^\circ = 110^\circ$  or  $1.920 \text{ rad}$ )



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6/100



$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.25\pi} = 2.19$$

Beam :  $\sum M_G = 0 :$

$$T_2 (1+x-1.5) = T_1 (1.5-x)$$

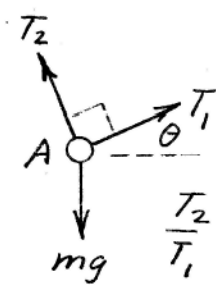
Combine to get

$$\frac{1.5-x}{x-0.5} = 2.19$$

$$\underline{x = 0.813 \text{ m}}$$

6/101 |  $L = 75(9.81) \text{ N}, T_2 = T_1 e^{\mu \theta}$   
 $75(9.81) \text{ N} \quad 75(9.81) - 10 = 10 e^{\mu(3 + \frac{1}{2})2\pi}$   
 $10 \text{ N}$   
 $72.6 = e^{21.99\mu}$   
 $21.99\mu = 4.285, \quad \underline{\mu = 0.195}$   
 $10 \text{ N}$

6/102



Equilibrium of A gives

$$T_1 = mg \sin \theta$$

$$T_2 = mg \cos \theta$$

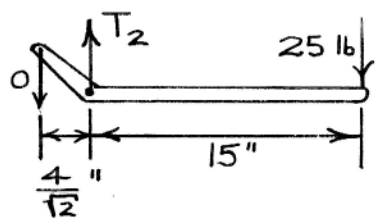
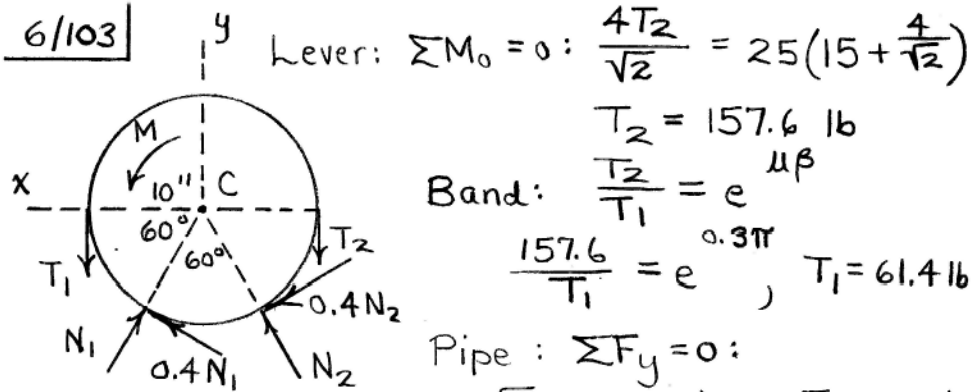
$$\frac{T_2}{T_1} = e^{\mu \beta} = \frac{mg \cos \theta}{mg \sin \theta} = e^{3\pi\mu/2}$$

$$\text{For } \theta = 20^\circ, \cot 20^\circ = e^{3\pi\mu/2} = 2.747$$

$$\text{or } \mu = \frac{2}{3\pi} \ln 2.747 = \underline{0.214}$$



6/103

Pipe:  $\sum F_y = 0:$ 

$$N_1 \left( \frac{\sqrt{3}}{2} + 0.4 \left( \frac{1}{2} \right) \right) + N_2 \left( \frac{\sqrt{3}}{2} - 0.4 \left( \frac{1}{2} \right) \right)$$

$$-157.6 - 61.4 = 0 \quad (a)$$

$$\sum F_x = 0: N_1 \left( \frac{1}{2} - 0.4 \frac{\sqrt{3}}{2} \right)$$

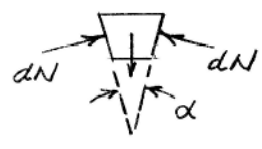
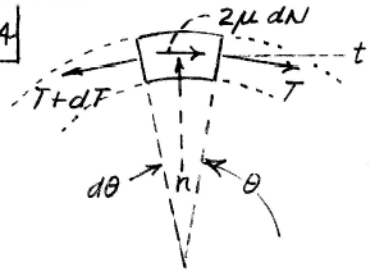
$$-N_2 \left( \frac{1}{2} + 0.4 \frac{\sqrt{3}}{2} \right) = 0 \quad (b)$$

Solve (a) & (b):  $N_1 = 184.5 \text{ lb}$ ,  $N_2 = 33.5 \text{ lb}$ 

$$\sum M_c = 0: M - 157.6(10) + 61.4(10)$$

$$-0.4(184.5 + 33.5)(10) = 0, \quad \underline{M = 1834 \text{ lb-in.}}$$

6/104



Cross section of belt

$$\Sigma F_n = 0; T \sin \frac{d\theta}{2} + (T+dT) \sin \frac{d\theta}{2} = 2dN \sin \frac{\alpha}{2}$$

$$\text{or } T d\theta = 2dN \sin \frac{\alpha}{2}$$

$$\Sigma F_t = 0; T \cos \frac{d\theta}{2} + 2\mu dN = (T+dT) \cos \frac{d\theta}{2}$$

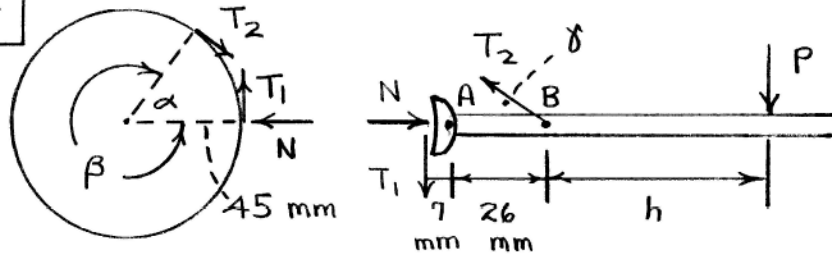
$$\text{or } 2\mu dN = dT$$

$$\text{combine \& get } \frac{dT}{T} = \frac{\mu}{\sin \frac{\alpha}{2}} d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu}{\sin \frac{\alpha}{2}} \int_0^{\beta} d\theta, \ln \frac{T_2}{T_1} = \frac{\mu\beta}{\sin \frac{\alpha}{2}}, \frac{T_2}{T_1} = e^{\frac{\mu\beta}{\sin \frac{\alpha}{2}}}$$

$$n = 1 / \sin 17.5^\circ = \underline{3.33}$$

6/105



$$\cos \alpha = \frac{45}{45+7+26}, \quad \alpha = 54.8^\circ, \quad \delta = 180 - 90 - \alpha = 35.2^\circ$$

$$\beta = 360 - \alpha = 305^\circ \text{ or } 5.33 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu \beta} = e^{0.25(5.33)} = 3.79 \quad (a)$$

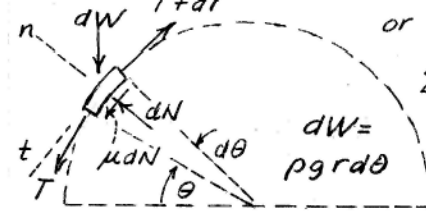
$$\text{Bar: } \sum M_P = 0: T_1(h+7+26) - T_2 \sin \delta (h) = 0$$

$$\text{or } \frac{T_2}{T_1} = \frac{h+33}{0.577h} \quad (b)$$

$$\text{From (a) \& (b), } \underline{h = 27.8 \text{ mm}}$$

(For actual wrench,  $h \cong 100 \text{ mm}$ )

6/106  $\Sigma F_t = 0; (T+dT) \cos \frac{d\theta}{2} - T \cos \frac{d\theta}{2} - \mu dN = pgr d\theta \cos \theta$



or  $dT - \mu dN = pgr \cos \theta d\theta$

$\Sigma F_n = 0; (T+dT) \sin \frac{d\theta}{2} + T \sin \frac{d\theta}{2} + pgr d\theta \sin \theta - dN = 0$

or  $T d\theta + pgr \sin \theta d\theta = dN$

Eliminate  $dN$  & get  $\frac{dT}{d\theta} - \mu T = pgr(\mu \sin \theta + \cos \theta)$

which is standard form for linear, nonhomogeneous Eq.

Sol. is  $T = Ce^{\mu\theta} + e^{\mu\theta} \int e^{-\mu\theta} pgr(\mu \sin \theta + \cos \theta) d\theta$

$= Ce^{\mu\theta} + \frac{pgr}{1+\mu^2} [0 - 2\mu(1)], C = \frac{2pgr\mu}{1+\mu^2}$

so  $T = \frac{pgr}{1+\mu^2} [2\mu e^{\mu\theta} + (1-\mu^2)\sin \theta - 2\mu \cos \theta]$

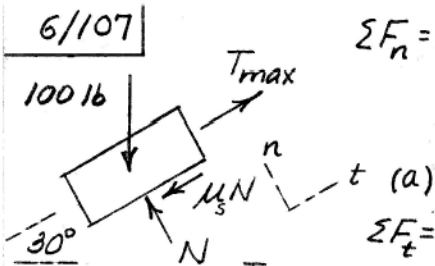
$T_{\theta=\pi} = pgh = \frac{pgr}{1+\mu^2} [2\mu e^{\mu\pi} + 0 - 2\mu(-1)]$

so  $h = \frac{2\mu r}{1+\mu^2} (1 + e^{\mu\pi})$



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6/107



$$\Sigma F_n = 0; N = 100 \cos 30^\circ = 86.6 \text{ lb}$$

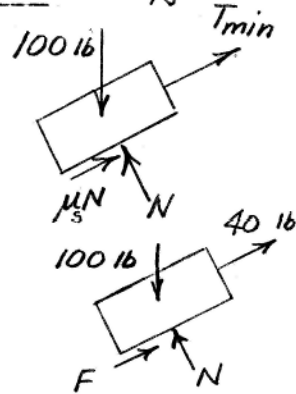
$$\mu_s N = 0.3(86.6) = 26.0 \text{ lb}$$

$$\Sigma F_t = 0; T_{\max} - 100 \sin 30^\circ - 26.0 = 0$$

$$T_{\max} = \underline{76.0 \text{ lb}}$$

$$T_{\min} - 100 \sin 30^\circ + 26.0 = 0$$

$$T_{\min} = \underline{24.0 \text{ lb}}$$



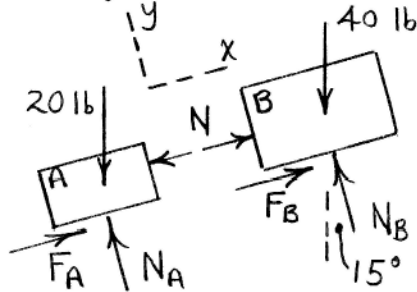
(b) For  $T = 40 \text{ lb}$ ,  $\Sigma F_t = 0$  gives

$$F - 100 \sin 30^\circ + 40 = 0$$

$$F = \underline{10 \text{ lb}} < \mu_s N$$

6/108 | From  $\theta_{\max} = \tan^{-1} \mu_s$ , we have

$$\left. \begin{aligned} (\theta_{\max})_A &= \tan^{-1} 0.30 = 16.70^\circ \\ (\theta_{\max})_B &= \tan^{-1} 0.20 = 11.31^\circ \\ (\theta_{\max})_C &= \tan^{-1} 0.35 = 19.29^\circ \end{aligned} \right\} \begin{array}{l} \text{So C remains stationary} \\ \text{By themselves, B} \\ \text{would slide, A would} \\ \text{not.} \end{array}$$



From  $\Sigma F_y = 0$ :

$$N_A = 19.32 \text{ lb}$$

$$N_B = 38.6 \text{ lb}$$

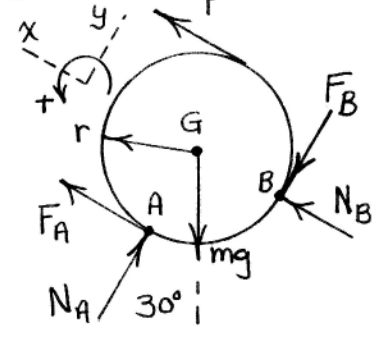
Assume that slipping impends for B:

$$\Sigma F_x = 0: N + 0.2(38.6) - 40 \sin 15^\circ = 0, \quad N = 2.63 \text{ lb}$$

$$\Sigma F_x = 0 \text{ for A: } F_A - 20 \sin 15^\circ - 2.63 = 0$$

$$F_A = 7.80 \text{ lb}; \quad (F_A)_{\max} = 0.30(19.32) = 5.80 \text{ lb}$$

Because  $(F_A)_{\max} < F$ , both A and B slip.



(1) Assume no slippage until contact at B is lost :

$$F_B = N_B = 0$$

$$\begin{cases} \sum F_x = 0 : P + F_A + mg \sin 30^\circ = 0 \\ \sum F_y = 0 : N_A - mg \cos 30^\circ = 0 \\ \sum M_G = 0 : Pr - F_A r = 0 \end{cases}$$

Solution :  $P = F_A = \frac{mg}{4}$  ,  $N_A = 0.866mg$

$(F_A)_{max} = \mu_s N_A = 0.25(0.866mg) = 0.217mg < F_A$ ;

Assumption invalid

(2) Assume rotational slippage impends :

$$F_A = \mu_s N_A = 0.25 N_A \quad , \quad F_B = \mu_s N_B = 0.25 N_B$$

$$\begin{cases} \sum F_x = 0 : P + 0.25 N_A - mg \sin 30^\circ + N_B = 0 \\ \sum F_y = 0 : N_A - mg \cos 30^\circ - 0.25 N_B = 0 \\ \sum M_G = 0 : Pr - 0.25 N_A r - 0.25 N_B r = 0 \end{cases}$$

Solution :  $P = 0.232mg$  ,  $N_A = 0.878mg$  ,  $N_B = 0.0487mg$

So rotational slippage occurs first, at  $P = 0.232mg$

$$\frac{6}{110} \quad \theta = \tan^{-1} \frac{20}{80} = 14.04^\circ$$

For the rope,

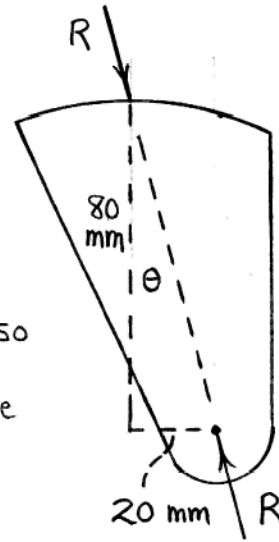
$$2R \sin 14.04^\circ = 900$$

$$\underline{R = 1855 \text{ N}}$$

$\theta < (\phi = \tan^{-1} 0.8 = 38.7^\circ)$ , so

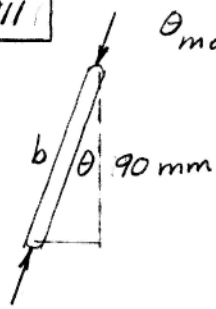
There is no slipping and the friction force is less than

$$F_{\max} = \mu_s N.$$





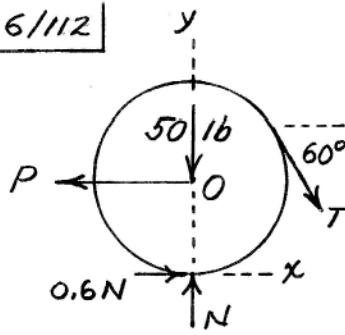
6/111



$$\theta_{max} = \phi = \tan^{-1} \mu = \tan^{-1} 0.40 = 21.80^\circ$$

$$b = 90 / \cos 21.80^\circ = \underline{96.9 \text{ mm}}$$

6/112



$$\sum M_O = 0; T = 0.6N \quad (a)$$

$$\sum F_x = 0; T \cos 60^\circ + 0.6N - P = 0 \quad (b)$$

$$\sum F_y = 0; T \sin 60^\circ + 50 - N = 0 \quad (c)$$

$$(a) \& (c) \quad 0.6N(0.866) + 50 - N = 0$$

$$0.4804N = 50, N = 104.1 \text{ lb}$$

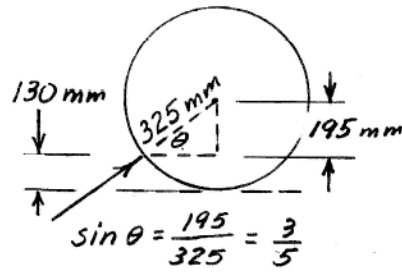
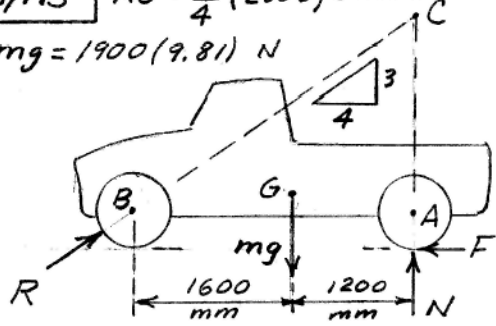
$$(c) \quad 0.866T + 50 - 104.1 = 0$$

$$T = 62.4 \text{ lb}$$

$$(b) \quad P = 62.4(0.5) + 0.6(104.1), \quad \underline{P = 93.7 \text{ lb}}$$

$$6/113 \quad \bar{AC} = \frac{3}{4} (2800) = 2100 \text{ mm}$$

$$mg = 1900(9.81) \text{ N}$$



$$\Sigma M_C = 0; F(2100 + 325) - 1200 mg = 0, F = 0.495 mg$$

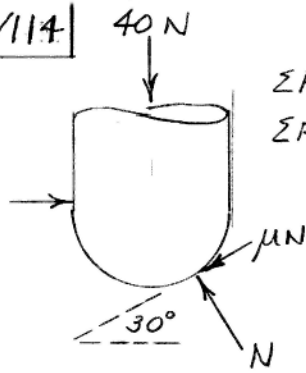
$$\Sigma M_B = 0; 1600 mg + 0.495 mg (325) - 2800 N = 0, N = 0.629 mg$$

$$\text{Thus } \mu_{\min} = F/N = 0.495/0.629 = \underline{0.787}$$

$$M = Fr = 0.495(1900)(9.81)(0.325) = 2998 \text{ N}\cdot\text{m}$$

$$\text{or } \underline{M = 3.00 \text{ kN}\cdot\text{m}}$$

6/114



Detent:

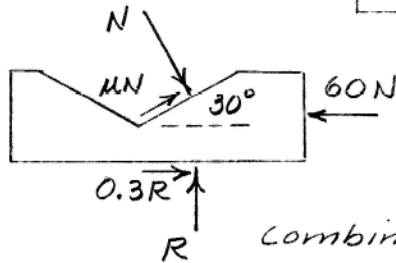
$$\sum F_x = 0; 0.3R + 0.866\mu N + 0.5N - 60 = 0$$

$$\sum F_y = 0; R + 0.5\mu N - 0.866N = 0$$

Eliminate R &amp; get

$$N = \frac{60}{0.7598 + 0.716\mu} \quad (a)$$

Plunger:



$$\sum F_y = 0;$$

$$0.866N - 0.5\mu N - 40 = 0$$

$$N(0.866 - 0.5\mu) = 40$$

$$N = \frac{40}{0.866 - 0.5\mu} \quad (b)$$

Combine (a) &amp; (b) &amp; get

$$60(0.866 - 0.5\mu) = 40(0.7598 + 0.716\mu)$$

$$\text{Solve for } \mu \text{ \& get } \underline{\mu = 0.368} \quad (N = 58.6 \text{ N})$$

$$\frac{6}{115} \quad \phi = \tan^{-1} \mu_s = \tan^{-1}(0.2) = 11.31^\circ$$

$$\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{2.5}{2\pi(5)} = 4.55^\circ$$

(a) Tighten:  $M = Pr \tan(\phi + \alpha)$

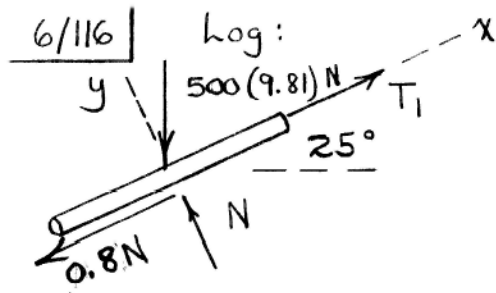
$$F(100) = 600 \left( \frac{10}{2} \right) \tan(11.31^\circ + 4.55^\circ)$$

$$\underline{F = 8.52 \text{ N}}$$

(b) Loosen:  $M = Pr \tan(\phi - \alpha)$

$$F(100) = 600 \left( \frac{10}{2} \right) \tan(11.31^\circ - 4.55^\circ)$$

$$\underline{F = 3.56 \text{ N}}$$



$$\sum F_y = 0: N - 500(9.81) \cos 25^\circ = 0, \quad N = 4450 \text{ N}$$

$$\sum F_x = 0: T_1 - 500(9.81) \sin 25^\circ - 0.8(4450) = 0$$

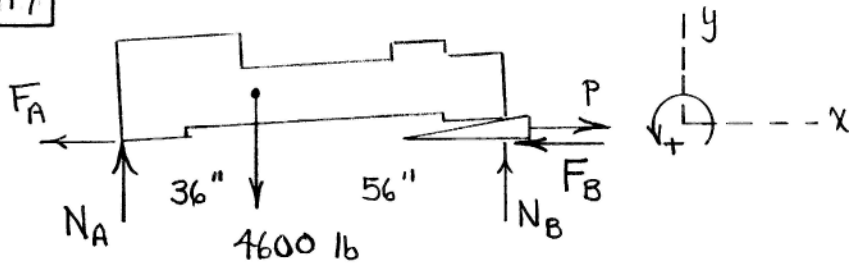
$$T_1 = 5630 \text{ N}$$

Rock:

$$\begin{aligned}
 T &= T_1 e^{\mu \beta} \\
 &= 5630 e^{0.5 \left[ 40 \frac{\pi}{180} \right]} \\
 &= \underline{7980 \text{ N}}
 \end{aligned}$$



6/117

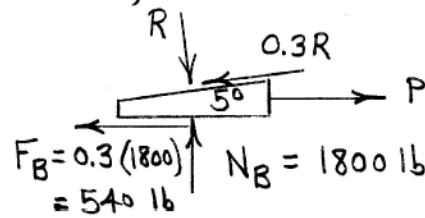


Lathe and wedge as a unit :

$$\sum M_A = 0 : 92 N_B - 4600(36) = 0, \quad N_B = 1800 \text{ lb}$$

$$\sum F_y = 0 : N_A - 4600 + 1800 = 0, \quad N_A = 2800 \text{ lb}$$

Wedge :



$$\sum F_y = 0 : 1800 - R \cos 5^\circ - 0.3R \sin 5^\circ = 0, \quad R = 1761 \text{ lb}$$

$$\sum F_x = 0 : P - 540 - 0.3(1761) \cos 5^\circ + 1761 \sin 5^\circ = 0$$

$$\underline{P = 913 \text{ lb}}$$

Lathe & wedge :  $\sum F_x = 0 : 913 - 540 - F_A = 0, \quad F_A = 373 \text{ lb}$   
 But  $373 < (0.3)(2800) = 840 \text{ lb}$ ; A cannot slip.

$$\frac{6}{118} \quad \alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{13}{2\pi(78)} = 3.04^\circ$$

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.25 = 14.04^\circ$$

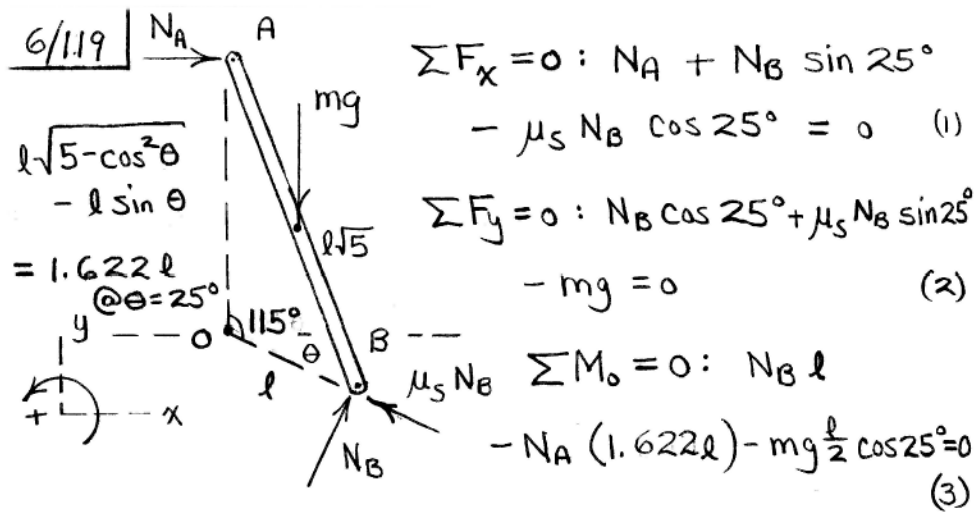
(a) To raise,  $M = Wr \tan(\alpha + \phi)$

$$= \frac{2.2(10^3)9.81}{2} \frac{(0.078)}{2} \tan(3.04^\circ + 14.04^\circ)$$
$$= \underline{129.3 \text{ N}\cdot\text{m}}$$

(b) To lower,  $M = Wr \tan(\phi - \alpha)$

$$= \frac{2.2(10^3)9.81}{2} \frac{(0.078)}{2} \tan(14.04^\circ - 3.04^\circ)$$
$$= \underline{81.8 \text{ N}\cdot\text{m}}$$





$$(2): N_B = \frac{mg}{\cos 25^\circ + \mu_s \sin 25^\circ} = \frac{mg}{0.906 + \mu_s (0.423)}$$

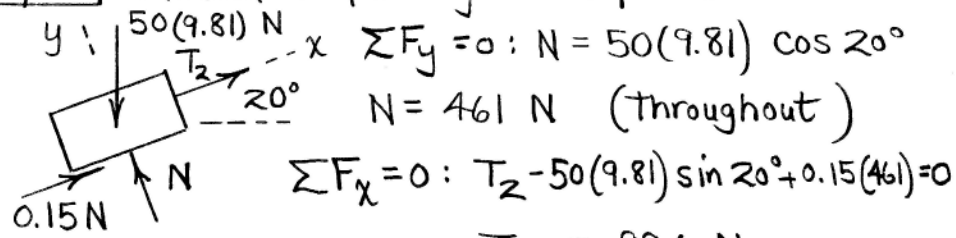
$$\left. \begin{aligned} \text{Rewrite (1) \& (3): } N_A + [0.423 - \mu_s (0.906)] N_B &= 0 \\ -1.622 N_A + N_B &= 0.453 mg \end{aligned} \right\}$$

$$\text{Multiply first eq. by } 1.622 \text{ \& add: } N_B = \frac{0.453 mg}{1.685 - 1.470 \mu_s}$$

Equate two expressions for  $N_B$  \& solve for  $\mu_s$ :

$$\underline{\mu_s = 0.767}$$

6/120 | Motion impending down plane:



$$\sum F_y = 0: N = 50(9.81) \cos 20^\circ$$

$$N = 461 \text{ N (throughout)}$$

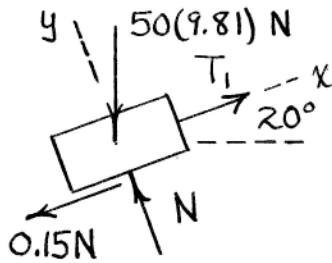
$$\sum F_x = 0: T_2 - 50(9.81) \sin 20^\circ + 0.15(461) = 0$$

$$T_2 = 98.6 \text{ N}$$

$$T_2 = T_1 e^{\mu\beta}: 98.6 = \frac{mg}{2} e^{0.25(110 \cdot \frac{\pi}{180})}$$

$$m = 12.44 \text{ kg}$$

Motion impending up plane:



$$\sum F_x = 0: T_1 - 50(9.81) \sin 20^\circ$$

$$- 0.15(461) = 0$$

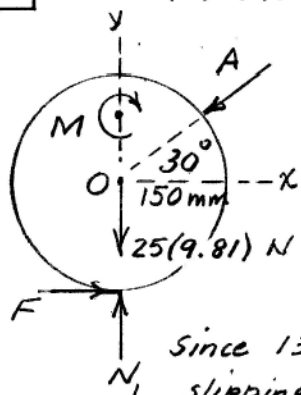
$$T_1 = 237 \text{ N}$$

$$T_2 = T_1 e^{\mu\beta}: \frac{mg}{2} = 237 e^{0.25(110 \cdot \frac{\pi}{180})}$$

$$m = 78.0 \text{ kg}$$

So range is  $12.44 \leq m \leq 78.0 \text{ kg}$

6/121



(a) Assume complete equilibrium

$$\Sigma M_O = 0; M - Fr = 0$$

$$F = \frac{20}{0.150} = 133.3 \text{ N}$$

$$\Sigma F_x = 0; 133.3 - A \cos 30^\circ = 0$$

$$A = 154.0 \text{ N}$$

$$\Sigma F_y = 0; N_1 - 154 \sin 30^\circ - 25(9.81) = 0$$

$$N_1 = 322.2 \text{ N}$$

Since  $133.3 < (\mu_s N_1 = 0.5 [322.2] = 161 \text{ N})$ ,

slipping does not occur & assumption is valid. Thus  $F = 133.3 \text{ N}$

(b) Assume wheel slips &  $\Sigma M_O \neq 0$  so  $F = \mu_k N_1 = 0.4 N_1$

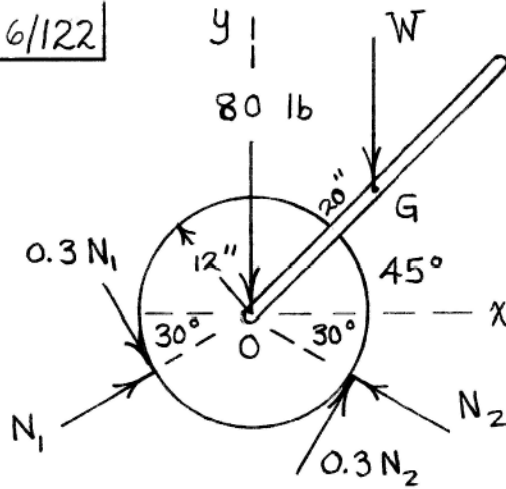
$$\Sigma F_x = 0; 0.4 N_1 - A \cos 30^\circ = 0$$

$$\Sigma F_y = 0; N_1 - 25(9.81) - A \sin 30^\circ = 0$$

Solve & get  $A = 147.3 \text{ N}$ ,  $N_1 = 318.9 \text{ N}$ ,  $F = 0.4(318.9) = 127.6 \text{ N}$

Assumption valid since  $(M = 40 \text{ N}\cdot\text{m}) > (127.6 [0.150] = 19.1 \text{ N}\cdot\text{m})$

6/122



(C = cos)  
(S = sin)

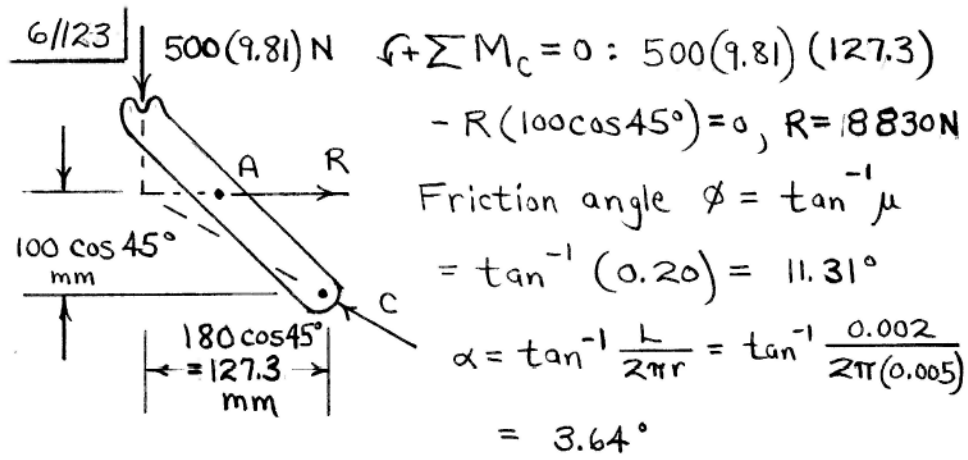
$$\sum F_x = 0: N_1 c 30^\circ + 0.3 N_1 s 30^\circ - N_2 c 30^\circ + 0.3 N_2 s 30^\circ = 0$$

$$\sum F_y = 0: N_1 s 30^\circ - 0.3 N_1 c 30^\circ + N_2 s 30^\circ + 0.3 N_2 c 30^\circ - 80 - W = 0$$

$$\sum M_O = 0: (0.3 N_1 + 0.3 N_2)(12) - W(20 c 45^\circ) = 0$$

Solve to obtain

$$\begin{cases} N_1 = 113.9 \text{ lb} \\ N_2 = 161.6 \text{ lb} \\ W = 70.1 \text{ lb} \end{cases}$$



Raise load:  $M = Rr \tan(\phi + \alpha)$

$$P(0.150) = 18830(0.005) \tan(11.31^\circ + 3.64^\circ)$$

$$P = \underline{78.6 \text{ N}}$$

Lower load:  $M = Rr \tan(\phi - \alpha)$

$$P(0.150) = 18830(0.005) \tan(11.31^\circ - 3.64^\circ)$$

$$P = \underline{39.6 \text{ N}}$$

6/124 | For the slab

$$\sum F_x = 0: \mu_k N \cos \alpha - N \sin \alpha = 0$$

$$\mu_k = \tan \alpha$$

$$\frac{d}{2} + \frac{a}{2} = \frac{b}{2} + \frac{d}{2} \cos \alpha$$

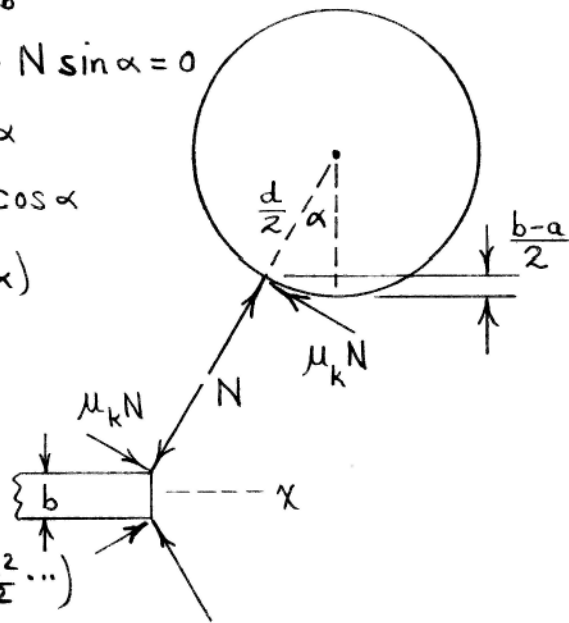
$$b = a + d(1 - \cos \alpha)$$

For small  $\alpha$ ,

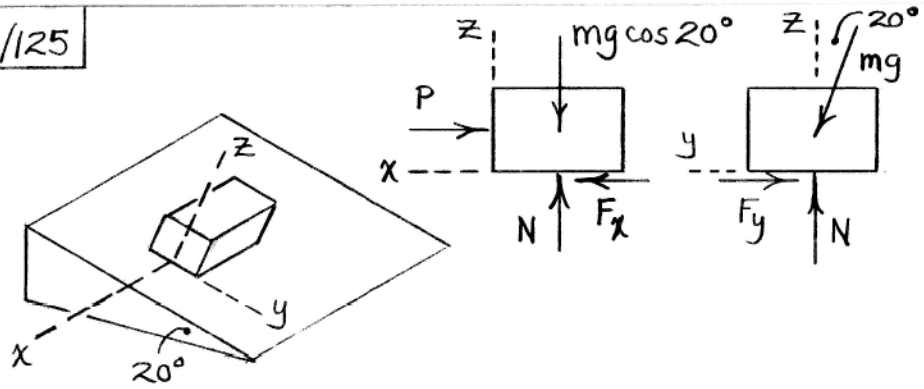
$$\left\{ \begin{array}{l} \cos \alpha \approx 1 - \frac{\alpha^2}{2} + \dots \\ \tan \alpha \approx \alpha \end{array} \right.$$

$$\text{So } b = a + d(1 - 1 + \frac{\alpha^2}{2} \dots)$$

$$= a + \frac{\mu_k^2 d}{2}$$



6/125



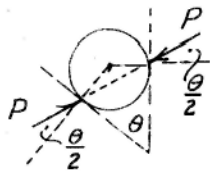
$$(x-z) \begin{cases} \sum F_z = 0 : N - 8(9.81) \cos 20^\circ = 0, & N = 73.7 \text{ N} \\ \sum F_x = 0 : F_x - P = 0, & F_x = P \end{cases}$$

$$(y-z) \begin{cases} \sum F_y = 0 : -F_y + 8(9.81) \sin 20^\circ = 0, & F_y = 26.8 \text{ N} \end{cases}$$

$$F = \sqrt{F_x^2 + F_y^2} = \mu_s N = \sqrt{P^2 + 26.8^2} = 0.5(73.7)$$

$$\underline{P = 25.3 \text{ N}}$$

Rollers can support equal & opposite forces  $P$  provided  $\frac{\theta}{2} < \tan^{-1} \mu$



Thus minimum  $d$  occurs when

$$\tan \frac{\theta}{2} = \mu$$

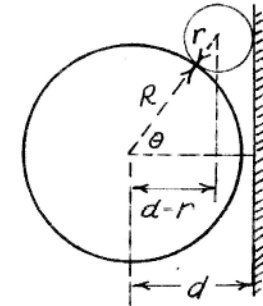
But  $\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$  so

$$\tan^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{1 + \cos \theta} \quad \& \quad \mu^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

or  $\cos \theta = \frac{1 - \mu^2}{1 + \mu^2}$  but  $\cos \theta = \frac{d-r}{R+r}$

so  $\frac{d-r}{R+r} = \frac{1 - \mu^2}{1 + \mu^2}$  which gives

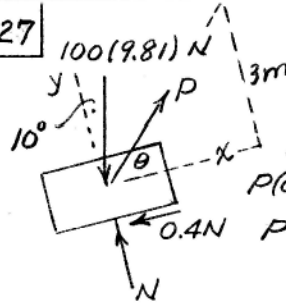
$$d_{min} = \frac{2r + (1 - \mu^2)R}{1 + \mu^2}$$



& clearly  $d_{max} = R + 2r$



\*6/127



$$\sum F_x = 0; P \cos \theta - 0.4N - 981 \sin 10^\circ = 0$$

$$\sum F_y = 0; N + P \sin \theta - 981 \cos 10^\circ = 0$$

Eliminate N & get

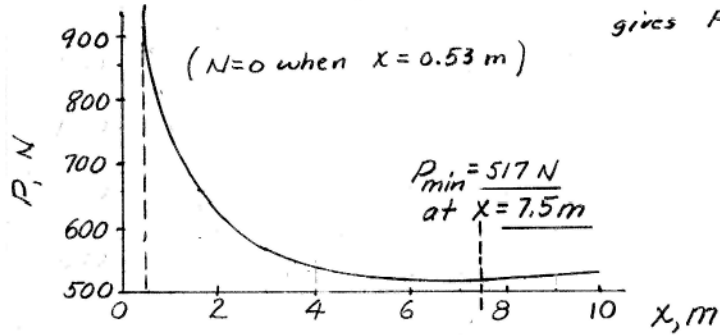
$$P(\cos \theta + 0.4 \sin \theta) = 981(0.4 \cos 10^\circ + \sin 10^\circ)$$

$$P = \frac{556.8}{\cos \theta + 0.4 \sin \theta}$$

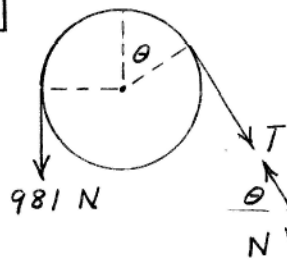
$$\frac{dP}{d\theta} (\cos \theta + 0.4 \sin \theta) + P(-\sin \theta + 0.4 \cos \theta) = 0$$

$$\frac{dP}{d\theta} = P \frac{-\sin \theta + 0.4 \cos \theta}{\cos \theta + 0.4 \sin \theta} = 0 \text{ for stationary value}$$

Thus  $\sin \theta = 0.4 \cos \theta$ ,  $\tan \theta = 0.4$ ,  $x = \frac{3}{\tan \theta} = \frac{3}{0.4} = 7.5 \text{ m}$   
gives  $P = 517 \text{ N}$



\*6/128



$$T = 981 e^{0.5(\pi/2 + \theta)}$$

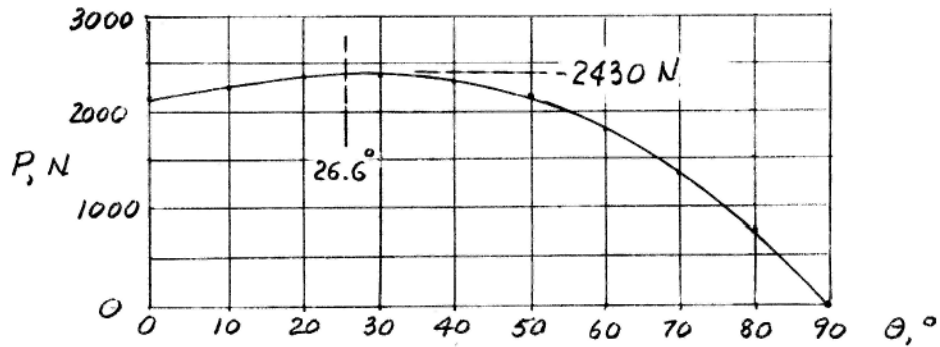
$$P = T \cos \theta$$

$$\text{so } P = 981 e^{(\frac{\pi}{4} + \frac{\theta}{2})} \cos \theta$$

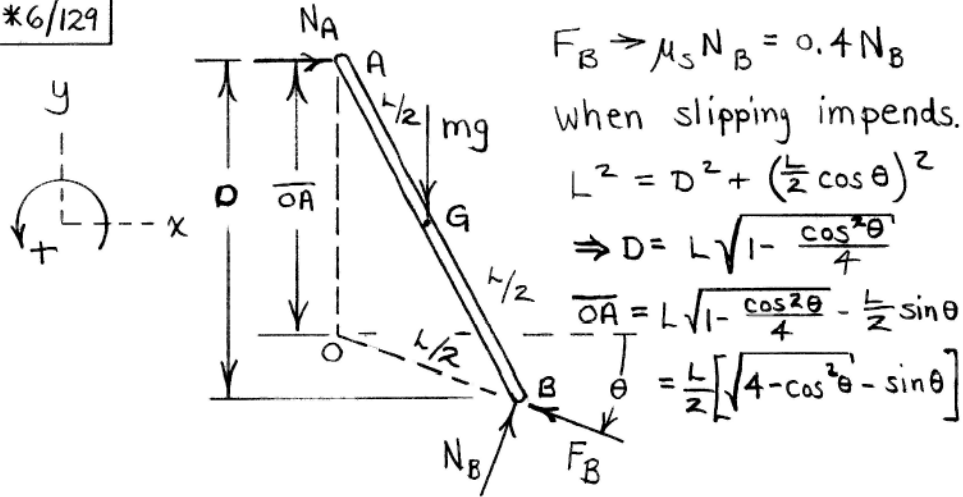
Carry out numerical  
solution & plot

$$\frac{dP}{d\theta} = 981 \left\{ -e^{\frac{\pi}{4} + \frac{\theta}{2}} \sin \theta + \frac{1}{2} e^{\frac{\pi}{4} + \frac{\theta}{2}} \cos \theta \right\} = 0 \text{ for max. } P$$

$$\theta = \tan^{-1} \frac{1}{2} = 26.6^\circ \text{ \& } P_{\max} = 981 e^{1.017} (0.894) = 2430 \text{ N}$$



\*6/129



$$\sum F_x = 0 : N_A + N_B \sin \theta - 0.4 N_B \cos \theta = 0 \quad (1)$$

$$\sum F_y = 0 : N_B \cos \theta + 0.4 N_B \sin \theta - mg = 0 \quad (2)$$

$$\sum M_o = 0 : N_B \left(\frac{L}{2}\right) - N_A \frac{L}{2} \left[ \sqrt{4 - \cos^2 \theta} - \sin \theta \right] - mg \frac{L}{4} \cos \theta = 0 \quad (3)$$

Numerical solution :

$$\begin{cases} N_A = 0.287 mg \\ N_B = 0.966 mg \\ \theta = 5.80^\circ \end{cases}$$

\*6/1.30



$$\bar{A}B = \frac{L}{\cos\theta}, \quad \bar{G}B = \frac{L}{2} \left( \frac{1}{\cos\theta} - 1 \right)$$

$$\sum M_B = 0; \quad W \frac{L}{2} \left( \frac{1}{\cos\theta} - 1 \right) \sin\theta + \mu_s N_1 \frac{L}{2} - N_1 \frac{L}{2} \tan\theta = 0$$

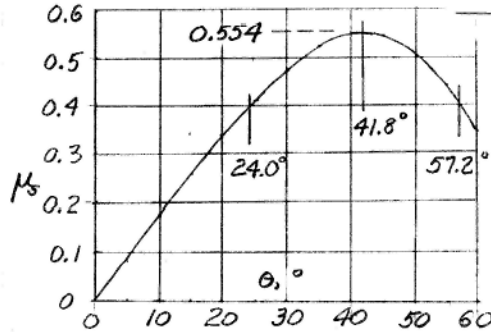
$$\therefore W (\tan\theta - \sin\theta) = N_1 (\tan\theta - \mu_s)$$

$$\sum M_C = 0; \quad W \frac{L}{2} \sin\theta - \mu_s N_1 \frac{L}{2} \frac{1}{\cos\theta} \frac{1}{\cos\theta} = 0$$

$$W \sin\theta = \mu_s N_1 / \cos^2\theta$$

Combine & get  $\frac{\tan\theta - \sin\theta}{\sin\theta} = \frac{\tan\theta - \mu_s \cos^2\theta}{\mu}$

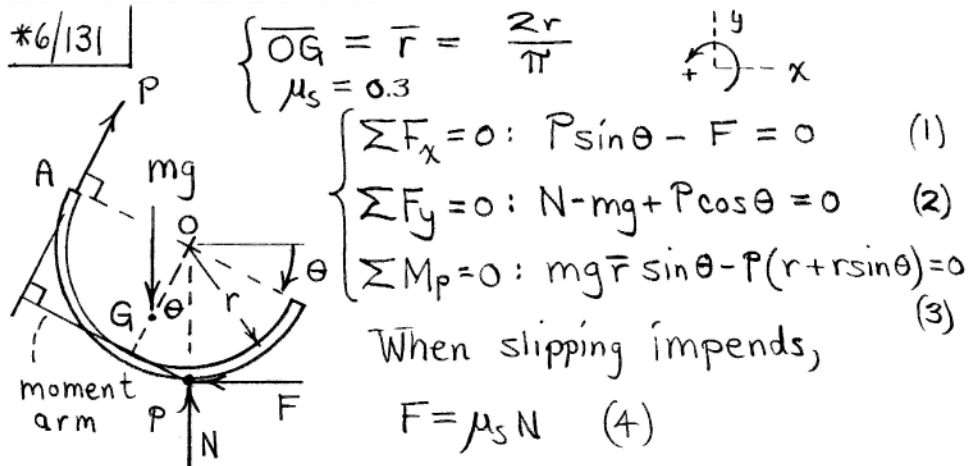
Solve for  $\mu_s$  & get  $\mu_s = \frac{\sin\theta}{1 + \tan\theta(\tan\theta - \sin\theta)}$



(a) For  $\mu_s = 0.4$ , pole is unstable for  $24.0^\circ < \theta < 57.2^\circ$

(b)  $\theta = 41.8^\circ$  requires  $\mu_s = 0.554$

\*6/131



$$\left\{ \begin{aligned} \overline{OG} = \bar{r} &= \frac{2r}{\pi} \\ \mu_s &= 0.3 \end{aligned} \right. \quad \begin{array}{c} \uparrow y \\ \leftarrow x \end{array}$$

$$\left\{ \begin{aligned} \Sigma F_x = 0: P \sin \theta - F &= 0 \quad (1) \\ \Sigma F_y = 0: N - mg + P \cos \theta &= 0 \quad (2) \\ \Sigma M_p = 0: mg \bar{r} \sin \theta - P(r + r \sin \theta) &= 0 \quad (3) \end{aligned} \right.$$

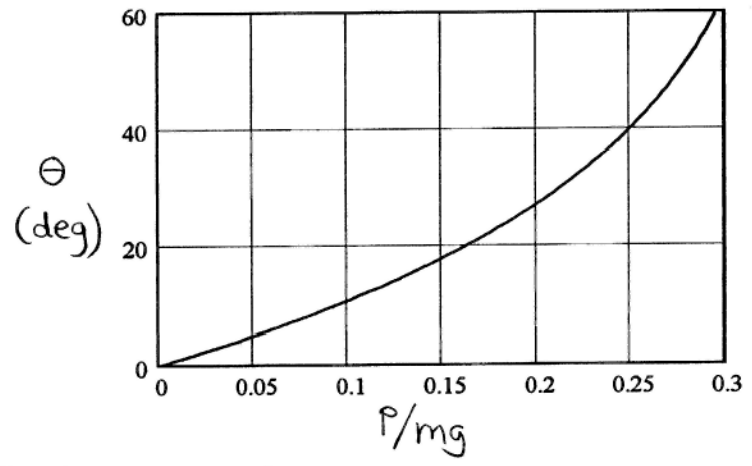
When slipping impends,

$$F = \mu_s N \quad (4)$$

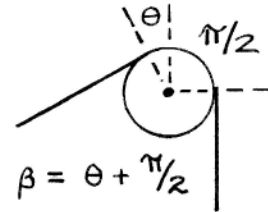
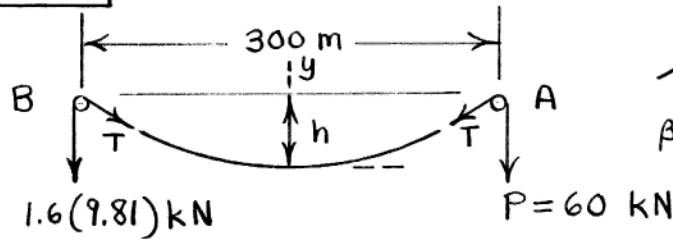
Numerical solution of (1) - (4):  $\theta_{max} = 59.9^\circ$ ,  $P_{max} = 0.295mg$

Eqs. (1) - (3) may be solved for P as

$$P = \frac{2 \sin \theta}{\pi (1 + \sin \theta)} \quad (\text{plotted below})$$



\*6/132



During slipping, Eq. 6/7 @ A:  $60 = T e^{\mu_k \beta}$

Eq. 6/7 @ B:  $T = 1.6(9.81) e^{\mu_k \beta}$

Eliminate  $\mu_k \beta$  & get  $T = 30.7$  kN

Then  $e^{\mu_k \beta} = 60/30.7$ ,  $\mu_k \beta = 0.670$

Eq. 5/21:  $T = T_0 \cosh \frac{\rho g x}{T_0}$ ;  $30.7 = T_0 \cosh \frac{12(9.81)10^{-3} \cdot 150}{T_0}$

or  $30.7 = T_0 \cosh \frac{17.66}{T_0}$ . Solve numerically:  $T_0 = 23.8$  kN

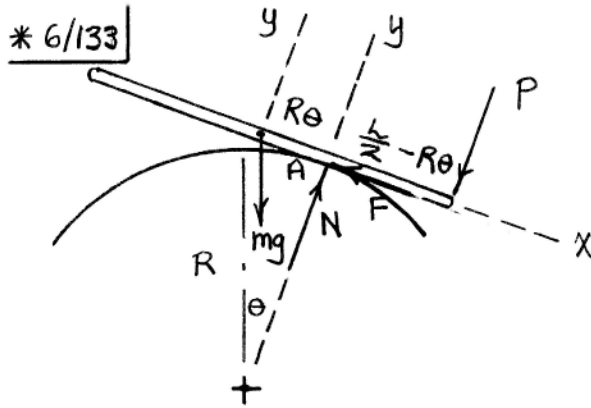
$\frac{dy}{dx} = \sinh \frac{\rho g x}{T_0}$ ;  $\tan \theta = \sinh \frac{17.66}{23.8} = 0.810$

$\theta = 0.681$  rad,  $\beta = \theta + \pi/2 = 2.25$  rad

$\mu_k = \frac{0.670}{2.25} = \underline{0.298}$

Eq. 5/22:  $T = T_0 + \rho g h$ ,  $h = \frac{T - T_0}{\rho g}$

$= \frac{30.7 - 23.8}{12(9.81)(10^{-3})} = \underline{58.1$  m



$$\Sigma F_x = 0 : mg \sin \theta - \mu_s N = 0 \quad (1)$$

$$\Sigma F_y = 0 : N - mg \cos \theta - P = 0 \quad (2)$$

$$\Sigma M_A = 0 : P \left( \frac{L}{2} - R \theta \right) - mg R \theta \cos \theta = 0 \quad (3)$$

$$(1) : N = \frac{mg \sin \theta}{\mu_s}$$

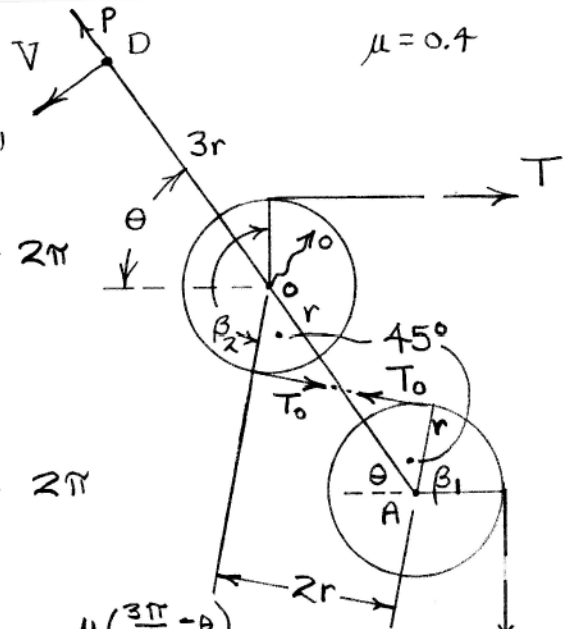
$$(2) : \frac{mg \sin \theta}{\mu_s} - mg \cos \theta = P$$

$$(3) : mg \left( \frac{\sin \theta}{\mu_s} - \cos \theta \right) \left( \frac{L}{2} - R \theta \right) - mg R \theta \cos \theta = 0$$

$$\text{Simplify : } \tan \theta = \mu_s \frac{1}{1 - \frac{2R\theta}{L}} = 0.15 \frac{1}{1 - 2(0.6)\theta}$$

$$\text{or } \tan \theta - \frac{0.15}{1 - 1.2\theta} = 0. \text{ Numerical solution: } \theta = 11.04^\circ$$

\*6/134



From geometry,  
upper wheel :

$$\theta + \frac{\pi}{4} + \frac{\pi}{2} + \beta_2 = 2\pi$$

$$\beta_2 = \frac{5\pi}{4} - \theta$$

lower wheel:

$$\theta + \frac{\pi}{4} + \beta_1 + \pi = 2\pi$$

$$\beta_1 = \frac{3\pi}{4} - \theta$$

Forces: 
$$\begin{cases} mg = T_0 e^{\mu(\frac{3\pi}{4} - \theta)} \\ T_0 = T e^{\mu(\frac{5\pi}{4} - \theta)} \end{cases}$$

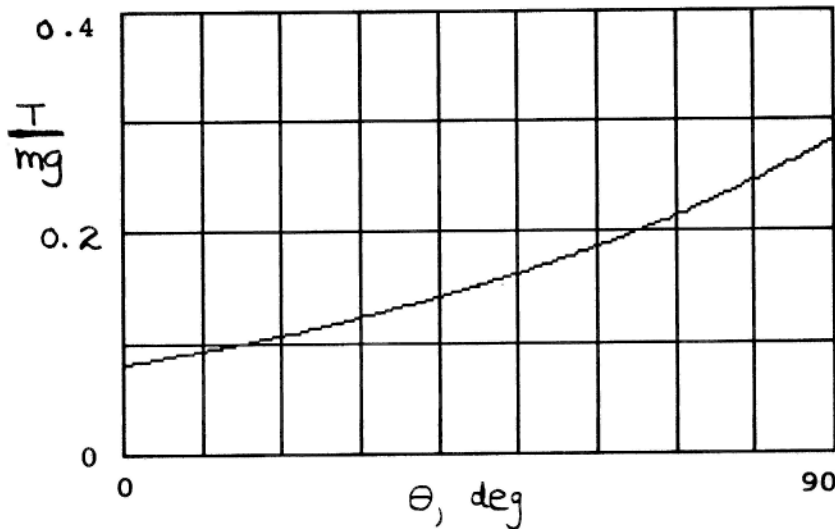
So 
$$mg = T e^{2\mu(\pi - \theta)} ; \quad \frac{T}{mg} = e^{-0.8(\pi - \theta)}$$

Device as a whole:

$$\sum M_0 = 0: 3rV - Tr - mg(r + 2r\sqrt{2}\cos\theta) = 0$$

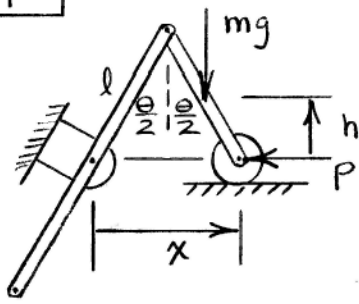
$$V = \frac{1}{3} [T + mg(1 + 2\sqrt{2}\cos\theta)]$$

For  $\theta = 60^\circ$ ,  $\frac{T}{mg} = 0.1872$ ,  $V = 0.867mg$





7/1



$$x = 2l \sin \frac{\theta}{2}$$

$$\delta x = l \cos \frac{\theta}{2} \delta \theta$$

$$h = \frac{l}{2} \cos \frac{\theta}{2}$$

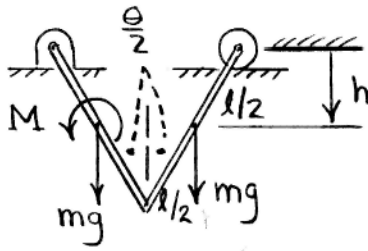
$$\delta h = -\frac{l}{4} \sin \frac{\theta}{2} \delta \theta$$

$$\delta U = 0: -P\delta x - mg\delta h = 0$$

$$-P(l \cos \frac{\theta}{2} \delta \theta) - mg(-\frac{l}{4} \sin \frac{\theta}{2} \delta \theta) = 0$$

$$\Rightarrow \theta = 2 \tan^{-1} \left( \frac{4P}{mg} \right)$$

7/2



$$h = \frac{l}{2} \cos \frac{\theta}{2}$$

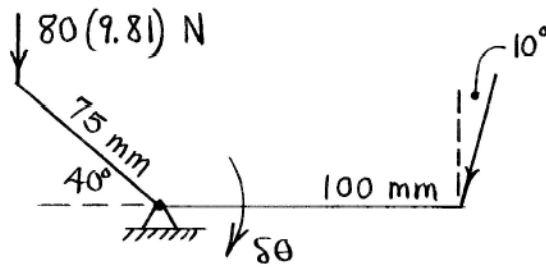
$$\delta h = -\frac{l}{4} \sin \frac{\theta}{2} \delta \theta$$

$$\delta U = 0: M \delta \left( \frac{\theta}{2} \right) + 2mg \delta h = 0$$

$$M \frac{\delta \theta}{2} + 2mg \left( -\frac{l}{4} \sin \frac{\theta}{2} \delta \theta \right) = 0$$

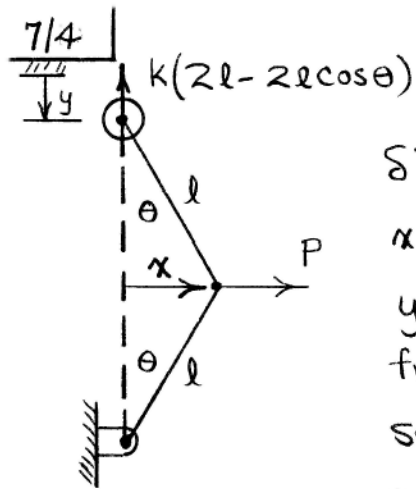
$$M = \underline{mgl \sin \frac{\theta}{2}}$$

7/3



For a virtual displacement  $\delta\theta$  of the lever,  
 $\delta U = 0 : P \cos 10^\circ (100 \delta\theta) - 80(9.81)[75 \delta\theta \cos 40^\circ] = 0$

$$\underline{P = 458 \text{ N}}$$



$$\delta U = 0: P \delta x - k(2l - 2l \cos \theta) \delta y = 0$$

$$x = l \sin \theta, \quad \delta x = l \cos \theta \delta \theta$$

$y = 2l - 2l \cos \theta$  (measured from wheel position when spring is unstretched)

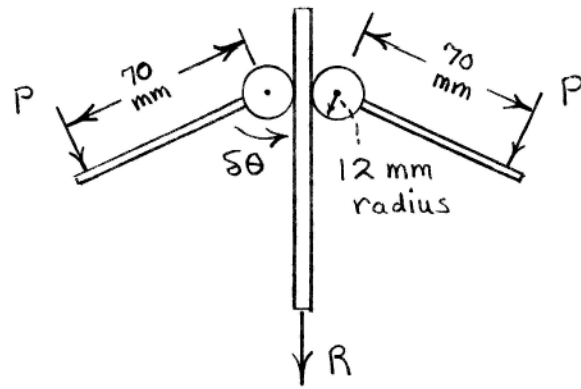
$$\delta y = 2l \sin \theta \delta \theta$$

$$\text{So } P (l \cos \theta \delta \theta) - k(2l - 2l \cos \theta) (2l \sin \theta \delta \theta) = 0$$

$$\Rightarrow P = \frac{4kl (\sin \theta - \sin \theta \cos \theta)}{\cos \theta}$$

$$\text{or } P = \underline{4kl (\tan \theta - \sin \theta)}$$

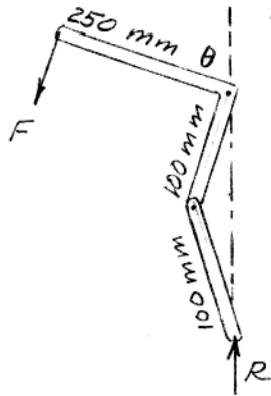
7/5



$$\delta U = 0: 2P(70)\delta\theta - R(12)\delta\theta = 0$$

$$\underline{R = 11.67 P}$$

7/6



$$\delta U = 0$$

$$F(250 \delta \theta) - R \delta(2[100 \sin \theta]) = 0$$

$$250 F \delta \theta = 200 R \cos \theta \delta \theta$$

$$\underline{F = 0.8 R \cos \theta}$$

$$\frac{7}{7} \quad e = \frac{\text{output work}}{\text{input work}}$$

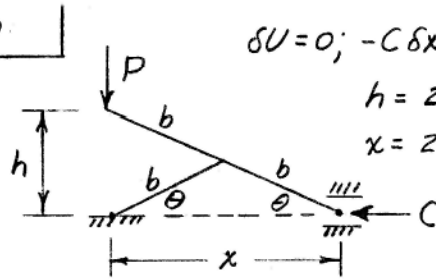
To raise,  $0.75 = \frac{250(1/4)}{P(1)}, \quad \underline{P = 83.3 \text{ lb}}$

To lower,  $0.75 = \frac{P'(1)}{250(1/4)}, \quad \underline{P' = 46.9 \text{ lb}}$

$$\underline{7/8} \quad \delta U = 0; -mg \delta (b \sin \theta + a \cos \theta) + M \delta \theta = 0$$
$$\underline{M = mg (b \cos \theta - a \sin \theta)}$$



7/9



$$\delta U = 0; -C\delta x - P\delta h = 0$$

$$h = 2b \sin \theta, \delta h = 2b \cos \theta \delta \theta$$

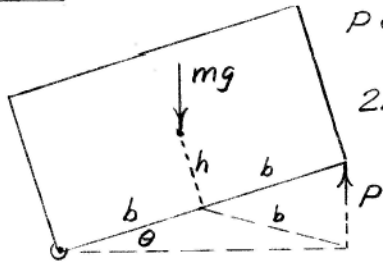
$$x = 2b \cos \theta, \delta x = -2b \sin \theta \delta \theta$$

$$\text{Thus } -C(-2b \sin \theta \delta \theta) - P(2b \cos \theta \delta \theta) = 0$$

$$C \sin \theta = P \cos \theta, C = P \cot \theta$$

$$\text{But } \cot \theta = \frac{\sqrt{4b^2 - h^2}}{h} = \sqrt{\left(\frac{2b}{h}\right)^2 - 1}, \text{ so } \underline{C = P\sqrt{\left(\frac{2b}{h}\right)^2 - 1}}$$

7/10



$$\delta U = 0;$$

$$P \delta(2b \sin \theta) - mg \delta(b \sin \theta + h \cos \theta) = 0$$

$$2Pb \cos \theta \delta \theta = mg(b \cos \theta - h \sin \theta) \delta \theta$$

$$P = \frac{mg}{2} \left(1 - \frac{h}{b} \tan \theta\right)$$

7/11

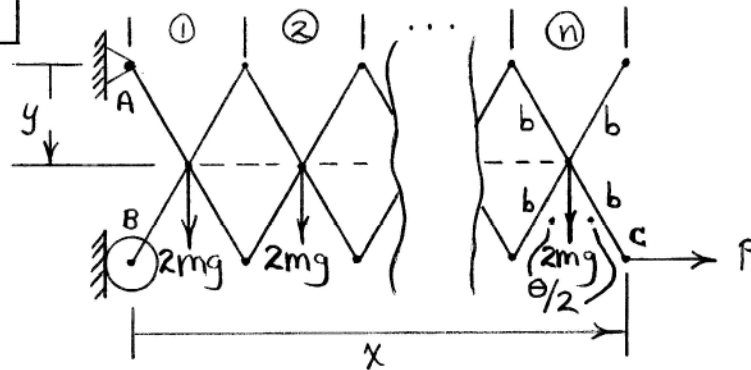
$$\delta U = 0: 160 F \delta \theta - 0.4(160 F \delta \theta) - 100(9.81) \left(150 \delta \frac{\theta}{25}\right) = 0$$

$$0.6(160)F = 981(6), \quad \underline{F = 61.3 \text{ N}}$$

7/12 | Let  $\delta\theta =$  virtual angle of input rotation  
Then  $\delta\theta/40 =$  " " " output "

$$e = \frac{\text{output work}}{\text{input work}} = \frac{1180(\delta\theta/40)}{30\delta\theta} = \frac{1180}{30(40)} = \underline{0.983}$$

7/13



$$y = b \cos \frac{\theta}{2}, \quad \delta y = -\frac{b}{2} \sin \frac{\theta}{2} \delta \theta$$

$$x = n (2b \sin \frac{\theta}{2}), \quad \delta x = nb \cos \frac{\theta}{2} \delta \theta$$

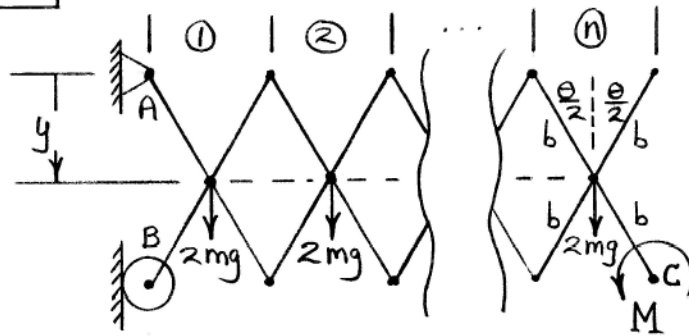
$$\delta U = 0: P \delta x + n (2mg) \delta y = 0$$

$$P (nb \cos \frac{\theta}{2} \delta \theta) = -2nmg \left(-\frac{b}{2} \sin \frac{\theta}{2} \delta \theta\right)$$

$$P = mg \tan \frac{\theta}{2}$$

$P$  does not depend on the number  $n$  of sections present.

7/14



$$y = b \cos \frac{\theta}{2}, \quad \delta y = -\frac{b}{2} \sin \frac{\theta}{2} \delta \theta$$

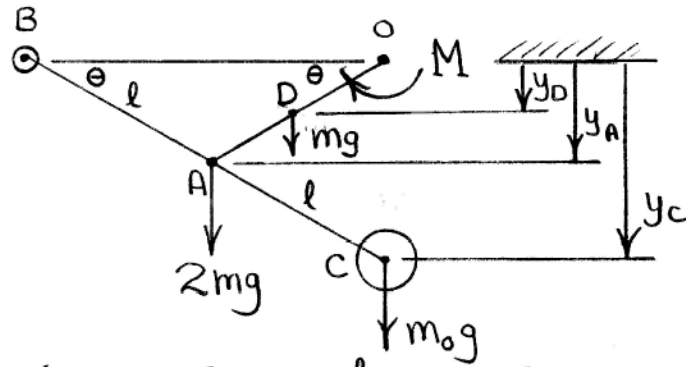
$$\delta U = 0 : M \delta \left( \frac{\theta}{2} \right) + n(2mg) \delta y = 0$$

$$\frac{M}{2} \delta \theta = -2nmg \left( -\frac{b}{2} \sin \frac{\theta}{2} \delta \theta \right)$$

$$\underline{M = 2nmg b \sin \frac{\theta}{2}}$$

M does depend on the number  $n$  of sections present.

7/15



$$y_D = \frac{l}{2} \sin \theta, \quad \delta y_D = \frac{l}{2} \cos \theta \delta \theta$$

$$y_A = l \sin \theta, \quad \delta y_A = l \cos \theta \delta \theta$$

$$y_C = 2l \sin \theta, \quad \delta y_C = 2l \cos \theta \delta \theta$$

$$\delta U = 0: -M \delta \theta + mg \delta y_D + 2mg \delta y_A + m_0 g \delta y_C = 0$$

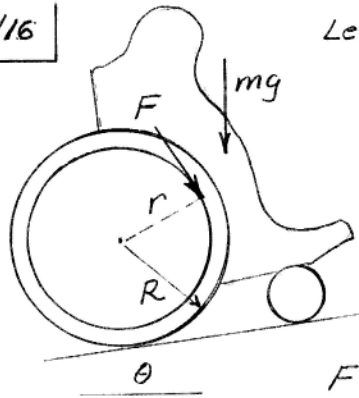
$$-M \delta \theta + mg \left( \frac{l}{2} \cos \theta \delta \theta \right) + 2mg (l \cos \theta \delta \theta)$$

$$+ m_0 g (2l \cos \theta \delta \theta) = 0$$

$$\Rightarrow M = \left( \frac{5}{2} m + 2m_0 \right) g l \cos \theta$$

$$\text{For } \theta = 30^\circ: \underline{M = \left( \frac{5}{4} m + m_0 \right) g l \sqrt{3}}$$

7/16



Let  $\beta$  = angle through which wheel turns

$s$  = corresponding displacement along incline.

$$s = R\beta \text{ so } \delta s = R \delta\beta$$

$$\delta U = 0$$

$$Fr \delta\beta - mg \delta s \sin\theta = 0$$

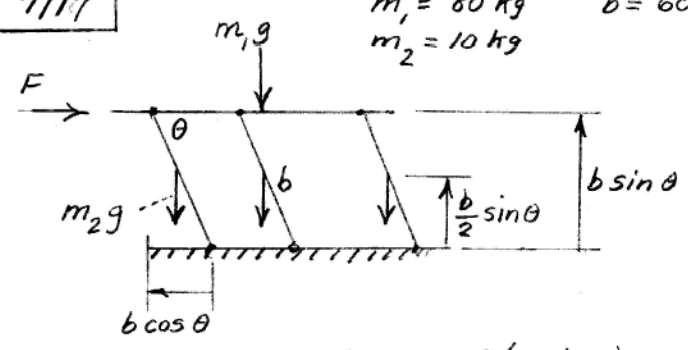
$$Fr \delta\beta = mg R \sin\theta \delta\beta$$

$$F = mg \frac{R}{r} \sin\theta$$



7/17

$m_1 = 80 \text{ kg}$      $b = 600 \text{ mm}$   
 $m_2 = 10 \text{ kg}$



$$\delta U = 0; -F \delta(b \cos \theta) - m_1 g \delta(b \sin \theta) - 3m_2 g \delta\left(\frac{b}{2} \sin \theta\right) = 0$$

$$F b \sin \theta \delta \theta = m_1 g b \cos \theta \delta \theta + \frac{3}{2} m_2 g b \cos \theta \delta \theta$$

$$F = g \cot \theta \left(m_1 + \frac{3}{2} m_2\right)$$

$$= 9.81 \left(80 + \frac{3}{2} 10\right) \cot \theta = \underline{932 \cot \theta \text{ N}}$$

Solution by force and moment equilibrium would require dismemberment with four FBD's and eventual elimination of unwanted forces and dimensions



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$$7/18 \quad \delta U = 0; \quad M \delta \theta - mg \delta h = 0$$

$$h = 2b \sin \theta, \quad \delta h = 2b \cos \theta \delta \theta$$

$$\text{So } M \delta \theta = mg (2b \cos \theta) \delta \theta$$

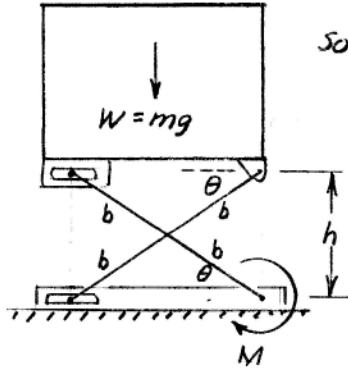
$$M = 2mg b \cos \theta$$

$$\text{But since } \sin \theta = \frac{h}{2b},$$

$$\cos \theta = \sqrt{1 - \left(\frac{h}{2b}\right)^2}$$

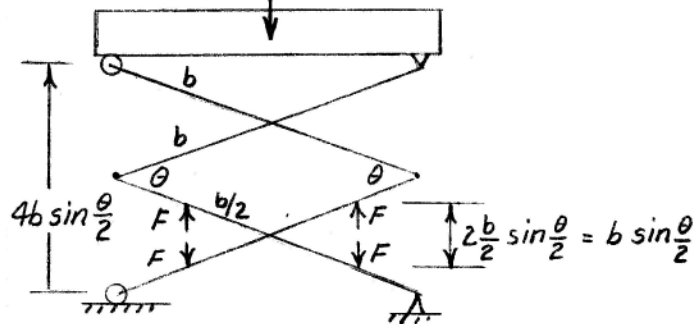
Thus

$$M = 2mg b \sqrt{1 - \left(\frac{h}{2b}\right)^2}$$





7/20

 $mg$  $F = pA$ 

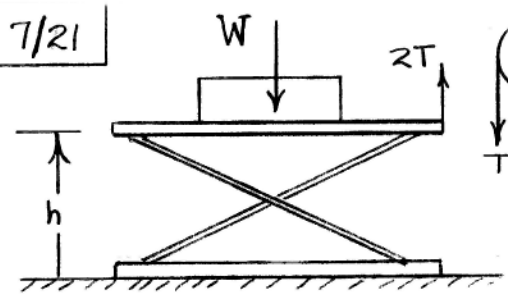
Work done by each cylinder is  $F \delta(b \sin \frac{\theta}{2})$   
 $= \frac{Fb}{2} \cos \frac{\theta}{2} \delta\theta$

$$\delta U = 0; -mg \delta(4b \sin \frac{\theta}{2}) + 2 \frac{Fb}{2} \cos \frac{\theta}{2} \delta\theta = 0$$

$$Fb \cos \frac{\theta}{2} = 2mgb \cos \frac{\theta}{2}, \quad F = pA = 2mg$$

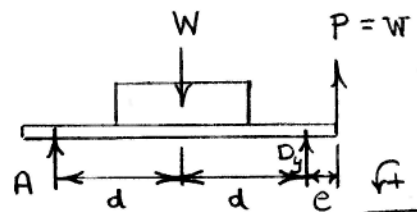
so  $p = 2mg/A$  independent of  $b$  &  $\theta$

7/21



$\delta U = 0: 2Tsh$   
 $- Wsh = 0$   
 $T = W/2$

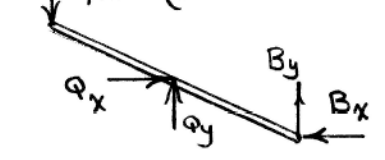
Pulley:  $P = 2T$ , so  $P = W$   
 (independent of h)



$D_y$  and A form a clockwise couple  
 $D_y = -A$  (Note  $D_x = 0$ )  
 $\sum M = 0: W(d+e) - A(2d) = 0$

$A/2$  (Note:  $d = \sqrt{l^2 - h^2}/2$ )

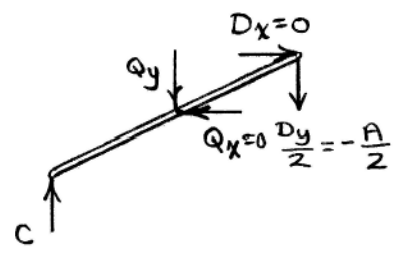
$A = \frac{W}{2} \left(1 + \frac{e}{d}\right)$   
 $= \frac{W}{2} \left(1 + \frac{2e}{\sqrt{l^2 - h^2}}\right)$



CD:  $\sum M_c = 0:$

$Q_y d - \frac{A}{2} (2d) = 0$

$Q = Q_y = A = \frac{W}{2} \left(1 + \frac{2e}{\sqrt{l^2 - h^2}}\right)$



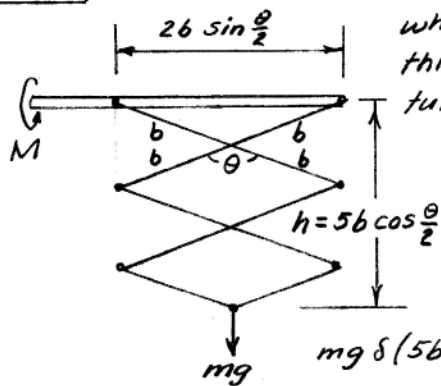
(From drawing,  $d > e$ ,)  
 So  $Q \approx \frac{W}{2}$

7/22

$$L = \text{lead of screw}; \delta(2b \sin \frac{\theta}{2}) = \frac{\delta\alpha}{2\pi} L$$

where  $\alpha$  is the angle through which the screw turns. Thus

$$\delta\alpha = \frac{2\pi b}{L} \cos \frac{\theta}{2} \delta\theta$$

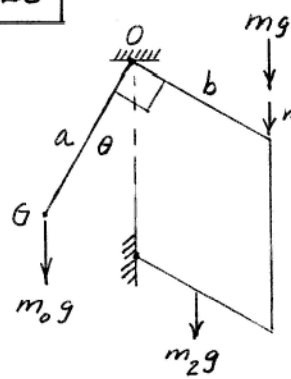


$$\delta U = 0; mg \delta h + M \delta\alpha = 0$$

$$mg \delta(5b \cos \frac{\theta}{2}) + M (\frac{2\pi b}{L} \cos \frac{\theta}{2} \delta\theta) = 0$$

$$-\frac{5}{2} mgb \sin \frac{\theta}{2} \delta\theta + \frac{2\pi b M}{L} \cos \frac{\theta}{2} \delta\theta = 0, \quad M = \frac{5m g L}{4\pi} \tan \frac{\theta}{2}$$

7/23



$$\delta U = 0; m_0 g \delta(a \cos \theta)$$

$$+ (m + m_1) g \delta(b \sin \theta)$$

$$+ m_2 g \delta\left(\frac{b}{2} \sin \theta\right) = 0$$

$$- m_0 a \sin \theta \delta \theta + (m + m_1) b \cos \theta \delta \theta$$

$$+ \frac{m_2 b}{2} \cos \theta \delta \theta = 0$$

$$m_0 a \tan \theta = mb + (m_1 + \frac{m_2}{2})b \quad \text{--- (1)}$$

When  $\theta = \theta_0$ ,  $mg = 0$  so

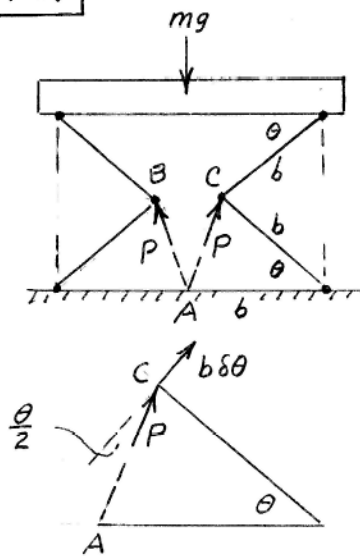
$$m_0 a \tan \theta_0 = (m_1 + \frac{m_2}{2})b \quad \text{--- (2)}$$

Eliminate  $m_1$  &  $m_2$  from Eqs. (1) & (2) & get

$$m_0 a \tan \theta = mb + m_0 a \tan \theta_0$$

$$m = \frac{a}{b} m_0 (\tan \theta - \tan \theta_0)$$

7/24



Work done by each  $P$  is

$$(P \cos \frac{\theta}{2}) b \delta \theta$$

Work done by  $mg$  is

$$-mg \delta (2b \sin \theta)$$

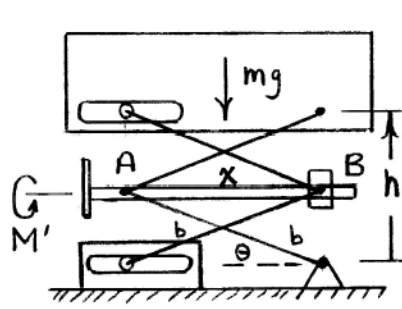
$$\delta U = 0; 2Pb \cos \frac{\theta}{2} \delta \theta - 2bmg \cos \theta \delta \theta = 0$$

$$P \cos \frac{\theta}{2} = mg \cos \theta$$

$$P = mg \frac{\cos \theta}{\cos \theta/2}$$



7/25 |  $M'$  = necessary moment without friction



Let  $\beta$  = angle through which screw turns

$$\delta U = 0: M' \delta \beta - mg \delta h = 0$$

$$\frac{L}{2\pi} = \frac{-\delta(x)}{\delta \beta}, \delta \beta = \frac{2\pi}{L} (-\delta x)$$

$$x = 2b \cos \theta, \delta x = -2b \sin \theta \delta \theta$$

$$\delta \beta = \frac{4\pi b}{L} \sin \theta \delta \theta$$

$$h = 4b \sin \theta, \delta h = 4b \cos \theta \delta \theta$$

$$\text{Thus } M' \frac{4\pi b}{L} \sin \theta \delta \theta - mg (4b \cos \theta \delta \theta) = 0$$

$$M' = \frac{mgL}{\pi} \cot \theta$$

$$M = M_f + \frac{mgL}{\pi} \cot \theta$$

$$7/26 \quad \delta U = 0; \quad 2C \delta a + P \delta c = 0$$

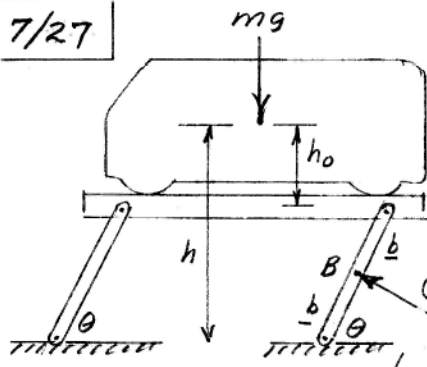
From rotation of jaw about fixed pivot

$\delta a = \frac{b}{d+c} \delta e$ ; also if  $l =$  length of  
connecting link,  $l^2 = e^2 + c^2$

$$0 = 2e \delta e + 2c \delta c, \quad \delta c = -\frac{e}{c} \delta e$$

$$\text{Thus } 2C \frac{b}{d+c} \delta e + P \left(-\frac{e}{c} \delta e\right) = 0, \quad C = \frac{P}{2} \frac{e(d+c)}{bc}$$

7/27



$$\text{Let } \bar{AB} = l$$

$$\delta U = 0; C \delta l - mg \delta h = 0$$

$$l^2 = (b \sin \theta)^2 + (L - b \cos \theta)^2$$

$$2l \delta l = 2b^2 \sin \theta \cos \theta \delta \theta$$

$$+ 2(L - b \cos \theta)(b \sin \theta \delta \theta)$$

$$= 2Lb \sin \theta \delta \theta$$

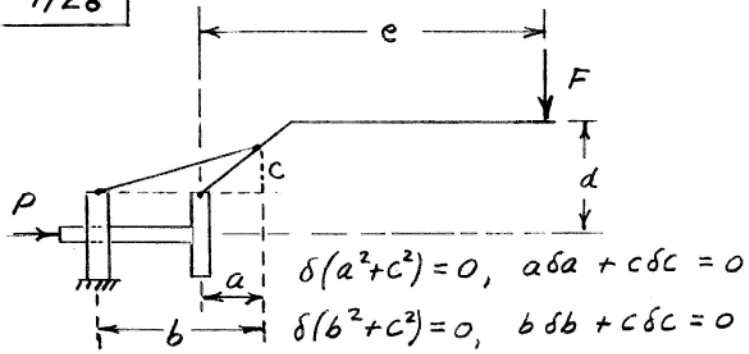
$$\delta l = \frac{Lb}{l} \sin \theta \delta \theta$$

$$h = 2b \sin \theta + h_0, \quad \delta h = 2b \cos \theta \delta \theta + 0$$

$$\text{Thus } C \frac{Lb}{l} \sin \theta \delta \theta - mg (2b \cos \theta \delta \theta) = 0$$

$$C = 2mg \frac{l}{L} \cot \theta = \frac{2mg}{L} \sqrt{(b \sin \theta)^2 + (L - b \cos \theta)^2} \cot \theta$$

$$C = 2mg \sqrt{1 + \left(\frac{b}{L}\right)^2 - 2\frac{b}{L} \cos \theta} \cot \theta$$

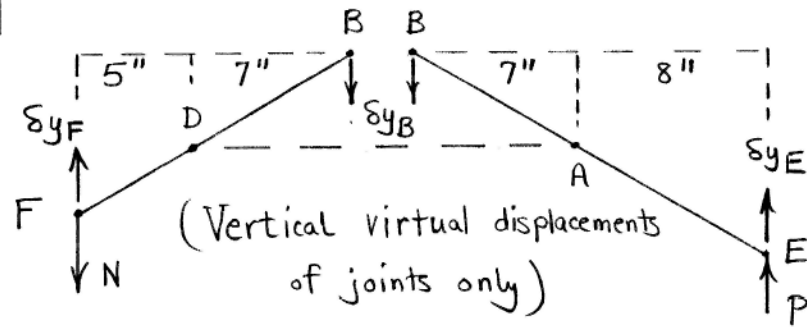


$$\delta U=0; P\delta(b-a) - 2F\delta d = 0 \text{ where } \delta d = \frac{e}{a}\delta c$$

$$\text{so } P\left(-\frac{c}{b}\delta c + \frac{c}{a}\delta c\right) - 2F\frac{e}{a}\delta c = 0$$

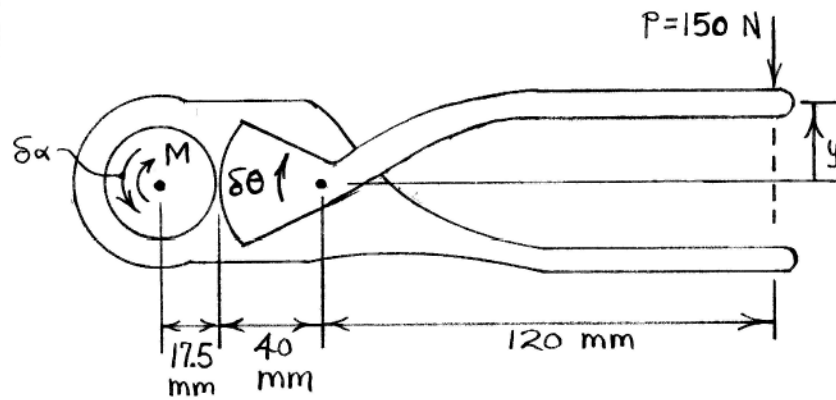
$$\frac{Pc}{ab}(b-a) = \frac{2Fe}{a}, \quad P = \frac{2Feb}{c(b-a)}$$

7/29



$$\delta y_B = \frac{7}{8} \delta y_E, \quad \delta y_F = \frac{5}{7} \delta y_B$$
$$\text{So } \delta y_F = \frac{5}{7} \cdot \frac{7}{8} \delta y_E = \frac{5}{8} \delta y_E$$
$$\delta U = 0: \quad P \delta y_E - N \delta y_F = 0$$
$$P \delta y_E = N \left( \frac{5}{8} \delta y_E \right)$$
$$N = \frac{8}{5} P = \underline{1.6 P}$$

7/30



$\delta\alpha$  = rotation of socket on bolt head  
 $\delta\theta$  = rotation of upper handle (lower handle and frame taken as fixed)

$$17.5 \delta\alpha = 40 \delta\theta, \quad \delta y = -120 \delta\theta$$

$$\delta U = 0 : -M \delta\alpha + P(-\delta y) = 0$$

$$M \left( \frac{40}{17.5} \delta\theta \right) = 150 (120 \delta\theta)$$

$$M = 7880 \text{ N}\cdot\text{mm} \text{ or } \underline{M = 7.88 \text{ N}\cdot\text{m}}$$

$$7/31 \quad \delta U = 0; \quad -P\delta x - W\delta h = 0$$

$$W = 250(9.81) \text{ N} \quad x^2 + y^2 = l^2, \quad x\delta x = -y\delta y$$

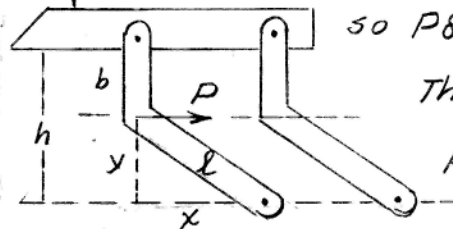
$$\delta h = \delta(b + y) = \delta y$$

$$\text{so } P\delta x = -W\delta y = -W\left(-\frac{x}{y}\right)\delta x$$

$$\text{Thus } P = W\frac{x}{y}$$

$$P = 250(9.81) \frac{500}{350} = 3500 \text{ N}$$

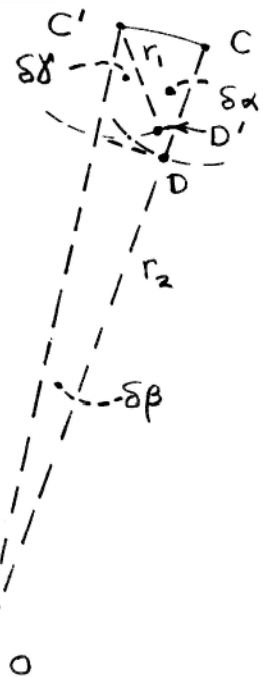
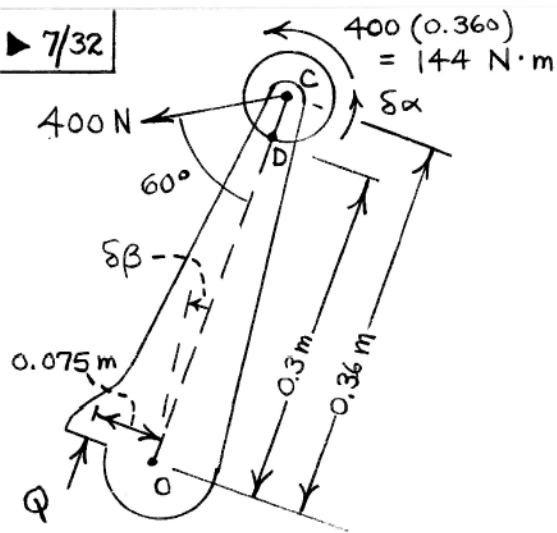
$$\text{or } \underline{P = 3.5 \text{ kN}}$$



$$h = 650 \text{ mm}, \quad b = 300 \text{ mm}$$

$$y = 350 \text{ mm}, \quad x = 500 \text{ mm}$$

► 7/32



During rotation  $\delta\beta$ , CD moves to  $C'D'$  & rotates through the

$$\begin{aligned} \text{absolute angle } \delta\alpha &= \delta\delta + \delta\beta = \frac{\text{arc}}{r_1} + \delta\beta \\ &= \frac{r_2 \delta\beta}{r_1} + \delta\beta = \left(\frac{r_2}{r_1} + 1\right) \delta\beta = \left(\frac{300}{60} + 1\right) \delta\beta = 6 \delta\beta \end{aligned}$$

$$\begin{aligned} \delta U = 0: & -Q(0.075) \delta\beta + 400 \cos 30^\circ (0.360 \delta\beta) \\ & + 144(6 \delta\beta) = 0 \quad , \quad \underline{Q = 13.18 \text{ kN}} \end{aligned}$$



$$\frac{7}{33} \mid V = 6x^4 - 3x^2 + 5$$

$$\frac{dV}{dx} = 24x^3 - 6x = 0 \text{ for equilibrium}$$

$$6x(4x^2 - 1) = 0 \Rightarrow x = 0, x = \pm \frac{1}{2}$$

$$\frac{d^2V}{dx^2} = 72x^2 - 6$$

$$\text{For } \underline{x = 0}, \quad \frac{d^2V}{d^2x} = -6 \quad \underline{\text{unstable}}$$

$$\underline{x = \frac{1}{2}}, \quad \frac{d^2V}{d^2x} = 18 - 6 = 12 \quad \underline{\text{stable}}$$

$$\underline{x = -\frac{1}{2}}, \quad \frac{d^2V}{d^2x} = 18 - 6 = 12 \quad \underline{\text{stable}}$$

7/34  $\delta =$  initial spring compression

$$V = V_g + V_e = mg \frac{L}{2} \cos \theta + \frac{1}{2} k (\delta + L \sin \theta)^2 + \frac{1}{2} k (\delta - L \sin \theta)^2$$

$$= \frac{1}{2} mgL \cos \theta + k (\delta^2 + L^2 \sin^2 \theta)$$

$$\frac{dV}{d\theta} = -\frac{1}{2} mgL \sin \theta + 2kL^2 \sin \theta \cos \theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2} mgL \cos \theta + 2kL^2 \cos 2\theta$$

For equilibrium,  $\frac{dV}{d\theta} = 0$ , so

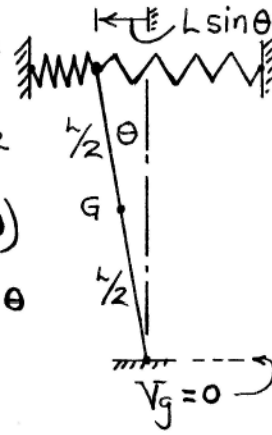
$$\left(-\frac{1}{2} mg + 2kL \cos \theta\right) \sin \theta = 0$$

$$\Rightarrow \theta = 0 \text{ or } \cos \theta = \frac{mg}{4kL}$$

$$\text{For } \theta = 0, \frac{d^2V}{d\theta^2} = -\frac{1}{2} mgL + 2kL^2$$


$> 0$  (Stable) if  $2kL^2 > \frac{1}{2} mgL$

$$\text{So } k_{\min} = \frac{mg}{4L}$$



7/35

$$V = V_g = mgb \cos \theta$$

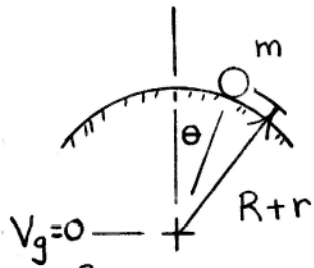

$$\frac{dV}{d\theta} = -mgb \sin \theta = 0 \text{ for equil.}$$
$$\theta = 0^\circ \text{ or } \theta = 180^\circ$$

$$V_g = 0 \quad \frac{d^2V}{d\theta^2} = -mgb \cos \theta$$

$$\theta = 0, \quad \frac{d^2V}{d\theta^2} = -mgb \text{ so } \underline{\text{unstable}}$$

$$\theta = 180^\circ, \quad \frac{d^2V}{d\theta^2} = -mgb(-1) = +mgb \quad \underline{\text{stable}}$$

7/36

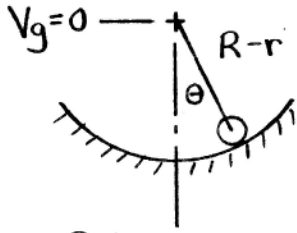


$$V = V_g = (R+r) \cos \theta$$

$$\frac{dV}{d\theta} = -(R+r) \sin \theta = 0 \quad \left( \begin{array}{l} \text{for} \\ \text{equil.} \end{array} \right)$$

$$\theta = 0, \pi \quad (\text{reject})$$

$$\frac{d^2V}{d\theta^2} = -(R+r) \cos \theta < 0 \quad @ \quad \theta = 0: \quad \underline{\text{unstable}}$$



$$V = V_g = -(R-r) \cos \theta$$

$$\frac{dV}{d\theta} = (R-r) \sin \theta = 0 \quad \left( \begin{array}{l} \text{for} \\ \text{equil.} \end{array} \right)$$

$$\theta = 0, \pi \quad (\text{reject})$$

$$\frac{d^2V}{d\theta^2} = (R-r) \cos \theta > 0 \quad @ \quad \theta = 0: \quad \underline{\text{stable}}$$

7/37

$$V = V_g + V_e$$

$$= mg(2b \cos \theta) + \frac{1}{2}k(2b \sin \theta)^2$$

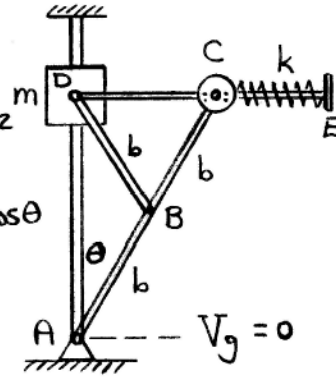
$$\frac{dV}{d\theta} = -2mgb \sin \theta + 4kb^2 \sin \theta \cos \theta$$

$$= 2b \sin \theta (-mg + 2kb \cos \theta)$$

$$= 0 \text{ for equilibrium}$$

$$\text{So } \sin \theta = 0 \text{ or } \theta = \cos^{-1} \frac{mg}{2kb}$$

$$\text{For } \theta_{\max} = 30^\circ, \quad k_{\min} = \frac{mg}{2b \cos 30^\circ} = \frac{mg}{b\sqrt{3}}$$



7/38 | Take  $V_g = 0$  through AO &  $V_e = 0$  when  $\theta = 0$

So  $V_g = -mgh = -60(9.81)(0.7 \sin \theta) = -412.0 \sin \theta$

$V_e = \frac{1}{2} kx^2 = \frac{1}{2} (160) [2(1.4) \sin \frac{\theta}{2}]^2 = 627.2 \sin^2 \frac{\theta}{2}$

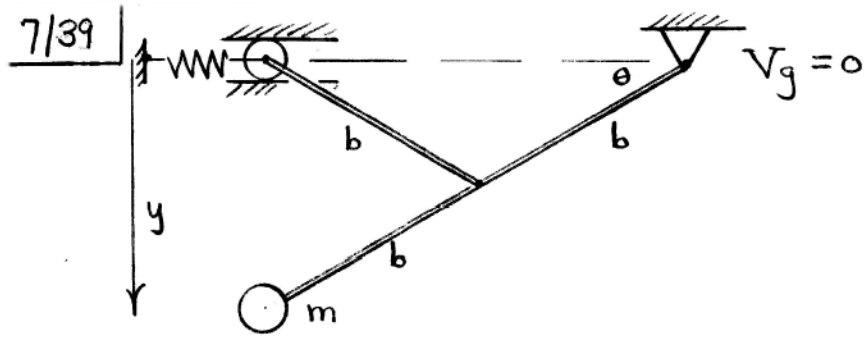
$V = V_e + V_g = 627.2 \sin^2 \frac{\theta}{2} - 412.0 \sin \theta$

$\frac{dV}{d\theta} = \frac{2}{2} (627.2) \sin \frac{\theta}{2} \cos \frac{\theta}{2} - 412.0 \cos \theta$

$= 313.6 \sin \theta - 412.0 \cos \theta = 0$  for equil.

or  $\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{412.0}{313.6} = 1.314$

$\theta = 52.7^\circ$



$$\text{Spring stretch} = 2b - 2b \cos \theta = 2b(1 - \cos \theta)$$

$$V_e = \frac{1}{2} k [2b(1 - \cos \theta)]^2 = 2kb^2(1 - \cos \theta)^2$$

$$V_g = -mg(2b \sin \theta) = -2mgb \sin \theta$$

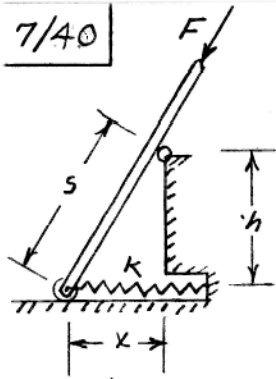
$$V = 2kb^2(1 - \cos \theta)^2 - 2mgb \sin \theta$$

$$\frac{dV}{d\theta} = 4kb^2(1 - \cos \theta) \sin \theta - 2mgb \cos \theta = 0$$

(for equilibrium)

$$2kb(1 - \cos \theta) \sin \theta = mg \cos \theta, \quad k = \frac{mg}{2b} \frac{\cot \theta}{1 - \cos \theta}$$

7/40



$$\delta U = F \delta s = F \delta (\sqrt{h^2 + x^2})$$

$$= \frac{F x \delta x}{\sqrt{h^2 + x^2}}$$

$$\delta V_e = k x \delta x$$

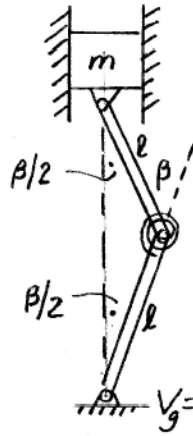
$$\delta U = \delta V_e ; \frac{F x \delta x}{\sqrt{h^2 + x^2}} = k x \delta x$$

Sol. is  $x=0$  or  $\frac{F}{\sqrt{h^2 + x^2}} = k, \left(\frac{F}{k}\right)^2 = h^2 + x^2$

$x = \sqrt{(F/k)^2 - h^2}$  provided  $k < F/h$



7/41



$$V_g = 2mgl \cos \frac{\beta}{2}, \quad V_e = \frac{1}{2} K \beta^2$$

$$V = V_g + V_e = 2mgl \cos \frac{\beta}{2} + \frac{1}{2} K \beta^2$$

$$\frac{dV}{d\beta} = -mgl \sin \frac{\beta}{2} + K\beta, \quad \frac{dV}{d\beta} = 0 \text{ for } \beta = 0$$

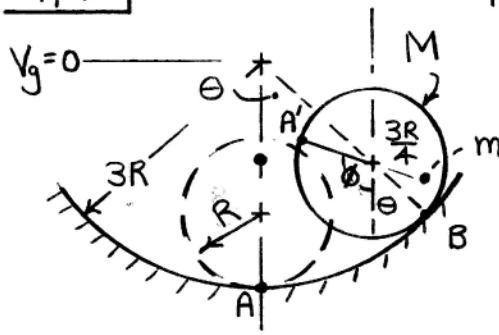
$$\frac{d^2V}{d\beta^2} = -\frac{1}{2} mgl \cos \frac{\beta}{2} + K$$

$$= -\frac{1}{2} mgl + K \text{ for } \beta = 0$$

$$= (+) \text{ stable if } K > \frac{1}{2} mgl$$

$$\text{Thus } \underline{K_{min} = \frac{1}{2} mgl}$$

7/42



$$\text{Arc } \overline{AB} = \text{Arc } \overline{A'B}$$

$$3R\theta = R(\theta + \phi)$$

$$\phi = 2\theta$$

$$V = V_g = -Mg(2R\cos\theta) - mg\left(2R\cos\theta - \frac{3R}{4}\cos\phi\right)$$

$$= -2(M+m)gR\cos\theta + \frac{3}{4}mgR\cos 2\theta$$

$$\frac{dV}{d\theta} = 2(M+m)gR\sin\theta - \frac{3}{2}mgR\sin 2\theta$$

= 0 for equilibrium;  $\theta = 0$  is desired solution.

$$\frac{d^2V}{d\theta^2} = 2(M+m)gR\cos\theta - 3mgR\cos 2\theta$$

For  $\theta = 0$ :  $2(M+m)gR - 3mgR > 0$  for stability

or  $M > \frac{m}{2}$

7/43 | Take  $V_g = 0$  through bearing

$$V = V_g = mg(2a \cos \theta) + mg(a \cos 2\theta) \\ = mga(2 \cos \theta + \cos 2\theta)$$

$$\frac{dV}{d\theta} = mga(-2 \sin \theta - 2 \sin 2\theta) = -2mga(\sin \theta + \sin 2\theta)$$

$$\frac{d^2V}{d\theta^2} = -2mga(\cos \theta + 2 \cos 2\theta)$$

For equil.  $\frac{dV}{d\theta} = 0$  so  $\sin \theta = -\sin 2\theta$

or  $\sin \theta(1 + 2 \cos \theta) = 0$ ;  $\sin \theta = 0$ ,  $\cos \theta = -1/2$

So sols. of interest are  $\theta = 0$ ,  $\theta = 180^\circ$ ,  
 $\theta = 120^\circ$ ,  $\theta = 240^\circ$

$$\theta = 0, \quad d^2V/d\theta^2 = -2mga(1 + 2) = (-) \text{ unstable}$$

$$\theta = 120^\circ, \quad d^2V/d\theta^2 = -2mga(-\frac{1}{2} - 2[\frac{1}{2}]) = (+) \text{ stable}$$

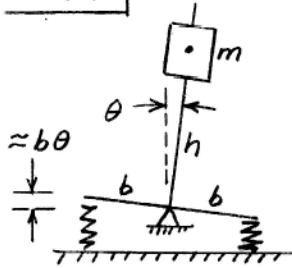
$$\theta = 180^\circ, \quad d^2V/d\theta^2 = -2mga(-1 + 2) = (-) \text{ unstable}$$

$$\theta = 240^\circ, \quad d^2V/d\theta^2 = -2mga(-1 + 2[-\frac{1}{2}]) = (+) \text{ stable}$$



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$$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta)^2 + \frac{1}{2}k(\Delta - b\theta)^2 + mgh \cos \theta$$

for  $\theta$  small

$$V = k(\Delta^2 + b^2\theta^2) + mgh \cos \theta$$

$$\frac{dV}{d\theta} = 2kb^2\theta - mgh \sin \theta$$

$$\frac{d^2V}{d\theta^2} = 2kb^2 - mgh \cos \theta$$

Let preset of springs be  $\Delta$  when  $\theta = 0$

For  $\theta \rightarrow 0$ ,  $\frac{d^2V}{d\theta^2}$  is (+) if  $2kb^2 > mgh$

Thus  $\theta = 0$  is stable if  $h < \frac{2kb^2}{mg}$

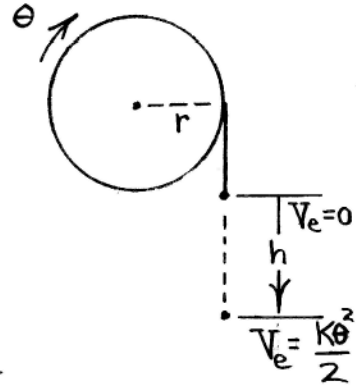
7/45

$$\delta U' = \delta V: 0 - mgr \delta \theta + \delta \left( \frac{1}{2} K \theta^2 \right)$$

$$mgr = K \theta$$

$$\text{With } h = r\theta : mgr = K \left( \frac{h}{r} \right)$$

$$h = \frac{mgr^2}{K}$$



$$\underline{7/46} \quad \delta U' = \delta V_g + \delta V_e$$

$$\delta U' = -P \delta (2b \sin \frac{\theta}{2}) = -Pb \cos \frac{\theta}{2} \delta \theta$$

$$\delta V_g = mg \delta (b \sin \frac{\theta}{2}) = mg \frac{b}{2} \cos \frac{\theta}{2} \delta \theta$$

$$\delta V_e = \delta \left[ \frac{1}{2} k (2b \cos \frac{\theta}{2} - 0)^2 \right]$$

$$= -2kb^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \delta \theta$$

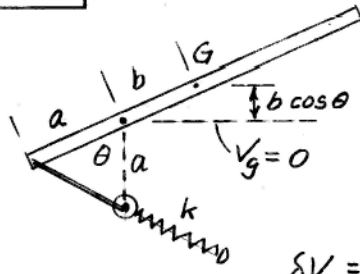
$$\text{Thus } -Pb \cos \frac{\theta}{2} \delta \theta = mg \frac{b}{2} \cos \frac{\theta}{2} \delta \theta - 2kb^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \delta \theta$$

$$\text{or } \left( P + \frac{mg}{2} \right) \cos \frac{\theta}{2} = 2kb \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\cos \frac{\theta}{2} = 0, \quad \theta = 180^\circ \text{ (collapsed position)}$$

$$\sin \frac{\theta}{2} = \left( P + \frac{mg}{2} \right) / 2kb \quad \text{or } \theta = 2 \sin^{-1} \frac{P + \frac{mg}{2}}{2kb}$$

7/47



Spring compression is

$$x = 2a \sin \frac{\theta}{2}$$

$$\begin{aligned} \delta V_e &= kx \delta x = 2ka \sin \frac{\theta}{2} \delta(2a \sin \frac{\theta}{2}) \\ &= 2ka^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \delta\theta \\ &= ka^2 \sin \theta \delta\theta \end{aligned}$$

$$\delta V_g = \delta(mgb \cos \theta) = -mgb \sin \theta \delta\theta$$

$$\delta V_e + \delta V_g = 0; \quad ka^2 \sin \theta \delta\theta - mgb \sin \theta \delta\theta = 0$$

$$(ka^2 - mgb) \sin \theta = 0$$

$$\text{Equil. for any } \theta \text{ if } \underline{k = \frac{mgb}{a^2}}$$

7/48

$$\text{Length } AB = 2(20) \cos \frac{\theta}{2} \text{ (in.)}$$

$$\text{Unstretched length} = 40 - 4 = 36 \text{ in.}$$

Spring stretch for arbitrary  $\theta$

$$\text{is } x = 40 \cos \frac{\theta}{2} - 36 \text{ in.}$$

$$V_e = 2 \frac{1}{2} k (40 \cos \frac{\theta}{2} - 36)^2$$

$$= k [1600 \cos^2 \frac{\theta}{2} - 2880 \cos \frac{\theta}{2} + 1296] \text{ in.-lb}$$

$$V_g = -3(20 \cos \theta) = -60 \cos \theta \text{ in.-lb}$$

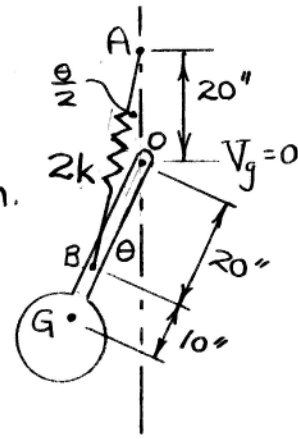
$$V = V_e + V_g = k [1600 \cos^2 \frac{\theta}{2} - 2880 \cos \frac{\theta}{2} + 1296] - 60 \cos \theta \text{ in.-lb}$$

$$\frac{dV}{d\theta} = k [-1600 \sin \frac{\theta}{2} \cos \frac{\theta}{2} + 1440 \sin \frac{\theta}{2}] + 60 \sin \theta$$

$$= k [-800 \sin \theta + 1440 \sin \frac{\theta}{2}] + 60 \sin \theta$$

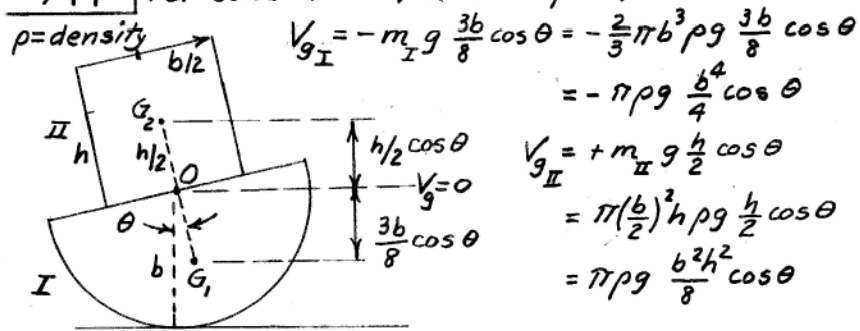
$$\frac{d^2V}{d\theta^2} = k [-800 \cos \theta + 720 \cos \frac{\theta}{2}] + 60 \cos \theta$$

$$\left( \frac{d^2V}{d\theta^2} \right)_{\theta=0} = k [-800 + 720] + 60 > 0 \text{ (Stable) if } k \text{ does not exceed } \frac{60}{80} = \underline{0.75 \text{ lb/in.} = k_{\max}}$$





7/49 | For solid hemisphere  $\overline{OG}_1 = \frac{3b}{8}$



$$\begin{aligned}
 \rho &= \text{density} \\
 V_{gI} &= -m_I g \frac{3b}{8} \cos \theta = -\frac{2}{3} \pi b^3 \rho g \frac{3b}{8} \cos \theta \\
 &= -\pi \rho g \frac{b^4}{4} \cos \theta \\
 V_{gII} &= +m_{II} g \frac{h}{2} \cos \theta \\
 &= \pi \left(\frac{b}{2}\right)^2 h \rho g \frac{h}{2} \cos \theta \\
 &= \pi \rho g \frac{b^2 h^2}{8} \cos \theta
 \end{aligned}$$

$$V = V_{II} + V_I = \pi \rho g \frac{b^2}{4} \left(\frac{h^2}{2} - b^2\right) \cos \theta$$

$$\frac{d^2V}{d\theta^2} = -\pi \rho g \frac{b^2}{4} \left(\frac{h^2}{2} - b^2\right) \cos \theta = +\pi \rho g \frac{b^2}{4} \left(b^2 - \frac{h^2}{2}\right) \cos \theta$$

= (+) stable if  $\frac{h^2}{2} < b^2$  or  $h < b\sqrt{2}$

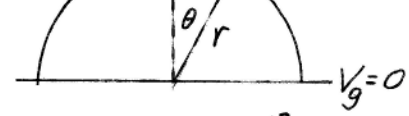
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cylinder

$$\bar{r} = \frac{4r}{3\pi}$$

shell

$$\bar{r} = \frac{2r}{\pi}$$



$$V = V_g = mg(2r \cos \theta - \bar{r} \cos 2\theta)$$

$$\frac{dV}{d\theta} = mg(-2r \sin \theta + 2\bar{r} \sin 2\theta)$$

$$= 2mg(-r \sin \theta + \bar{r} \sin 2\theta)$$

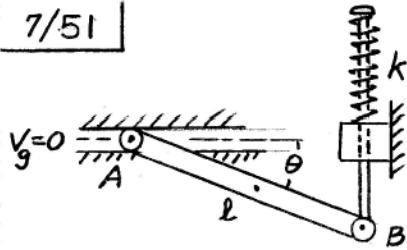
$$\frac{d^2V}{d\theta^2} = 2mg(-r \cos \theta + 2\bar{r} \cos 2\theta)$$

For  $\theta=0$ ,  $\frac{d^2V}{d\theta^2} = 2mrg(-1 + \frac{2\bar{r}}{r})$

For cylinder  $\bar{r}/r = \frac{4}{3\pi}$ ,  $\frac{d^2V}{d\theta^2} = 2mrg(-1 + \frac{8}{3\pi}) = (-)$   
unstable

For shell  $\bar{r}/r = \frac{2}{\pi}$ ,  $\frac{d^2V}{d\theta^2} = 2mrg(-1 + \frac{4}{\pi}) = (+)$   
stable

7/51

Spring compressed  $l \sin \theta$ 

$$\text{so } V_e = \frac{1}{2} k (l \sin \theta)^2$$

$$V_g = -mg \frac{l}{2} \sin \theta$$

$$V = V_e + V_g = \frac{k}{2} l^2 \sin^2 \theta - mg \frac{l}{2} \sin \theta$$

$$\frac{dV}{d\theta} = k l^2 \sin \theta \cos \theta - mg \frac{l}{2} \cos \theta = \frac{k l^2}{2} \sin 2\theta - mg \frac{l}{2} \cos \theta$$

$$= l \cos \theta (k l \sin \theta - mg/2) = 0 \text{ for equil.}$$

$$(1) \cos \theta = 0, \theta = \pi/2$$

$$(2) \sin \theta = \frac{mg}{2kl}$$

$$\frac{d^2V}{d\theta^2} = k l^2 \cos 2\theta + mg \frac{l}{2} \sin \theta$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(1)} = k l^2 \cos \pi + mg \frac{l}{2} (1) = k l^2 \left(-1 + \left[\frac{mg}{2kl}\right]\right)$$

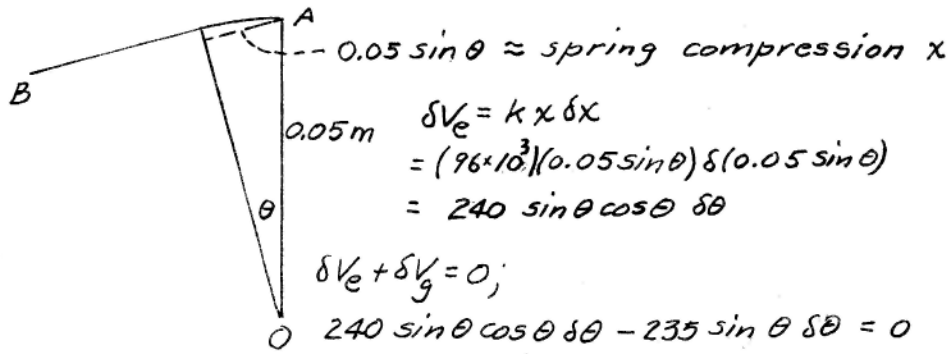
$$= (+) \text{ stable if } k < \frac{mg}{2l}$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(2)} = k l^2 \left(1 - 2 \left[\frac{mg}{2kl}\right]^2\right) + \frac{mg l}{2} \frac{mg}{2kl} = k l^2 \left[1 - \left(\frac{mg}{2kl}\right)^2\right]$$

$$= (+) \text{ stable if } k > \frac{mg}{2l}$$

$$\frac{7}{52} \quad \delta V_g = \delta(0.3 mg \cos \theta) = -0.3(80)(9.81) \sin \theta \delta \theta$$

$$= -235 \sin \theta \delta \theta$$



$$\delta V_e = k x \delta x$$

$$= (96 \times 10^3)(0.05 \sin \theta) \delta(0.05 \sin \theta)$$

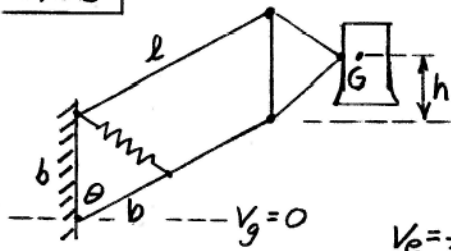
$$= 240 \sin \theta \cos \theta \delta \theta$$

$$\delta V_e + \delta V_g = 0;$$

$$240 \sin \theta \cos \theta \delta \theta - 235 \sin \theta \delta \theta = 0$$

$$\sin \theta = 0 \text{ or } \cos \theta = \frac{235}{240} = 0.9810, \quad \theta = 11.19^\circ$$

7/53



$$V_g = mg(l \cos \theta + h)$$

$$\text{Spring length} = 2b \sin \frac{\theta}{2}$$

$$\text{stretch } x = 2b \sin \frac{\theta}{2} - \frac{b}{2}$$

$$= \frac{b}{2} (4 \sin \frac{\theta}{2} - 1)$$

$$V_e = \frac{1}{2} k x^2 = \frac{k b^2}{8} (4 \sin \frac{\theta}{2} - 1)^2$$

$$V = V_e + V_g; \quad \frac{dV}{d\theta} = -mg l \sin \theta + \frac{k b^2}{4} (4 \sin \frac{\theta}{2} - 1) 2 \cos \frac{\theta}{2}$$

$$= (k b^2 - mg l) \sin \theta - \frac{k b^2}{2} \cos \frac{\theta}{2} = 0 \text{ for equil.}$$

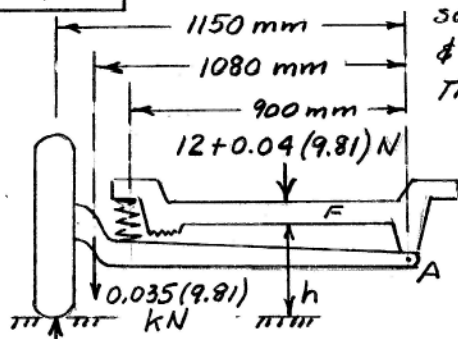
$$\text{Thus } [2(k b^2 - mg l) \sin \frac{\theta}{2} - \frac{k b^2}{2}] \cos \frac{\theta}{2} = 0$$

$$\& \text{ [ ]} = 0 \text{ gives } k = \frac{mg l}{b^2} \frac{1}{1 - \frac{1}{4} \csc \frac{\theta}{2}}$$

$$\frac{d^2V}{d\theta^2} = (k b^2 - mg l) \cos \theta + \frac{k b^2}{4} \sin \frac{\theta}{2}; \quad \text{substitute } \sin \frac{\theta}{2} = \frac{k b^2}{4(k b^2 - mg l)}$$

$$\& \text{ get } \frac{d^2V}{d\theta^2} = + \text{ (stable) within range of } \theta = 29^\circ, k = \infty \text{ to } \theta = 180^\circ, k = \frac{4mg l}{3b^2}$$

► 7/54 | Let  $x$  = compression of spring (most easily seen by considering A fixed & wheels moving up)  
 Thus  $x = \frac{900}{1150} (0.35 - h)$  meters



With F & hence A fixed,

$$\begin{aligned} \delta U' &= -2(6.54) \delta h \\ &\quad + 2(0.035) 9.81 \frac{1080}{1150} \delta h \\ &= (-13.08 + 0.645) \delta h \\ &= -12.43 \delta h \end{aligned}$$

$$\begin{aligned} P &= 6 + 0.035(9.81) + 0.02(9.81) \\ &= 6.54 \text{ kN} \end{aligned}$$

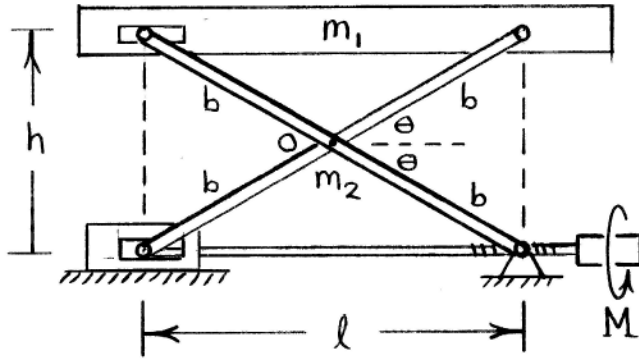
$$\begin{aligned} \delta V_e &= 2(kx \delta x) = 2(120) \frac{900}{1150} (0.35 - h) \frac{900}{1150} (-\delta h) \\ &= -147.0 (0.35 - h) \delta h \end{aligned}$$

$$\delta U' = \delta V_e; \quad -12.43 \delta h = -147.0 (0.35 - h) \delta h$$

$$h = 0.35 - \frac{12.43}{147.0} = 0.265 \text{ m or } \underline{h = 265 \text{ mm}}$$

► 7/55

Let  $\beta =$  rotation angle of screw  
 $p =$  screw pitch



$$\delta U' = \delta V_g$$

$$\delta U' = M \delta \beta$$

$$\delta V_g = m_1 g \delta h + m_2 g \delta \left(\frac{h}{2}\right) = \left(m_1 + \frac{m_2}{2}\right) g \delta h$$

$$\delta h = \delta (2b \sin \theta) = 2b \cos \theta \delta \theta$$

$$l = 2b \cos \theta, \quad \delta l = -2b \sin \theta \delta \theta$$

$$\left| \frac{\delta l}{\delta \beta} \right| = \frac{p}{2\pi}, \quad \text{so } |\delta \beta| = \frac{2\pi}{p} |\delta l| = \frac{4\pi b}{p} \sin \theta \delta \theta$$

$$\text{So } M \frac{4\pi b}{p} \sin \theta \delta \theta = \left(m_1 + \frac{m_2}{2}\right) g (2b \cos \theta \delta \theta)$$

$$M = \frac{(2m_1 + m_2) p g}{4\pi} \cot \theta$$

► 7/56 For  $\frac{1}{2}$  of door  $V_g = \frac{mgr}{2} (1 - \cos \theta)$

Spring length  $l = \sqrt{r^2 + a^2 + 2ra \cos \theta}$

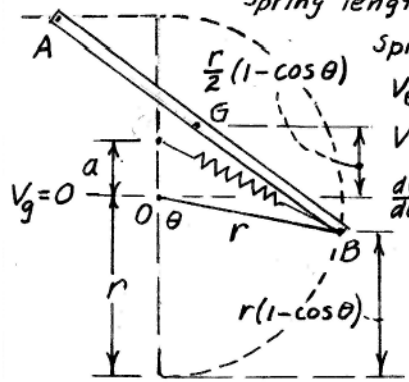
Spring stretch is  $l - (r - a)$

$$V_e = \frac{1}{2} k (l - [r - a])^2$$

$$V = V_g + V_e = \frac{mgr}{4} (1 - \cos \theta) + \frac{k}{2} (l - [r - a])^2$$

$$\frac{dV}{d\theta} = \frac{mgr}{4} \sin \theta + k (l - [r - a]) \frac{-2ra \sin \theta}{2l}$$

$$= \left(\frac{mgr}{4} - kar\right) \sin \theta + \frac{kar(r-a) \sin \theta}{\sqrt{r^2 + a^2 + 2ra \cos \theta}}$$



$$\frac{d^2V}{d\theta^2} = \left(\frac{mgr}{4} - kar\right) \cos \theta + kar(r-a) \frac{l \cos \theta - \sin \theta (-ra \sin \theta)}{l^2}$$

For  $\theta = 0$ ,  $\frac{d^2V}{d\theta^2} = \frac{mgr}{4} - kar + kar(r-a) \left(\frac{1}{r+a} + 0\right)$

= 0 for neutral equil. (insensitive response)

Thus  $\frac{mgr}{4} = kar \left(1 - \frac{r-a}{r+a}\right)$ ,  $k = \frac{mg(r+a)}{8a^2}$



$$\frac{7}{57} \quad x = k\theta, \quad 0.060 = k(2\pi), \quad k = \frac{0.030}{\pi} \frac{\text{m}}{\text{rad}}$$

$$\delta U = 0; \quad M\delta\theta - P\delta x = 0, \quad P = M \frac{\delta\theta}{\delta x} = M/k$$

$$\text{so } P = \frac{\pi}{0.030} 10 = \underline{1047 \text{ N}}$$

7/58 | Force & moment equilibrium (A)

(a), (b), (d)

Virtual work (B)

(c), (e), (f)

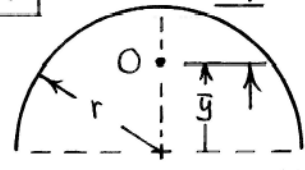
(c)  $\delta U' = \delta V_g$

(e)  $\delta U' = \delta V_g + \delta V_e$

(f)  $d^2V/d\theta^2$  must be (+)

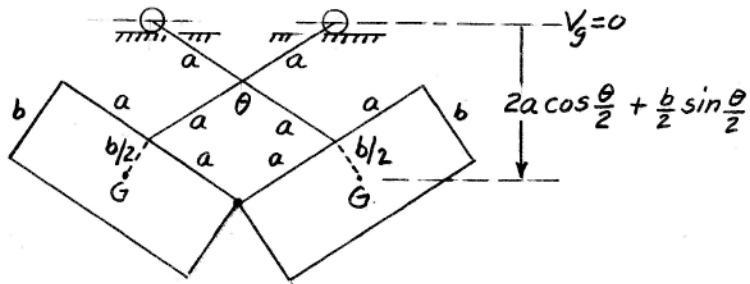
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The pivot  $O$  must be at or above the mass center  $G$  of the shell, which is located at  $\bar{y} = \frac{2r}{\pi}$ . So



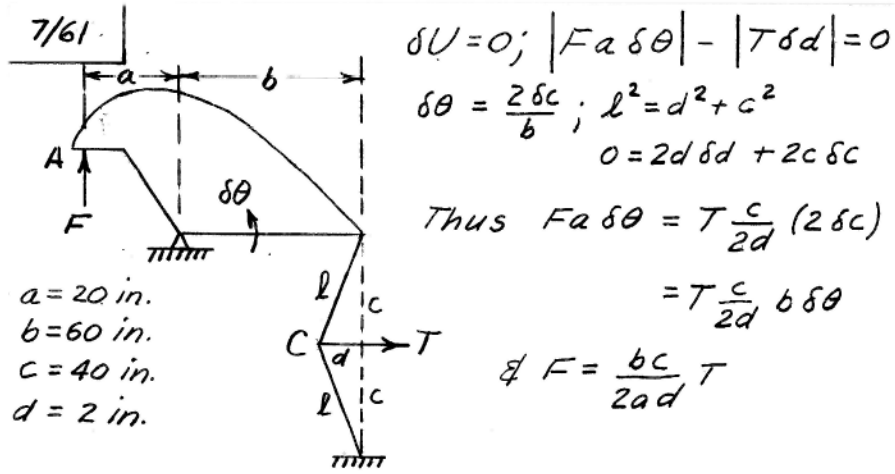
$$h_{\max} = r - \bar{y} = r \left(1 - \frac{2}{\pi}\right) = \underline{0.363r}$$

$$7/60 \quad V = V_g = -2mg \left[ 2a \cos \frac{\theta}{2} + \frac{b}{2} \sin \frac{\theta}{2} \right]$$



$$\frac{dV}{d\theta} = -2mg \left[ -a \sin \frac{\theta}{2} + \frac{b}{4} \cos \frac{\theta}{2} \right] = 0 \text{ for equil.}$$

$$\tan \frac{\theta}{2} = \frac{b}{4a}, \quad \theta = 2 \tan^{-1} \frac{b}{4a}; \quad \text{For } b=a, \quad \theta = 2 \tan^{-1} \frac{1}{4} = 28.1^\circ$$

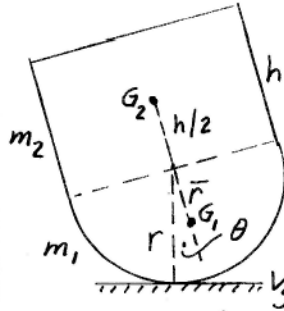


$$\text{So } F = \frac{60(40)}{2(20)(2)} (2000)(16) = \underline{\underline{960,000 \text{ lb}}}$$

7/62 | Let  $\rho =$  mass per unit area of shell

$$m_1 = 2\pi r^2 \rho, \quad m_2 = 2\pi r h \rho$$

$$\bar{r} = r/2 \text{ for hemispherical shell}$$



$$V = V_{g_1} + V_{g_2}$$

$$= 2\pi r^2 \rho g (r - \bar{r} \cos \theta) + 2\pi r h \rho g (r + \frac{h}{2} \cos \theta)$$

$$= 2\pi r \rho g [(r^2 + hr) - \frac{1}{2}(r^2 - h^2) \cos \theta]$$

$$\frac{dV}{d\theta} = 2\pi r \rho g [0 + \frac{1}{2}(r^2 - h^2) \sin \theta]$$

$$\frac{d^2V}{d\theta^2} = \pi r \rho g (r^2 - h^2) \cos \theta$$

For equil.  $\frac{dV}{d\theta} = 0$  gives  $\theta = 0$  &  $h = r$

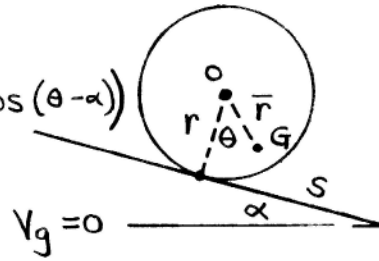
For  $\theta = 0$ ,  $\frac{d^2V}{d\theta^2} = (+)$  if  $h < r$

For  $h = r$ , neutral equilibrium

7/63

$$V_g = W(s \sin \alpha + r \cos \alpha - \bar{r} \cos(\theta - \alpha))$$

$$\text{But } s = s_0 + r\theta, \quad \frac{ds}{d\theta} = r$$



$$V_g = 0$$

$$\text{So } \frac{dV_g}{d\theta} = W(r \sin \alpha + \bar{r} \sin(\theta - \alpha))$$

$$\text{For } \alpha = 10^\circ, \quad r = 100 \text{ mm}, \quad \bar{r} = 60 \text{ mm}:$$

$$\frac{dV_g}{d\theta} = W[0.1 \sin 10^\circ + 0.060 \sin(\theta - 10^\circ)]$$

$$= 0 \quad \text{for equilibrium}$$

$$-0.1 \sin 10^\circ = 0.060 \sin(\theta - 10^\circ)$$

$$\theta - 10^\circ = \sin^{-1}\left(-\frac{0.1}{0.06} \sin 10^\circ\right) = -16.82^\circ \text{ or } 196.8^\circ$$

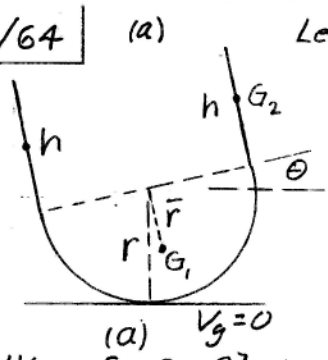
$$\Rightarrow \underline{\theta = -6.82^\circ} \quad \text{or} \quad \underline{\theta = 207^\circ}$$

$$\frac{d^2V_g}{d\theta^2} = W[0 + \bar{r} \cos(\theta - \alpha)]$$

$$\theta = -6.82^\circ: \quad \frac{d^2V_g}{d\theta^2} = W\bar{r} \cos(-16.82^\circ) > 0 \quad \underline{\text{Stable}}$$

$$\theta = 207^\circ: \quad \frac{d^2V_g}{d\theta^2} = W\bar{r} \cos(196.8^\circ) < 0 \quad \underline{\text{Unstable}}$$

7/64



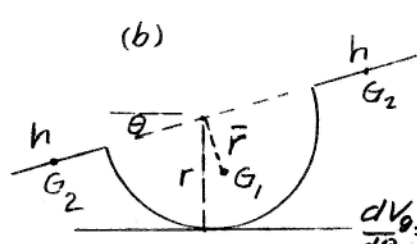
Let  $\rho$  = mass per unit periphery of shell  
 $\bar{r} = 2r/\pi$

$$V_g = \rho \left[ \pi r \left( r - \frac{2r}{\pi} \cos \theta \right) + h \left( r + r \sin \theta + \frac{h}{2} \cos \theta \right) + h \left( r - r \sin \theta + \frac{h}{2} \cos \theta \right) \right]$$

$$= \rho \left[ \pi r^2 - 2r^2 \cos \theta + 2hr + h^2 \cos \theta \right]$$

$$\frac{dV_g}{d\theta} = \rho [2r^2 - h^2] \sin \theta, \quad \frac{d^2V_g}{d\theta^2} = \rho [2r^2 - h^2] \cos \theta$$

Equil. at  $\theta = 0$  stable if  $h < r\sqrt{2}$   
 unstable if  $h > r\sqrt{2}$   
 Neutral equil. if  $h = r\sqrt{2}$  for any  $\theta$



$$V_g = \rho \left[ \pi r \left( r - \frac{2r}{\pi} \cos \theta \right) + h \left( r + \left[ r + \frac{h}{2} \sin \theta \right] \right) + h \left( r - \left[ r + \frac{h}{2} \sin \theta \right] \right) \right]$$

$$= \rho \left[ \pi r^2 - 2r^2 \cos \theta + 2hr \right]$$

$$\frac{dV_g}{d\theta} = 2pr^2 \sin \theta, \quad \theta = 0 \text{ for stable equil. independent of } \theta$$

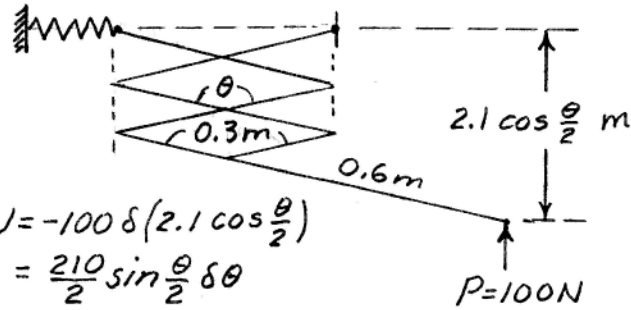


ارائه دهنده خدمات متنوع مهندسی



7/65 | Spring compression  $x = 0.6(\sin \frac{\theta}{2} - \sin 15^\circ)$

$$\delta V_e = kx \delta x; \delta V_e = k(0.6)(\sin \frac{\theta}{2} - \sin 15^\circ) \frac{0.6}{2} \cos \frac{\theta}{2} \delta \theta$$



$$\delta U = -100 \delta(2.1 \cos \frac{\theta}{2})$$

$$= \frac{210}{2} \sin \frac{\theta}{2} \delta \theta$$

$$\delta U = \delta V_e; 105 \sin \frac{\theta}{2} \delta \theta = 0.18k (\sin \frac{\theta}{2} - \sin 15^\circ) \cos \frac{\theta}{2} \delta \theta$$

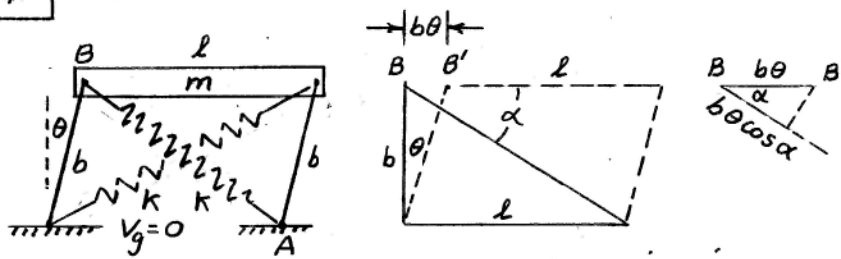
$$\text{For } \frac{\theta}{2} = 60^\circ, 105 \frac{\sqrt{3}}{2} = 0.18k (\frac{\sqrt{3}}{2} - 0.2588) \frac{1}{2}$$

$$k = 1664 \text{ N/m or } \underline{k = 1.664 \text{ kN/m}}$$



7/67

$\Delta =$  initial tensile deflection of each spring



For small  $\theta$ , change in length of each spring is approx.  $\pm b\theta \cos \alpha$

$$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta \cos \alpha)^2 + \frac{1}{2}k(\Delta - b\theta \cos \alpha)^2 + mgb \cos \theta$$

$$= k(\Delta^2 + b^2\theta^2 \cos^2 \alpha) + mgb \cos \theta$$

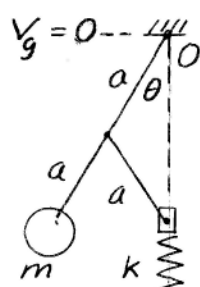
$$\frac{dV}{d\theta} = 2kb^2\theta \cos^2 \alpha - mgb \sin \theta$$

$$\frac{d^2V}{d\theta^2} = 2kb^2 \cos^2 \alpha - mgb \cos \theta; \text{ for } \theta = 0, \frac{d^2V}{d\theta^2} = 2kb^2 \cos^2 \alpha - mgb$$

Thus  $\left(\frac{d^2V}{d\theta^2}\right)_{\theta=0}$  is (+) stable if  $2kb^2 \cos^2 \alpha > mgb$

$$\text{or } k_{\min} = \frac{mgb}{2b \cos^2 \alpha} = \frac{mgb}{2b} \left(1 + \frac{b^2}{l^2}\right) \text{ where } \cos^2 \alpha = \frac{l^2}{b^2 + l^2}$$

► 7/68 | Take  $V_g = 0$  through pt. O



$$V_g = -2mga \cos \theta$$

$$\text{Compression in spring } x = 2a \cos \theta - a = 0 \text{ for } \theta = 60^\circ$$

$$V_e = \frac{1}{2} kx^2 = \frac{ka^2}{2} (2 \cos \theta - 1)^2$$

$$V = V_e + V_g = \frac{ka^2}{2} (2 \cos \theta - 1)^2 - 2mga \cos \theta$$

$$\frac{dV}{d\theta} = 2a [(mg + ka) \sin \theta - 2ka \sin \theta \cos \theta]$$

$$\frac{d^2V}{d\theta^2} = 2a [(mg + ka) \cos \theta - 2ka (2 \cos^2 \theta - 1)]$$

$$\text{For equil. } \frac{dV}{d\theta} = 0, [(mg + ka) - 2ka \cos \theta] \sin \theta = 0$$

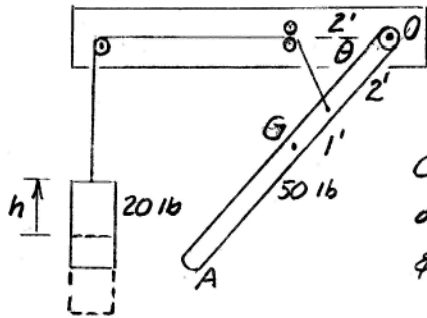
$$\sin \theta = 0 \text{ \& \; } \cos \theta = \frac{mg + ka}{2ka} = \frac{1}{2} \left( 1 + \frac{mg}{ka} \right)$$

$$\text{For } \sin \theta = 0, \theta = 0; \frac{d^2V}{d\theta^2} = 2a(mg - ka) = + \text{ stable if } k < \frac{mg}{a}$$

$$= - \text{ unstable if } k > \frac{mg}{a}$$

$$\text{For } \theta = \cos^{-1} \frac{1}{2} \left( 1 + \frac{mg}{ka} \right), k > \frac{mg}{a} \text{ stable}$$

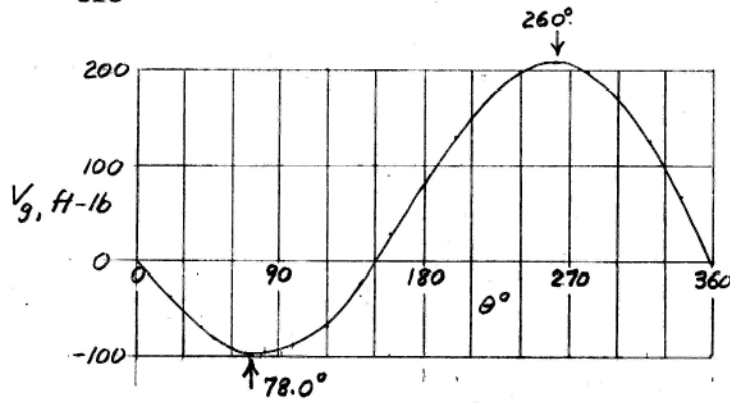
\*7/69 |  $h = 2(2 \sin \frac{\theta}{2}) = 4 \sin \frac{\theta}{2}$



$$V_g = 20(4 \sin \frac{\theta}{2}) - 50(3 \sin \theta)$$

$$= 80 \sin \frac{\theta}{2} - 150 \sin \theta \text{ ft-lb}$$

Compute  $V_g$  as a function of  $\theta$  from  $\theta = 0$  to  $\theta = 360^\circ$  & plot



$\theta = 78.0^\circ$   
stable

$\theta = 260^\circ$   
unstable

\*7/70

$$y = 2(150) \cos \theta$$

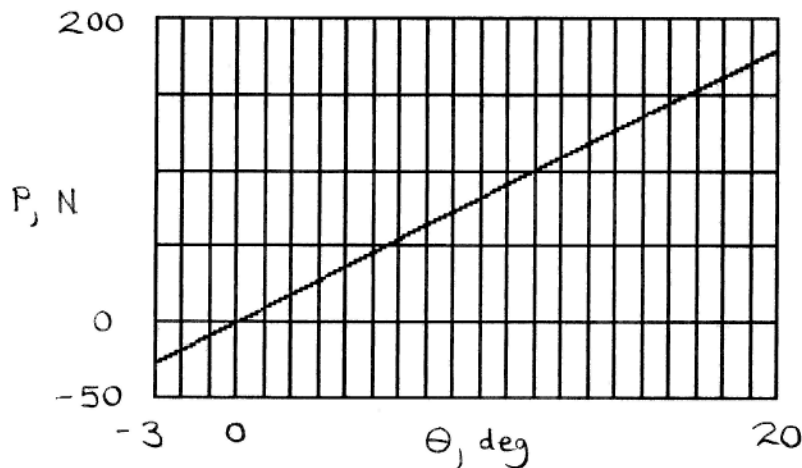
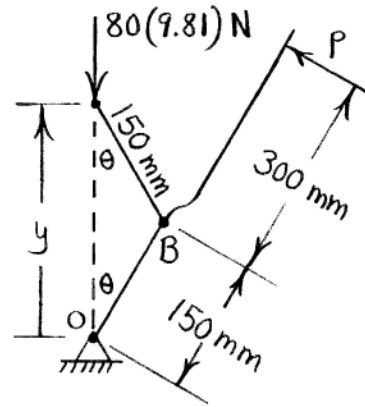
$$\delta y = -300 \sin \theta \delta \theta$$

$$\delta U = 0 :$$

$$-P(450 \delta \theta) - 80(9.81) \delta y = 0$$

$$P = \frac{80(9.81)(300 \sin \theta)}{450}$$

$$= \underline{523 \sin \theta \text{ (in newtons)}}$$



$$\left( \begin{array}{l} \text{At } \theta = -3^\circ, \quad P = -27.4 \text{ N} \\ \text{At } \theta = 20^\circ, \quad P = 178.9 \text{ N} \end{array} \right)$$

\* 7/71 |

Spring stretch =  $0.15(1 - \sin \theta)$  m

$$V_e = \frac{1}{2} (1600) [0.15(1 - \sin \theta)]^2$$

$\left. \begin{array}{l} 60(0.15) \\ = 9 \text{ N}\cdot\text{m} \end{array} \right\}$

$$= 18(1 - \sin \theta)^2 \text{ J}$$

$$\delta V_e = 36(1 - \sin \theta)(-\cos \theta) \delta \theta$$

$$\delta U' = -60 \cos \theta \delta(0.15 \sin \theta) - 9 \delta \theta$$

$$= -9 \cos^2 \theta \delta \theta - 9 \delta \theta = -9[1 + \cos^2 \theta] \delta \theta$$

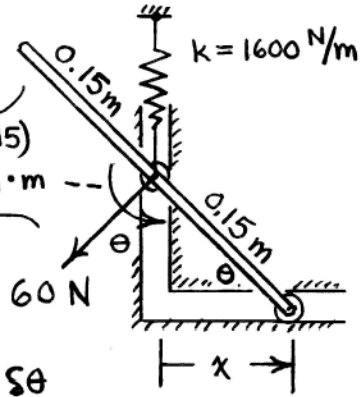
$$\delta U' = \delta V_e : -9[1 + \cos^2 \theta] \delta \theta = -36 \cos \theta (1 - \sin \theta) \delta \theta$$

$$(1 + \cos^2 \theta) - 4 \cos \theta (1 - \sin \theta) = 0$$

Numerical solution :  $\theta = 29.7^\circ$

so that  $x = 0.15 \cos 29.7^\circ = 0.1303 \text{ m}$

or  $x = \underline{130.3 \text{ mm}}$



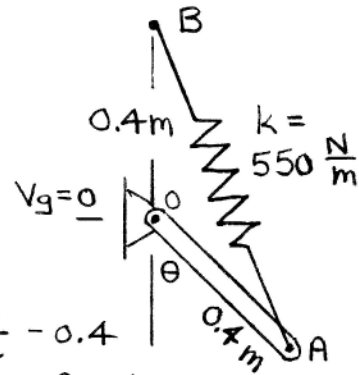
\*7/72

$$V_g = 20(9.81)(-0.2 \cos \theta)$$

$$= -39.24 \cos \theta \text{ J}$$

$$\overline{AB} = 2(0.4) \sin\left(\frac{180-\theta}{2}\right)$$

$$= 0.8 \cos \frac{\theta}{2}$$



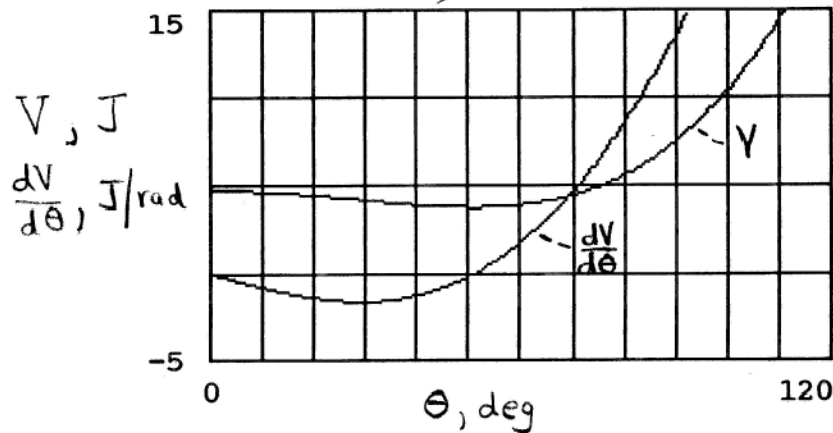
$$\text{Stretch of spring} = 0.8 \cos \frac{\theta}{2} - 0.4$$

$$= 0.4(2 \cos \frac{\theta}{2} - 1)$$

$$V_e = \frac{1}{2} 550 [0.4(2 \cos \frac{\theta}{2} - 1)]^2 = 44(2 \cos \frac{\theta}{2} - 1)^2$$

$$V = V_e + V_g = 44(2 \cos \frac{\theta}{2} - 1)^2 - 39.24 \cos \theta$$

$$\frac{dV}{d\theta} = 88(2 \cos \frac{\theta}{2} - 1)(-\sin \frac{\theta}{2}) + 39.24 \sin \theta$$



$\theta = 0$ , unstable  
 $\theta = 51.1^\circ$ , stable



\* 7/73

$$\text{Spring stretch} = 2(16 \sin \frac{\theta}{2}) - 8$$

$$= 8(4 \sin \frac{\theta}{2} - 1) \text{ in.}$$

$$V_e = \frac{1}{2} (12)(8)^2 (4 \sin \frac{\theta}{2} - 1)^2$$

$$= 384 (4 \sin \frac{\theta}{2} - 1)^2 \text{ in.-lb}$$

$$V_g = 10(8 \cos \theta) + 10(16 + 8 \cos \theta)$$

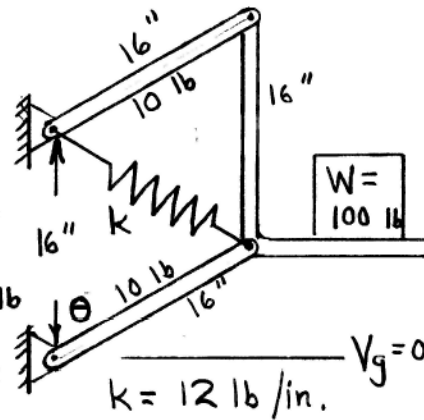
$$+ 100(16 \cos \theta) = 1760 \cos \theta + 160 \text{ in.-lb}$$

$$V = V_e + V_g = 384 (4 \sin \frac{\theta}{2} - 1)^2 + 1760 \cos \theta + 160 \text{ in.-lb}$$

$$\frac{dV}{d\theta} = 768 (4 \sin \frac{\theta}{2} - 1) 2 \cos \frac{\theta}{2} - 1760 \sin \theta$$

$$= 0 \text{ for equilibrium}$$

Numerical solution :  $\theta = 71.7^\circ$



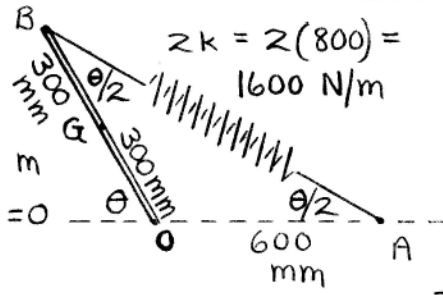
\*7/74|

Springs undeflected @  $\theta = \frac{\pi}{2}$

Deflection  $x = \overline{AB} - 0.6\sqrt{2}$  m

$\overline{AB} = 2(0.6) \cos \frac{\theta}{2}$   $V_g = 0$

So  $x = 0.6(2 \cos \frac{\theta}{2} - \sqrt{2})$  m



$$V = V_g + V_e = 25(9.81)(0.3 \sin \theta) + \frac{1}{2} 2k [0.6(2 \cos \frac{\theta}{2} - \sqrt{2})]^2$$

$$= 73.6 \sin \theta + 800(0.36)(4 \cos^2 \frac{\theta}{2} - 4\sqrt{2} \cos \frac{\theta}{2} + 2)$$

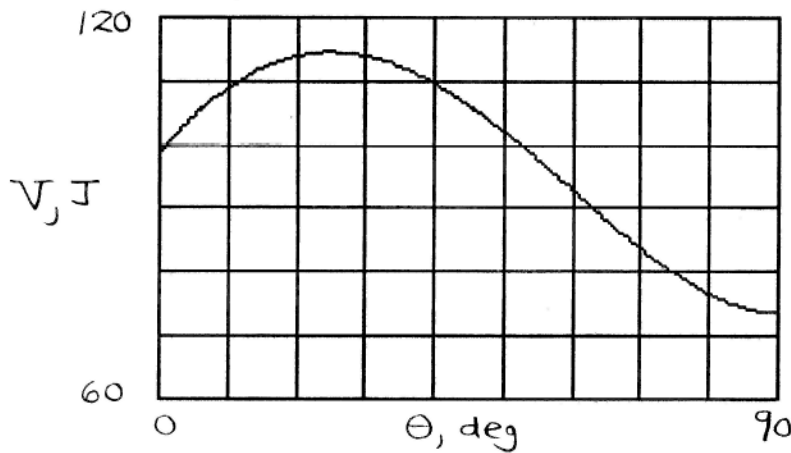
$$\frac{dV}{d\theta} = 73.6 \cos \theta + 288(-4 \cos \frac{\theta}{2} \sin \frac{\theta}{2} + 2\sqrt{2} \sin \frac{\theta}{2})$$

$$= 73.6 \cos \theta - 576 \sin \theta + 815 \sin \frac{\theta}{2}$$

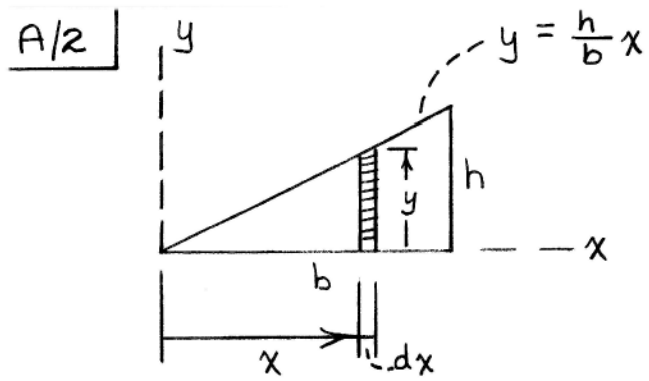
$$\frac{d^2V}{d\theta^2} = -73.6 \sin \theta - 576 \cos \theta + 407 \cos \frac{\theta}{2}$$

Set  $\frac{dV}{d\theta} = 0$  & solve numerically:  $\theta = 24.8^\circ$

$\left(\frac{d^2V}{d\theta^2}\right)_{\theta=24.8^\circ} = -156 < 0$  so unstable.



$$\frac{A}{1} \quad I_x \cong Ay^2, A = \frac{2.56(10^6)}{40^2} = \underline{1600 \text{ mm}^2}$$



$$\begin{aligned}
 I_y &= \int x^2 dA = \int x^2 y dx = \int x^2 \left( \frac{h}{b} x \right) dx \\
 &= \frac{h}{b} \int_0^b x^3 dx = \frac{h}{4b} x^4 \Big|_0^b = \underline{\underline{\frac{hb^3}{4}}}
 \end{aligned}$$

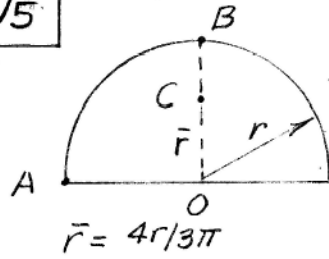
$$\frac{A}{3} \mid I_p = I_c + A(3)^2, \quad I_{p'} = I_c + A(2)^2$$
$$I_p - I_{p'} = 50 = A(3^2 - 2^2), \quad \underline{A = 10 \text{ in.}^2}$$

A/4 |  $I_x + I_y = I_z$ , but  $I_x = I_y$  so  $I_x = \frac{I_z}{2}$

where  $I_z \cong Ar^2 = 1600 (100)^2 = 16(10^6) \text{ mm}^4$

So  $I_x = 8(10^6) \text{ mm}^4$

A/5



For complete circle

$$I_A = I_O + Ar^2 = \frac{1}{2}Ar^2 + Ar^2 = \frac{3}{2}Ar^2$$

For half circle

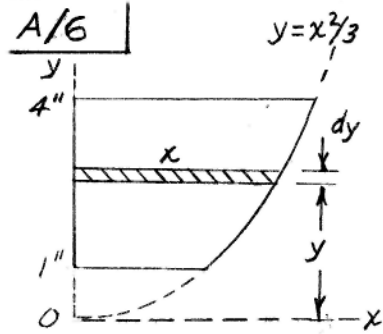
$$I_A = \frac{1}{2} \left( \frac{3}{2} \pi r^4 \right) = \frac{3}{4} \pi r^4$$

For half circle,  $I_O = \frac{1}{4} \pi r^4$ 

$$I_B = I_C + A(r - \bar{r})^2 = I_O - A\bar{r}^2 + A(r - \bar{r})^2$$

$$= I_O + A(r^2 - 2r\bar{r})$$

$$= \frac{1}{4} \pi r^4 + \frac{\pi r^4}{2} \left( 1 - \frac{8}{3\pi} \right) = r^4 \left( \frac{3\pi}{4} - \frac{4}{3} \right)$$



$$dI_y = \frac{1}{3} x^3 dy = \frac{1}{3} (3y)^{3/2} dy$$

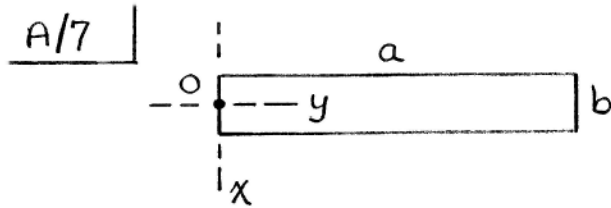
$$= \sqrt{3} y^{3/2} dy$$

$$I_y = \sqrt{3} \int_1^4 y^{3/2} dy = \sqrt{3} \frac{2}{5} (4^{5/2} - 1^{5/2})$$

$$= \frac{2\sqrt{3}}{5} (32 - 1)$$

$$= \underline{\underline{21.5 \text{ in.}^4}}$$





$$I_x = \frac{1}{3} A a^2 = \frac{1}{3} a^3 b, \quad I_y = \frac{1}{12} a b^3$$

$$I_o = I_x + I_y = \frac{1}{3} a b \left( a^2 + \frac{b^2}{4} \right)$$

$$n = \% \text{ error} = \frac{I_x - I_o}{I_o} (100) = \frac{-\frac{1}{12} a b^3}{\frac{1}{3} a b \left( a^2 + \frac{b^2}{4} \right)} 100$$

$$= -\frac{1}{4} \frac{b^2}{a^2 + \frac{b^2}{4}} 100$$

$$\text{For } \frac{b}{a} = \frac{1}{10}, \quad n = -25 \frac{1}{10^2 + \frac{1}{4}} = \underline{\underline{-0.249\%}}$$

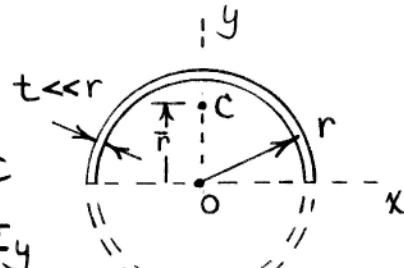
A/8

For complete ring,

$$I_o = Ar^2 = 2\pi r t r^2 = 2\pi r^3 t$$

and  $I_o = I_x + I_y$ ,  $I_x = I_y$

So for complete ring,  $I_x = \frac{I_o}{2} = \pi r^3 t$



For half-ring,  $I_x = \frac{1}{2} \pi r^3 t$  and  $I_y = I_x$

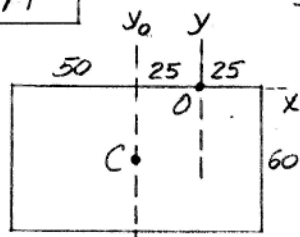
by symmetry so  $I_y = \frac{1}{2} \pi r^3 t$

For half-ring,  $I_o = \frac{1}{2} (2\pi r^3 t) = \pi r^3 t$

$$I_c = I_o - A\bar{r}^2 = \pi r^3 t - \pi r t \left(\frac{2r}{\pi}\right)^2$$

$$= \underline{\underline{\pi r^3 t \left(1 - \frac{4}{\pi^2}\right)}}$$

A/9



Dimensions in mm

$$I_{y_0} = \frac{1}{12} b d^3 = \frac{1}{12} (60)(100)^3 = 5(10^6) \text{ mm}^4$$

$$I_y = I_{y_0} + A d^2 = 5(10^6) + 6(10^3)(25)^2 = 8.75(10^6) \text{ mm}^4$$

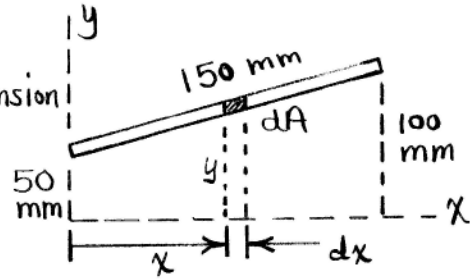
$$I_x = \frac{1}{3} b d^3 = \frac{1}{3} (100)(60^3) = 7.2(10^6) \text{ mm}^4$$

$$I_o = I_x + I_y = (7.2 + 8.75)10^6 = \underline{15.95(10^6) \text{ mm}^4}$$

A/10

Area per unit  $x$ -dimension  
is  $\frac{750}{\sqrt{150^2 - (100-50)^2}}$

$$= \frac{750}{141.4} = 5.30 \frac{\text{mm}^2}{\text{mm}}$$



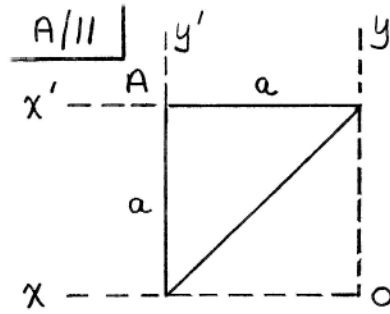
$$dA = 5.30 dx, \quad y = 50 + \frac{50}{141.4} x = 50 \left(1 + \frac{x}{141.4}\right)$$

$$dI_x = y^2 dA = 50^2 \left(1 + \frac{x}{141.4}\right)^2 \frac{750}{141.4} dx$$

$$I_x = 50^2 \frac{750}{141.4} \int_0^{141.4} \left(1 + \frac{2x}{141.4} + \frac{x^2}{20000}\right) dx$$

$$= \underline{4.38 (10^6) \text{ mm}^4}$$

Erroneous result  $I_x \neq A\bar{y}^2 = 750(75)^2 = \underline{4.22(10^6) \text{ mm}^4}$



From Sample Problem A/2,

$$I_x = I_y = \frac{bh^3}{4} = \frac{aa^3}{4} = \frac{a^4}{4}$$

$$I_z = I_x + I_y = \frac{a^4}{4} + \frac{a^4}{4} = \frac{a^4}{2}$$

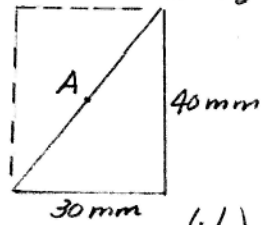
$$k_o = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{a^4/2}{a^2/2}} = \frac{a}{\sqrt{2}}$$

Also,  $I_{y'} = I_{x'} = \frac{bh^3}{12} = \frac{a^4}{12}$

$$I_{z'} = I_{x'} + I_{y'} = \frac{a^4}{6}$$

$$k_A = \sqrt{\frac{I_{z'}}{A}} = \sqrt{\frac{a^4/6}{a^2/2}} = \frac{a}{\sqrt{3}}$$

$$A/12 \quad (J_A)_{\text{triangle}} = \frac{1}{2} (J_A)_{\text{rectangle}}$$



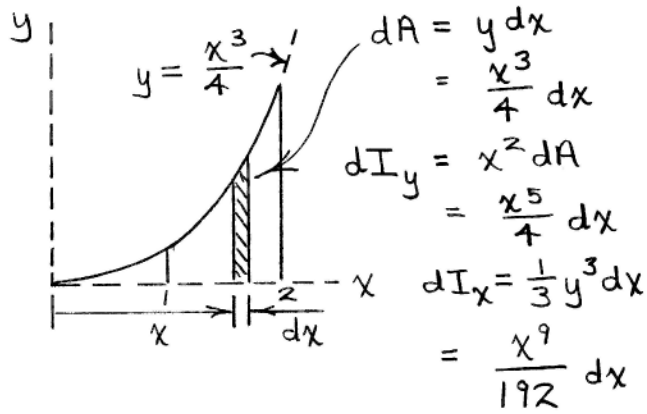
$$= \frac{1}{2} \left[ \frac{1}{12} A (b^2 + h^2) \right] \quad \text{from Sample Prob. A/1}$$

$$= \frac{1}{24} (30)(40)(30^2 + 40^2) = 12.5(10^4) \text{ mm}^4$$

$$(J_A)_{\text{triangle}} = k_A^2 A$$

$$\text{So } k_A = \sqrt{\frac{12.5(10^4)}{30(40)/2}} = \sqrt{208.4} = \underline{14.43 \text{ mm}}$$

A/13



$$A = \int dA = \int_0^2 \frac{x^3}{4} dx = \frac{1}{4} \frac{x^4}{4} \Big|_0^2 = \frac{15}{16}$$

$$I_y = \int dI_y = \int_0^2 \frac{x^5}{4} dx = \frac{1}{4} \frac{x^6}{6} \Big|_0^2 = \frac{63}{24}$$

$$I_x = \int dI_x = \int_0^2 \frac{x^9}{192} dx = \frac{1}{192} \frac{x^{10}}{10} \Big|_0^2 = \frac{1023}{1920}$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{63/24}{15/16}} = \sqrt{14/5} = \underline{1.673}$$

$$k_x = \sqrt{I_x/A} = \sqrt{\frac{1023/1920}{15/16}} = \underline{0.754}$$

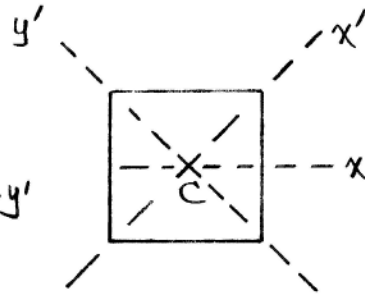
$$k_z = \sqrt{k_x^2 + k_y^2} = \sqrt{1.673^2 + 0.754^2} = \underline{1.835}$$

A/14

$$I_x + I_y = I_z = I_{x'} + I_{y'}$$

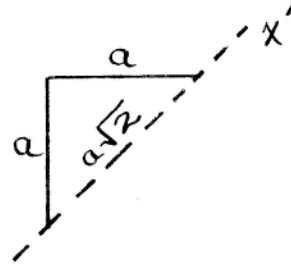
Because  $I_x = I_y$  &  $I_{x'} = I_{y'}$

Therefore  $I_x = I_{x'}$



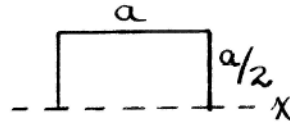
Triangle : From Sample Prob.

$$\begin{aligned} A/2 : I_{x'} &= \frac{bh^3}{12} = \frac{a\sqrt{2} (a/\sqrt{2})^3}{12} \\ &= \frac{a^4}{24} \end{aligned}$$



Rectangle : From Sample Prob.

$$\begin{aligned} A/1 : I_x &= \frac{1}{3} Ah^2 = \frac{1}{3} \frac{a^2}{2} \left(\frac{a}{2}\right)^2 \\ &= \frac{a^4}{24} \end{aligned}$$





A/15

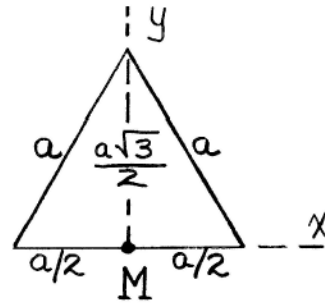
$$I_z = I_x + I_y, \quad I_z = A k_z^2$$

$$\therefore k_M = \sqrt{(I_x + I_y)/A}$$

$$I_x = \frac{1}{12} b h^3 = \frac{1}{12} a \left( \frac{a\sqrt{3}}{2} \right)^3$$
$$= \frac{\sqrt{3}}{32} a^4$$

$$I_y = 2 \left( \frac{1}{12} \frac{a\sqrt{3}}{2} \left( \frac{a}{2} \right)^3 \right) = \frac{\sqrt{3}}{96} a^4$$

$$k_M = \left[ \frac{\frac{\sqrt{3}}{32} a^4 + \frac{\sqrt{3}}{96} a^4}{\frac{a}{2} a \frac{\sqrt{3}}{2}} \right]^{1/2} = \frac{a}{\sqrt{6}}$$

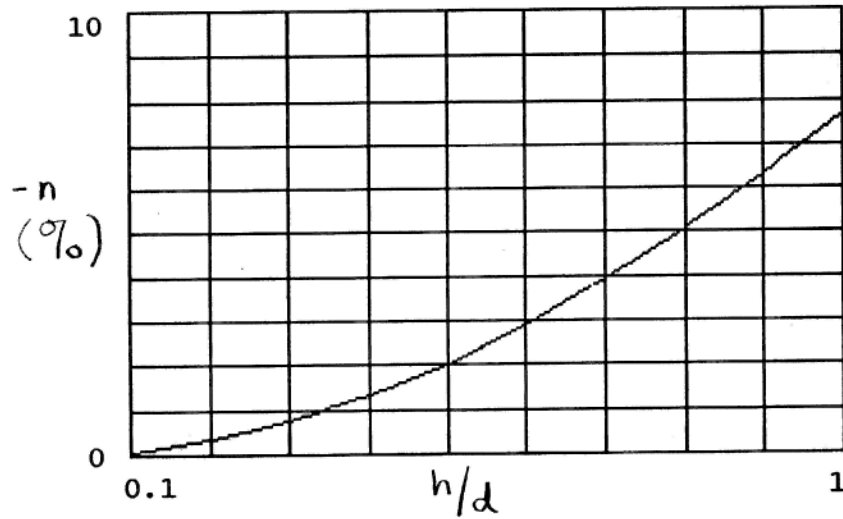


A/16 | Exact :  $I_x = I_{x_0} + Ad^2 = \frac{1}{12}bh^3 + bhd^2$

Approximate :  $I_x' = Ad^2 = bhd^2$

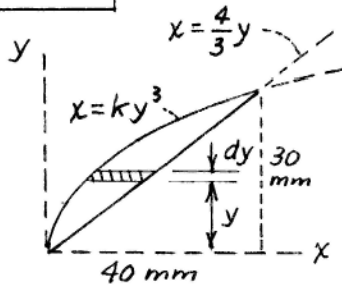
$$n = \frac{I_x' - I_x}{I_x} 100 = \frac{-\frac{1}{12}bh^3}{\frac{1}{12}bh^3 + bhd^2} 100 = \frac{-1}{1 + \frac{12}{(h/d)^2}} 100$$

For  $h = \frac{d}{4}$  ,  $n = -0.518\%$



A/17

For  $x = 40 \text{ mm}$  &  $y = 30 \text{ mm}$ ,  $k = \frac{40}{27(10^3)}$

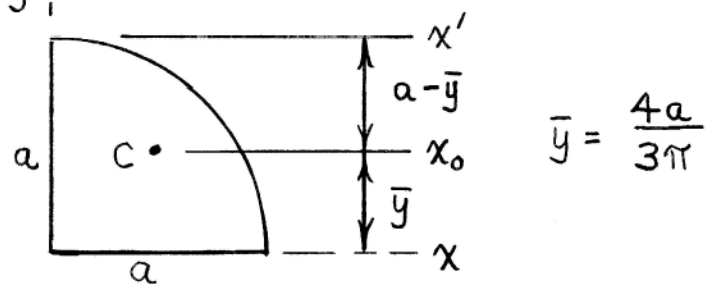


$$dI_x = y^2 dA = y^2 \left( \frac{4}{3} y - \frac{4}{2700} y^3 \right) dy$$

$$I_x = \int_0^{30} \left( \frac{4}{3} y^3 - \frac{4}{2700} y^5 \right) dy$$

$$= \left[ \frac{y^4}{3} - \frac{y^6}{4050} \right]_0^{30} = \underline{9(10^4) \text{ mm}^4}$$

A/18 | y :

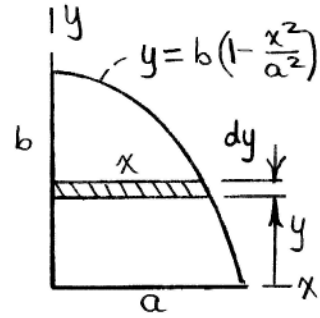


$$\begin{aligned} I_{x'} &= I_{x_0} + A(a - \bar{y})^2 = I_x - A\bar{y}^2 + A(a - \bar{y})^2 \\ &= I_x + A(a^2 - 2a\bar{y}) \\ &= \frac{1}{4} \left( \frac{1}{4} \pi a^4 \right) + \frac{\pi a^2}{4} \left( a^2 - 2a \frac{4a}{3\pi} \right) \\ &= \frac{\pi a^4}{16} + \frac{\pi a^4}{4} \left( 1 - \frac{8}{3\pi} \right) = \underline{0.315 a^4} \end{aligned}$$

A/19

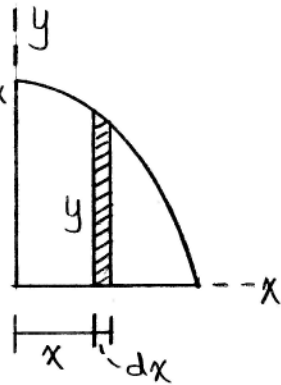
(a) Horizontal strip

$$\begin{aligned} I_x &= \int y^2 dA = \int_0^b y^2 x dy \\ &= \int_0^b y^2 a \sqrt{1 - y/b} dy \\ &= \frac{a}{\sqrt{b}} \int_0^b y^2 \sqrt{b-y} dy \\ &= \frac{a}{\sqrt{b}} \frac{2}{105(-1)} (8b^2 + 12by + 15y^2) \sqrt{(b-y)^3} \Big|_0^b = \frac{16ab^3}{105} \end{aligned}$$



(b) Vertical strip

$$\begin{aligned} I_x &= \int_0^a \frac{1}{3} y^2 (y dx) = \frac{1}{3} \int_0^a b^3 \left(1 - \frac{x^2}{a^2}\right)^3 dx \\ &= \frac{b^3}{3} \frac{1}{a^6} \int_0^a (a^6 - 3a^4 x^2 + 3a^2 x^4 - x^6) dx \\ &= \frac{b^3}{3a^6} \left[ a^6 x - a^4 x^3 + \frac{3a^2 x^5}{5} - \frac{x^7}{7} \right]_0^a \\ &= \frac{16ab^3}{105} \end{aligned}$$

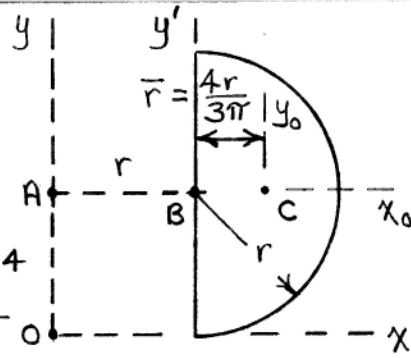


A/20

$$I_{x_0} = \frac{1}{2} \left( \frac{\pi r^4}{4} \right)$$

$$I_x = I_{x_0} + Ar^2$$

$$= \frac{\pi r^4}{8} + \frac{\pi r^2}{2} r^2 = \frac{5}{8} \pi r^4$$



$$I_y = I_{y_0} + A(r + \bar{r})^2 = I_{y'} - A\bar{r}^2 + A(r + \bar{r})^2$$

$$= I_{y'} + Ar^2 + 2Ar\bar{r} = \frac{1}{8} \pi r^4 + \frac{\pi r^4}{2} + 2 \frac{\pi r^2}{2} r \frac{4r}{3\pi}$$

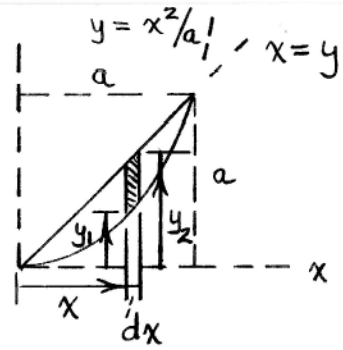
$$= \pi r^4 \left( \frac{1}{8} + \frac{1}{2} + \frac{4}{3\pi} \right) = \pi r^4 \left( \frac{5}{8} + \frac{4}{3\pi} \right)$$

$$I_z = I_x + I_y = \frac{5}{8} \pi r^4 + \frac{5\pi r^4}{8} + \frac{4\pi r^4}{3\pi} = \pi r^4 \left( \frac{5}{8} + \frac{4}{3\pi} \right)$$

$$I_z = Ak_z^2, \text{ so } k_z = \sqrt{I_z/A}$$

$$\text{or } k_0 = \sqrt{\frac{\pi r^4}{\pi r^2/2} \left( \frac{5}{8} + \frac{4}{3\pi} \right)} = \underline{1.830 r}$$

A/21



$$dA = (y_2 - y_1) dx$$

$$= \left(x - \frac{x^2}{a}\right) dx$$

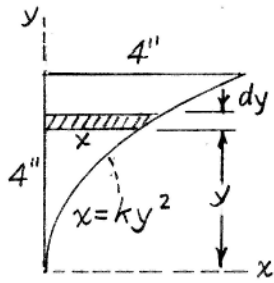
$$I_x = \int \frac{1}{3} y_2^3 dx - \int \frac{1}{3} y_1^3 dx = \frac{1}{3} \int_0^a \left(x^3 - \frac{x^6}{a^3}\right) dx$$

$$= \frac{1}{3} \left[ \frac{x^4}{4} - \frac{x^7}{7a^3} \right]_0^a = \frac{a^4}{3} \left( \frac{1}{4} - \frac{1}{7} \right) = \underline{\underline{a^4/28}}$$

$$I_y = \int x^2 dA = \int x^2 \left(x - \frac{x^2}{a}\right) dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right) dx$$

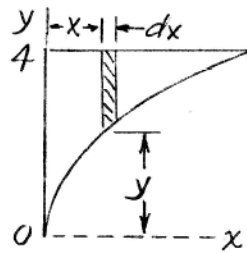
$$= \left[ \frac{x^4}{4} - \frac{x^5}{5a} \right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{\underline{a^4/20}}$$

$$A/22 \quad dI_x = y^2 dA = y^2 x dy = y^2 (y^2/4) dy = (y^4/4) dy$$



$$I_x = \frac{1}{4} \int_0^4 y^4 dy = \frac{1}{4} \left[ \frac{y^5}{5} \right]_0^4 = \frac{256}{5} = \underline{51.2 \text{ in.}^4}$$

$$k = 1/4, \quad x = y^2/4$$



$$dI_x = \frac{1}{3} (dx) (4^3 - y^3)$$

$$= \frac{1}{3} (4^3 - [2\sqrt{x}]^3) dx$$

$$I_x = \frac{1}{3} \int_0^4 (4^3 - 8x^{3/2}) dx$$

$$= \frac{1}{3} \left[ 4^3 x - 8 \frac{2}{5} x^{5/2} \right]_0^4$$

$$= \frac{256}{3} \left( 1 - \frac{2}{5} \right) = \frac{256}{5} = \underline{51.2 \text{ in.}^4}$$



A/23 (a)  $k_o^2 = k_c^2 + \bar{oc}^2$  where  $k_c^2 = I_c/A = \frac{40(10^4)}{1600} \text{ mm}^2$   
 $= 250 \text{ mm}^2$

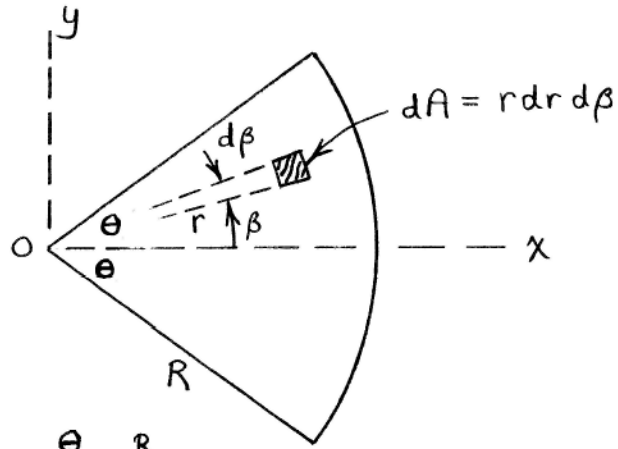
$$k_o^2 = 250 + (30\sqrt{2})^2 = 2050 \text{ mm}^2$$

$$k_o = \sqrt{2050} = \underline{45.3 \text{ mm}}$$

(b)  $I_{x_o} = k_{x_o}^2 A$  &  $I_{x_o} + I_{y_o} = I_c$  &  $I_{y_o} = I_{x_o}$  so  $I_{x_o} = \frac{1}{2} I_c$

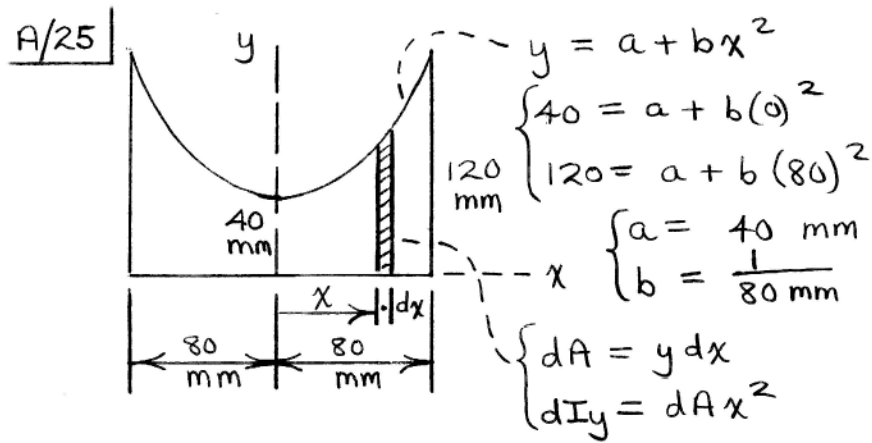
$$\text{so } k_{x_o}^2 = I_c/2A, \quad k_{x_o} = \sqrt{\frac{40(10^4)}{2(1600)}} = \underline{11.18 \text{ mm}}$$

A/24



$$\begin{aligned} I_x &= \int y^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \sin \beta)^2 r dr d\beta \\ &= \frac{R^4}{4} \left( \frac{\beta}{2} - \frac{\sin 2\beta}{4} \right)_{-\theta}^{\theta} = \frac{R^4}{4} \left( \theta - \frac{1}{2} \sin 2\theta \right) \end{aligned}$$

$$\begin{aligned} I_y &= \int x^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \cos \beta)^2 r dr d\beta \\ &= \frac{R^4}{4} \left( \frac{\beta}{2} + \frac{\sin 2\beta}{4} \right)_{-\theta}^{\theta} = \frac{R^4}{4} \left( \theta + \frac{1}{2} \sin 2\theta \right) \end{aligned}$$



$$A = \int dA = \int y dx = 2 \int_0^{80} \left(40 + \frac{1}{80} x^2\right) dx$$

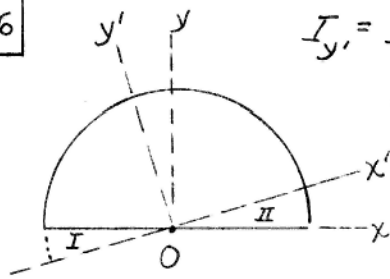
$$= 2 \left[40x + \frac{x^3}{240}\right]_0^{80} = 10670 \text{ mm}^2$$

$$I_y = \int dI_y = \int x^2 y dx = 2 \int_0^{80} \left[40x^2 + \frac{x^4}{80}\right] dx$$

$$= 2 \left[\frac{40}{3} x^3 + \frac{x^5}{400}\right]_0^{80} = 30.0 (10^6) \text{ mm}^4$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{30.0 (10^6)}{10670}} = \underline{53.1 \text{ mm}}$$

A/26



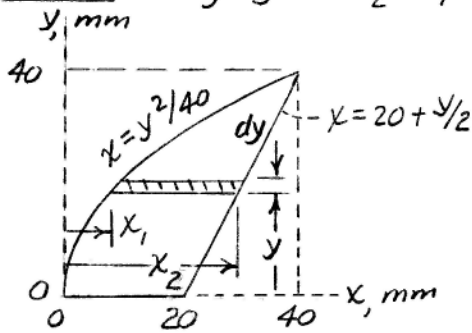
$$I_{y'} = I_{\text{half circle}} - I_I + I_{II}$$

But  $I_I = I_{II}$  by symmetry

Similarly for  $I_{x'}$

$$\text{So } \underline{I_{x'} = I_{y'} = I_x = I_y}$$

$$\frac{A/2.7}{y, \text{ mm}} \quad dI_y = \frac{1}{3} dy (x_2^3 - x_1^3) = \frac{1}{3} \left[ \left(20 + \frac{y}{2}\right)^3 - \left(\frac{y^2}{40}\right)^3 \right] dy$$

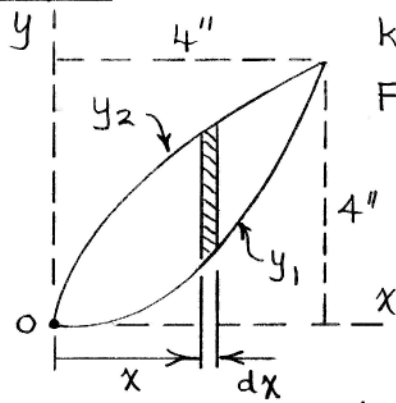


$$\int_0^{40} \left(20 + \frac{y}{2}\right)^3 dy = \frac{1}{2} \left(20 + \frac{y}{2}\right)^4 \Big|_0^{40} = 120(10^4) \text{ mm}^4$$

$$\int_0^{40} \frac{y^6}{40^3} dy = \frac{1}{40^3} \frac{y^7}{7} \Big|_0^{40} = 36.57(10^4) \text{ mm}^4$$

$$I_y = \frac{1}{3} (120 - 36.6) 10^4 = \underline{27.8 (10^4) \text{ mm}^4}$$

A/28



$$\text{From } y_1 = k_1 x^3 : 4 = k_1 4^3$$

$$k_1 = \frac{1}{16} \text{ in.}^{-2} \quad \& \quad y_1 = \frac{x^3}{16}$$

$$\text{From } y_2 = k_2 \sqrt{x} : 4 = k_2 \sqrt{4}$$

$$k_2 = 2 \text{ in.}^{1/2} \quad \& \quad y_2 = 2\sqrt{x}$$

$$dA = (y_2 - y_1) dx$$

$$= \left( 2\sqrt{x} - \frac{x^3}{16} \right) dx$$

$$I_y = \int x^2 dA = \int_0^4 x^2 \left( 2x^{1/2} - \frac{x^3}{16} \right) dx$$

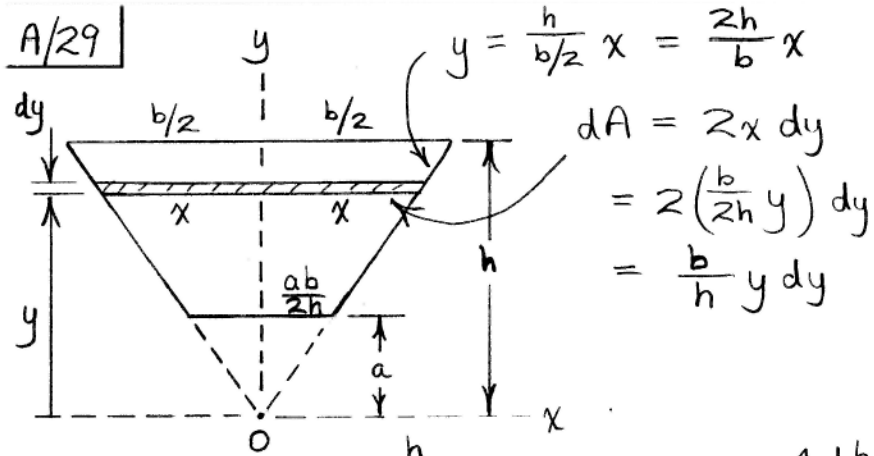
$$= \left[ \frac{4}{7} x^{7/2} - \frac{x^6}{96} \right]_0^4 = \underline{30.5 \text{ in.}^4}$$

$$I_x = \int \left[ \frac{1}{3} y_2^3 dx - \frac{1}{3} y_1^3 dx \right] = \frac{1}{3} \int_0^4 \left[ (2\sqrt{x})^3 - \left( \frac{x^3}{16} \right)^3 \right] dx$$

$$= \frac{1}{3} \left[ 8 \cdot \frac{2}{5} x^{5/2} - \frac{x^{10}}{10 \cdot 16^3} \right]_0^4 = \underline{25.6 \text{ in.}^4}$$

$$I_o = I_x + I_y = \underline{56.1 \text{ in.}^4}$$

A/29



$$I_x = \int y^2 dA = \int_a^h y^2 \frac{b}{h} y dy = \frac{b}{h} \frac{y^4}{4} \Big|_a^h$$

$$= \frac{b}{4h} (h^4 - a^4)$$

$$I_y = \int dI_y = \int \frac{1}{12} (2x)^3 dy = \frac{2}{3} \int \left(\frac{b}{2h} y\right)^3 dy$$

$$= \frac{1}{12} \frac{b^3}{h^3} \frac{y^4}{4} \Big|_a^h = \frac{1}{48} \frac{b^3}{h^3} (h^4 - a^4)$$

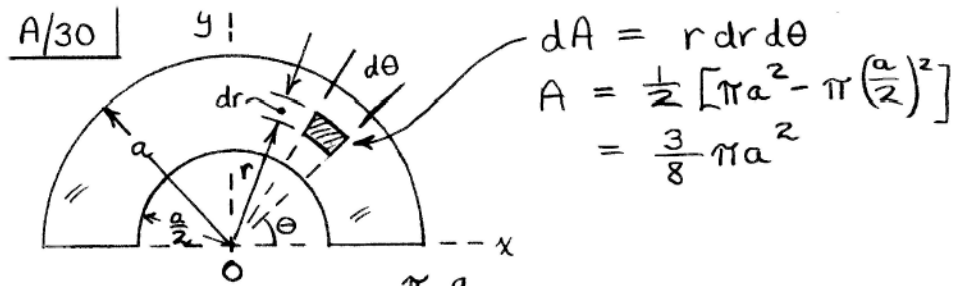
$$I_z = I_x + I_y = \frac{b}{4h} \left(1 + \frac{b^2}{12h^2}\right) (h^4 - a^4)$$

$$A = \frac{bh}{2} - \frac{1}{2} \left(\frac{ab}{h}\right) (a) = \frac{b}{2} \left(h - \frac{a^2}{h}\right)$$

$$k_o = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{1}{2} \left(1 + \frac{b^2}{12h^2}\right) (h^2 + a^2)}$$



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$$I_x = \int y^2 dA = \int_0^{\pi/2} \int_{a/2}^a (r \sin \theta)^2 r dr d\theta$$

$$= \int_0^{\pi/2} \frac{15}{64} a^4 \sin^2 \theta d\theta = \frac{15}{128} \pi a^4$$

$$I_y = \int x^2 dA = 2 \int_0^{\pi/2} \int_{a/2}^a (r \cos \theta)^2 r dr d\theta$$

$$= 2 \int_{a/2}^a \frac{15}{64} a^4 \cos^2 \theta d\theta = \frac{15}{128} \pi a^4$$

$$\underline{k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{15}{128} \frac{\pi a^4}{\frac{3}{8} \pi a^2}} = \frac{\sqrt{5}}{4} a = k_y}$$

$$k_z^2 = k_x^2 + k_y^2 = 2 \left( \frac{5}{16} a^2 \right)$$

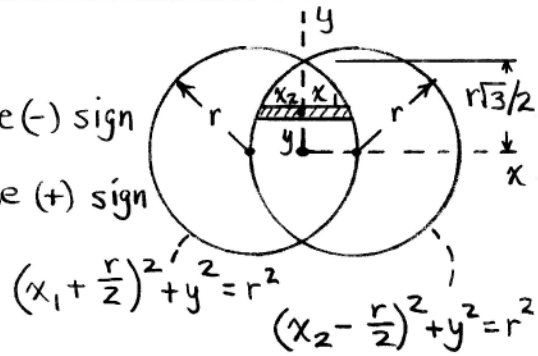
$$\underline{k_z = \frac{\sqrt{10}}{4} a}$$



► A/31

$$x_2 = \frac{r}{2} \pm \sqrt{r^2 - y^2} \quad \text{use (-) sign}$$

$$x_1 = -\frac{r}{2} \pm \sqrt{r^2 - y^2} \quad \text{use (+) sign}$$



$$(x_1 - x_2) = -\frac{r}{2} + \sqrt{r^2 - y^2} - \frac{r}{2} + \sqrt{r^2 - y^2} = 2\sqrt{r^2 - y^2} - r$$

$$dA = (2\sqrt{r^2 - y^2} - r) dy$$

$$I_x = \int y^2 dA = 2 \int_0^{r\sqrt{3}/2} y^2 (2\sqrt{r^2 - y^2} - r) dy$$

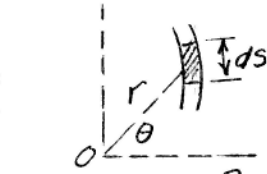
$$= 4 \left\{ -\frac{y}{4} \sqrt{(r^2 - y^2)^3} + \frac{r^2}{8} (y\sqrt{r^2 - y^2} + r^2 \sin^{-1} \frac{y}{r}) \right\} - \frac{2r^3}{3} y \Big|_0^{r\sqrt{3}/2}$$

$$= 4 \left\{ -\frac{r\sqrt{3}}{8} \frac{r^3}{8} + \frac{r^2}{8} \left( \frac{r\sqrt{3}}{2} \frac{r}{2} + r^2 \frac{\pi}{3} \right) \right\} - \frac{2\sqrt{3}}{8} r^4 - 0$$

$$= \frac{r^4}{2} \left\{ -\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right\} = \frac{r^4}{2} \left\{ \frac{\pi}{3} - \frac{3\sqrt{3}}{8} \right\}$$

$$= \underline{0.1988 r^4}$$

► A/32  $\overline{ds}^2 = \overline{dr}^2 + r^2 \overline{d\theta}^2$ ,  $ds = \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2} d\theta$



$r = k\theta$ ,  $k = \frac{R}{2\pi}$   
 $r = \frac{R}{2\pi}\theta$

$$= \sqrt{\left(\frac{R}{2\pi}\right)^2 + \left(\frac{R}{2\pi}\right)^2 \theta^2} d\theta$$

$$= \frac{R}{2\pi} \sqrt{1 + \theta^2} d\theta$$

$$\text{Area} = \int b ds = \frac{Rb}{2\pi} \int_0^{2\pi} \sqrt{1 + \theta^2} d\theta$$

$$= \frac{Rb}{2\pi} \left[ \frac{1}{2} \left( \theta \sqrt{1 + \theta^2} + \ln(\theta + \sqrt{1 + \theta^2}) \right) \right]_0^{2\pi}$$

$$= \frac{Rb}{4\pi} \left[ 2\pi \sqrt{1 + 4\pi^2} + \ln(2\pi + \sqrt{1 + 4\pi^2}) \right] = 3.383 Rb$$

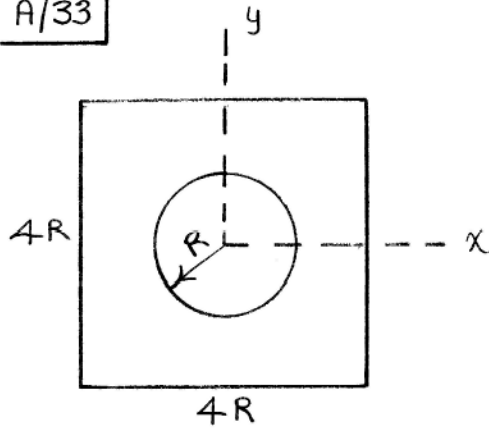
$$I_0 = \int r^2 b ds = \int \frac{R^2}{4\pi^2} b \theta^2 \left( \frac{R}{2\pi} \sqrt{1 + \theta^2} d\theta \right) = \int_0^{2\pi} \frac{R^3 b}{8\pi^3} \theta^2 \sqrt{1 + \theta^2} d\theta$$

$$I_0 = \frac{R^3 b}{8\pi^3} \left[ \frac{\theta}{4} \sqrt{(1 + \theta^2)^3} - \frac{\theta}{8} \sqrt{1 + \theta^2} - \frac{1}{8} \ln(\theta + \sqrt{1 + \theta^2}) \right]_0^{2\pi}$$

$$= \frac{R^3 b}{8\pi^3} \left[ \frac{\pi}{2} \sqrt{(1 + 4\pi^2)^3} - \frac{\pi}{4} \sqrt{1 + 4\pi^2} - \frac{1}{8} \ln(2\pi + \sqrt{1 + 4\pi^2}) \right] = 1.609 R^3 b$$

$$k_0 = \sqrt{I_0 / A} = R \sqrt{\frac{1.6094}{3.383}} = 0.690 R$$

A/33

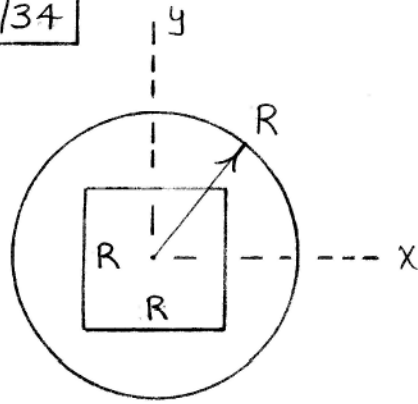


$$\text{Without hole, } I_x = \frac{1}{12} (4R)(4R)^3 = \frac{64}{3} R^4$$
$$(21.3 R^4)$$

$$\text{With hole, } I_x = \frac{64}{3} R^4 - \frac{1}{4} (\pi R^2) R^2$$
$$= \underline{20.6 R^4}$$

(a 3.68% reduction)

A/34



Without square hole:

$$I_z = 2I_x = 2 \left( \frac{1}{4} \pi R^2 \cdot R^2 \right) = \underline{1.571 R^4}$$

With hole:

$$I_z = 1.571 R^4 - 2 \left( \frac{1}{12} R \cdot R^3 \right) = \underline{1.404 R^4}$$

(a reduction of 10.61%)

$$\frac{A/35}{I_z} = \frac{1}{2} \left[ \frac{\pi a^4}{2} - \frac{\pi \left(\frac{a}{2}\right)^4}{2} \right] = \frac{15}{64} \pi a^4$$

$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{\frac{15}{64} \pi a^4}{\frac{3}{8} \pi a^2}} = \underline{\frac{\sqrt{10}}{4} a}$$

From  $k_x^2 + k_y^2 = k_z^2$  and the fact that

$k_x = k_y$  for the present case,

$$2k_x^2 = \left(\frac{\sqrt{10}}{4} a\right)^2, \quad k_x = k_y = \underline{\frac{\sqrt{5}}{4} a}$$

A/36

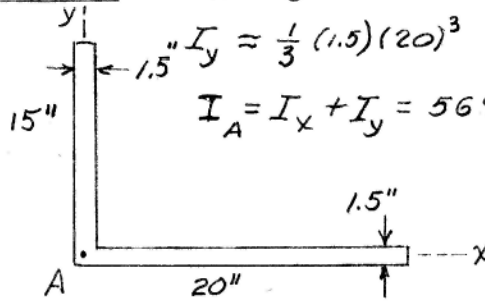
$$I_x \approx \frac{1}{3}(1.5)(15)^3 + 0 = 1690 \text{ in.}^4$$

$$I_y \approx \frac{1}{3}(1.5)(20)^3 + 0 = 4000 \text{ in.}^4$$

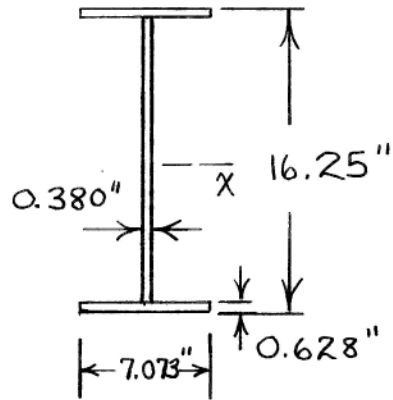
$$I_A = I_x + I_y = 5690 \text{ in.}^4$$

$$k_A = \sqrt{I_A / A} = \sqrt{\frac{5690}{1.5(15+20)}}$$

$$= 10.4 \text{ in.}$$



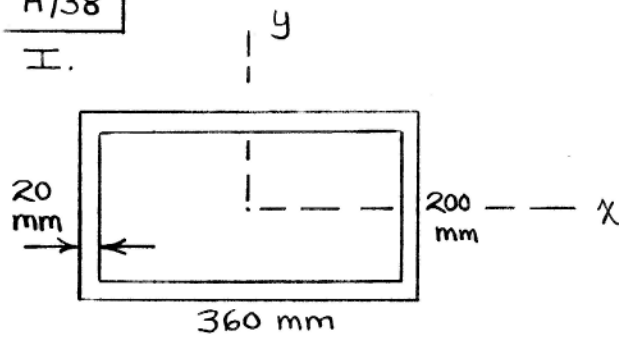
A/37



$$\begin{aligned} I_x &= \frac{1}{12} (0.380) [16.25 - 2(0.628)]^3 \\ &+ 2 \left\{ \frac{1}{12} (7.073)(0.628)^3 + 7.073(0.628) \left[ \frac{16.25}{2} - \frac{0.628}{2} \right]^2 \right\} \\ &= \underline{649 \text{ in.}^4} \end{aligned}$$

A/38

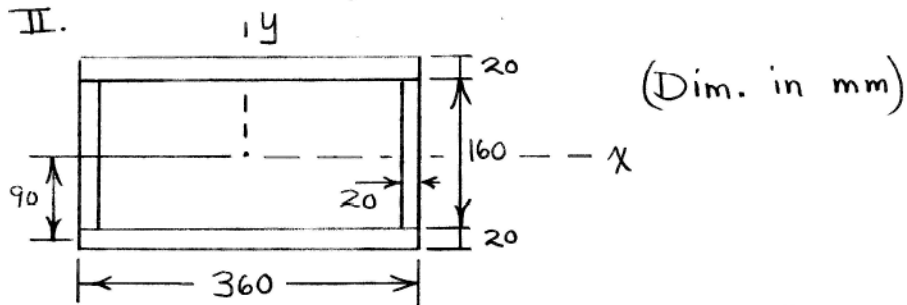
I.



$$I_x = \frac{1}{12} (360)(200)^3 - \frac{1}{12} (320)(160)^3$$

$$= \underline{130.8 (10^6) \text{ mm}^4}$$

II.

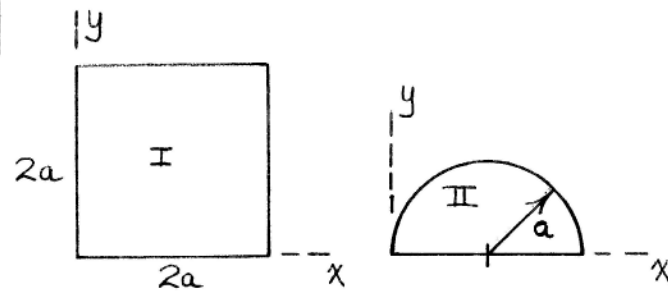


$$I_x = 2 \left[ \frac{1}{12} (360) (20)^3 + 360 (20) (90)^2 \right]$$

$$+ 2 \left[ \frac{1}{12} (20) (160)^3 \right] = \underline{130.8 (10^6) \text{ mm}^4}$$



A/39



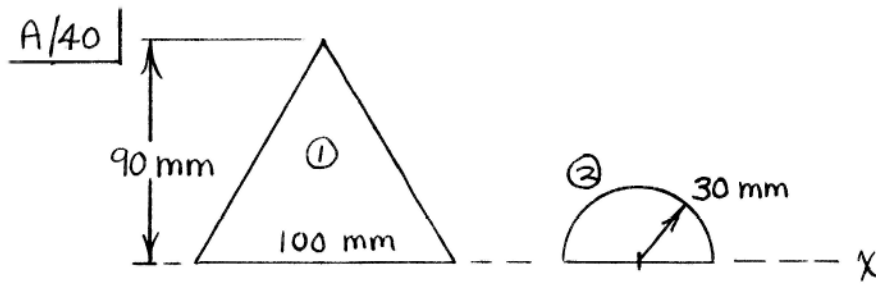
I. Square  $I_x = I_y = \frac{1}{3} (4a^2)(2a)^2 = \frac{16}{3} a^4$

II. Semicircle  $I_x = \frac{1}{8} \pi a^4$

$$I_y = \frac{1}{8} \pi a^4 + \frac{1}{2} \pi a^2 (a^2) = \frac{5}{8} \pi a^4$$

Combined:  $I_x = \frac{16}{3} a^4 - \frac{\pi}{8} a^4 = 4.94 a^4$

$$I_y = \frac{16}{3} a^4 - \frac{5}{8} \pi a^4 = \underline{\underline{3.37 a^4}}$$

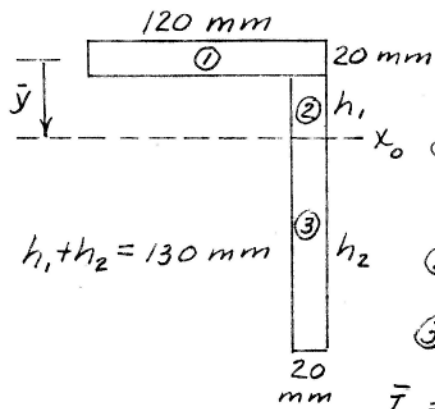


$$I_{x_1} = \frac{1}{12} (100) (90^3) = 6.08 (10^6) \text{ mm}^4$$

$$I_{x_2} = - \frac{\pi (30^4)}{8} = -0.318 (10^6) \text{ mm}^4$$

$$\text{So } I_x = (6.08 - 0.318) 10^6 = \underline{5.76 (10^6) \text{ mm}^4}$$

$$\frac{A}{A} \bar{y} = \frac{\sum Ay}{\sum A} = \frac{0 + (130)(20)(65+10)}{250(20)} = 39.0 \text{ mm}$$



$$\text{so } h_1 = 29.0 \text{ mm}$$

$$h_2 = 101.0 \text{ mm}$$

$$\textcircled{1} \bar{I}_x = \frac{1}{12}(120)(20^3) + (120)(20)(39.0)^2 = 3.73(10^6) \text{ mm}^4$$

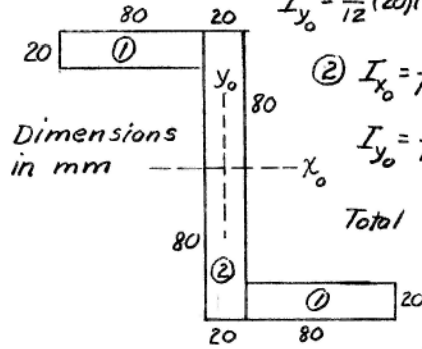
$$\textcircled{2} \bar{I}_x = \frac{1}{3}(20)(29.0)^3 = 0.163(10^6) \text{ mm}^4$$

$$\textcircled{3} \bar{I}_x = \frac{1}{3}(20)(101.0)^3 = 6.87(10^6) \text{ mm}^4$$

$$\bar{I}_x = (3.73 + 0.163 + 6.87)10^6 = 10.76(10^6) \text{ mm}^4$$

A/42  $\textcircled{1} I_{x_o} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6) \text{ mm}^4$

$I_{y_o} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6) \text{ mm}^4$



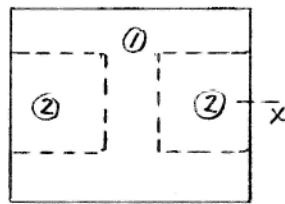
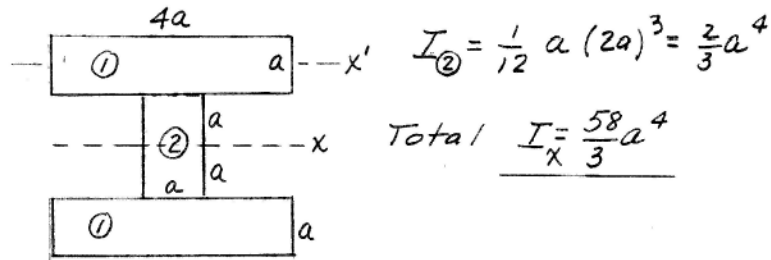
$\textcircled{2} I_{x_o} = \frac{1}{12}(20)(160)^3 = 6.83(10^6) \text{ mm}^4$

$I_{y_o} = \frac{1}{12}(160)(20)^3 = 0.1067(10^6) \text{ mm}^4$

Total  $\bar{I}_x = [2(7.89) + 6.83](10^6)$   
 $= 22.6(10^6) \text{ mm}^4$

$\bar{I}_y = [2(4.85) + 0.1067](10^6)$   
 $= 9.81(10^6) \text{ mm}^4$

A/43 Sol. I  $I_{\text{O}} = 2 \left[ \frac{1}{12} 4a(a^3) + 4a^2 \left( \frac{3a}{2} \right)^2 \right] = \frac{56}{3} a^4$



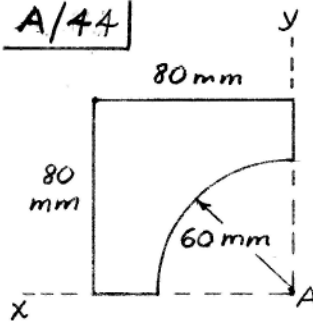
Sol. II

$$I_{\text{O}} = \frac{1}{12} (4a)(4a)^3 = \frac{64}{3} a^4$$

$$I_{\text{O}} = -\frac{1}{12} (3a)(2a)^3 = -2a^4$$

$$\text{Total} = \left( \frac{64}{3} - \frac{6}{3} \right) a^4 = \frac{58}{3} a^4$$

A/44



Square :

$$I_x = I_y = \frac{1}{3}(80)(80)^3 = 13.65(10^6) \text{ mm}^4$$

Quarter - circular area :

$$I_x = I_y = -\frac{1}{4} \left( \frac{1}{4} \pi [60]^4 \right) = -2.54(10^6) \text{ mm}^4$$

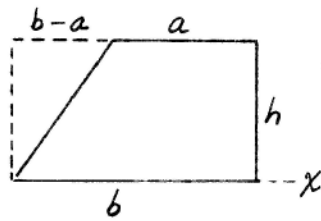
$$\text{Area } A = (80)^2 - \frac{1}{4} \pi (60)^2 = 3573 \text{ mm}^2$$

$$I_z = I_x + I_y = 2(13.65 - 2.54)10^6 \\ = 22.22(10^6) \text{ mm}^4$$

$$k_A^2 = I_z / A = 22.22(10^6) / 3573 = 6219 \text{ mm}^2, k_A = \sqrt{6219} = \underline{78.9 \text{ mm}}$$

A/45 | Distort to a rectangle and a triangle without altering y-distribution of area

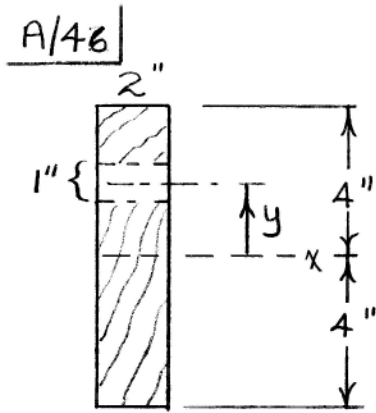
Rectangle  $I_x = \frac{1}{3} b h^3$



Triangle (Sample Problem A2)

$$I_x = -\frac{(b-a)h^3}{4}$$

For trapezoid,  $I_x = \frac{b h^3}{3} - \frac{b-a}{4} h^3 = \frac{1}{12} (b+3a) h^3$



Without hole,

$$I_x = \frac{1}{12} bh^3 = \frac{1}{12} (2)(8)^3$$

$$= 85.3 \text{ in.}^4$$

With hole,  $I'_x = I_x - (\bar{I}_{\text{hole}} + Ay^2)$

$$I'_x = 85.3 - \left[ \frac{1}{12} (2)(1)^3 + (2)(1)y^2 \right]$$

$$= 85.2 - 2y^2 \quad (y \text{ in in.}, I'_x \text{ in in.}^4)$$

Percent reduction  $n = \frac{I_x - I'_x}{I_x} (100\%)$

$$n = \frac{85.3 - (85.2 - 2y^2)}{85.3} (100\%) = \underline{0.1953 + 2.34y^2}$$

(in percent)

For  $y = 2 \text{ in.}$ ,

$$n = 0.1953 + 2.34(2)^2 = \underline{9.57\%}$$

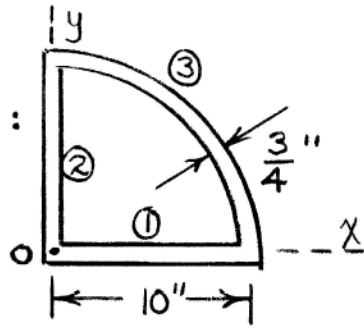


A/47

Parts 1 and 2 separately:

$$I_o = \frac{1}{3} \left( 10 \times \frac{3}{4} \right) 10^2 = \frac{10^3}{4} \text{ in.}^4$$

(polar); Area =  $10 \left( \frac{3}{4} \right) \text{ in.}^2$



Part 3:  $I_o = Ar^2 = \frac{\pi(10)}{2} \left( \frac{3}{4} \right) 10^2 = \frac{3\pi}{8} 10^3 \text{ in.}^4$

$$\text{Area} = 10 \left( \frac{3\pi}{8} \right) \text{ in.}^2$$

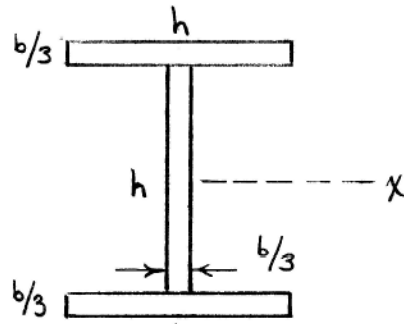
Combined:  $I_o = 2 \left( \frac{10^3}{4} \right) + \frac{3\pi}{8} (10^3)$

$$\text{Area} = 2(10) \left( \frac{3}{4} \right) + 10 \left( \frac{3\pi}{8} \right)$$

$$k_o = \sqrt{\frac{I_o}{A}} = \sqrt{\frac{\frac{1}{2} + \frac{3\pi}{8}}{\frac{3}{2} + \frac{3\pi}{8}}} 10 = \underline{7.92 \text{ in.}}$$

A/48

For area (a),  
 $I_x = \frac{1}{12} b h^3$



For area (b),

$$I_x = \frac{1}{12} \frac{b}{3} h^3 + 2 \left[ \frac{1}{12} h \left( \frac{b}{3} \right)^3 + h \frac{b}{3} \left( \frac{h}{2} + \frac{b}{6} \right)^2 \right]$$
$$= \frac{hb}{9} \left( \frac{7}{4} h^2 + \frac{2}{9} b^2 + hb \right)$$

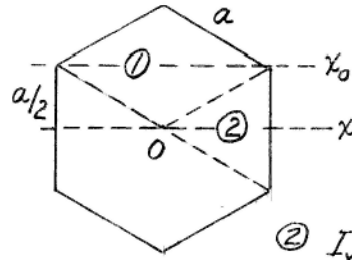
If  $h = 200 \text{ mm}$  and  $b = 60 \text{ mm}$ , we have

$$(a) I_x = \frac{1}{12} (60) (200)^3 = 40 (10^6) \text{ mm}^4$$

$$(b) I_x = \frac{200(60)}{9} \left( \frac{7}{4} (200)^2 + \frac{2}{9} (60)^2 + 200(60) \right)$$
$$= 110.4 (10^6) \text{ mm}^4$$

$$\text{Percent increase } n = \frac{110.4 - 40}{40} (100\%) = \underline{\underline{176.0\%}}$$

A/49 | ①  $I_{x_0} = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}(2a\sqrt{3}/2)(a/2)^3 = \frac{\sqrt{3}}{48}a^4$



$$I_x = I_{x_0} + Ad^2$$

$$= \frac{\sqrt{3}}{48}a^4 + (2a\sqrt{3}/2)(a/2)(a/2)^2$$

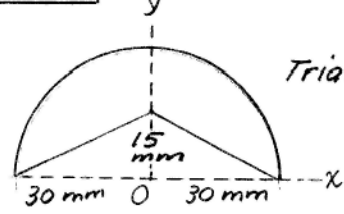
$$= \frac{7\sqrt{3}}{48}a^4$$

②  $I_x = 2\left(\frac{1}{12}bd^3\right) = \frac{1}{6}(a\sqrt{3}/2)(a/2)^3 = \frac{\sqrt{3}}{96}a^4$

Total  $I_x = 2\left(\frac{7\sqrt{3}}{48}a^4 + \frac{\sqrt{3}}{96}a^4\right) = \frac{5\sqrt{3}}{16}a^4$



A/51



Semi-circle:  $I_z = \frac{1}{4} \pi r^4 = \frac{1}{4} \pi (30)^4$

$$= 0.6362(10^6) \text{ mm}^4$$

Triangle:  $I_x = \frac{1}{12} b h^3 = \frac{1}{12} (60)(15)^3$

$$= -0.01688(10^6) \text{ mm}^4$$

$$I_y = -\frac{2}{12} (15)(30)^3$$

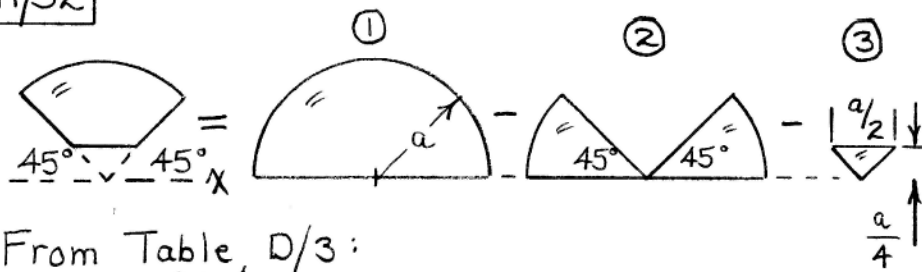
$$= -0.06750(10^6) \text{ mm}^4$$

$$I_z = I_x + I_y = -(0.01688 + 0.06750)(10^6)$$

$$= -0.0844(10^6) \text{ mm}^4$$

$$\text{Total } I_z = (0.6362 - 0.0844)(10^6) = \underline{0.552(10^6) \text{ mm}^4}$$

A/52



From Table D/3:

$$I_{x_1} = \frac{\pi a^4}{8}$$

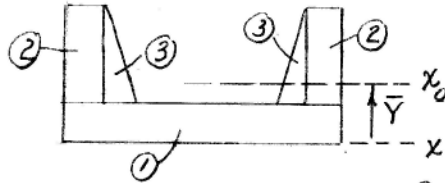
$$I_{x_2} = 2 \frac{a^4}{8} \left[ \frac{\pi}{4} - \frac{1}{2} \sin \left( 2 \cdot \frac{\pi}{4} \right) \right] = a^4 \left( \frac{\pi}{16} - \frac{1}{8} \right)$$

$$I_{x_3} = \frac{a}{2} \left( \frac{a}{4} \right)^3 / 4 = \frac{a^4}{512}$$

$$\begin{aligned} \text{So } I_x &= I_{x_1} - I_{x_2} - I_{x_3} \\ &= \frac{\pi a^4}{8} - a^4 \left( \frac{\pi}{16} - \frac{1}{8} \right) - \frac{a^4}{512} \\ &= a^4 \left( \frac{\pi}{16} + \frac{63}{512} \right) = \underline{0.319a^4} \end{aligned}$$

A/53

Part	A in. <sup>2</sup>	$\bar{y}$ in.	$\bar{y}A$ in. <sup>3</sup>	$\bar{I}_x$ in. <sup>4</sup>	d in.	$Ad^2$ in. <sup>4</sup>
1	8.40	0.35	2.94	0.343	0.696	4.07
2	4.29	2.35	10.08	3.894	1.304	7.30
3	0.33	1.80	0.59	0.200	0.754	0.19
Totals	13.02		13.61	4.437		11.56



$$\textcircled{1} A_1 = 0.70 \times 12 = 8.40 \text{ in.}^2$$

$$\textcircled{2} A_2 = 0.65 \times 3.30 \times 2 = 4.29 \text{ in.}^2$$

$$\textcircled{3} A_3 = 2 \left( \frac{1}{2} \right) (0.10) (3.30) = 0.33 \text{ in.}^2$$

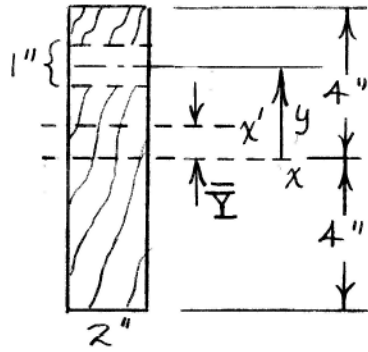
$$\bar{Y} = \frac{13.61}{13.02} = 1.046 \text{ in.}$$

$$\begin{aligned} \bar{I}_x = I_{x_0} &= \sum \bar{I}_x + \sum Ad^2 \\ &= 4.437 + 11.56 \\ &= \underline{16.00 \text{ in.}^4} \end{aligned}$$

►A/54

Without hole,  $x' = x$  and  $I_x = I_{x'} =$

$$\frac{1}{12}bh^3 = \frac{1}{12}(2)(8)^3 = 85.3 \text{ in.}^4$$



Centroid location :

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{8(2)(0) - 2(1)y}{8(2) - 2(1)}$$

$$= -0.1429y \quad (\text{or } -\frac{1}{7}y)$$

$$I'_{x'} = \frac{1}{3}(2)(4 + \bar{Y})(4 + \bar{Y})^2 + \frac{1}{3}(2)(4 - \bar{Y})(4 - \bar{Y})^2$$

$$- \left[ \frac{1}{12}(2)(1)^3 + 2(1)(y - \bar{Y})^2 \right]$$

$$= 85.2 + 16\bar{Y}^2 - 2(y - \bar{Y})^2$$

$$\text{With } \bar{Y} = -\frac{1}{7}y : I'_{x'} = 85.2 + 16\left(-\frac{1}{7}y\right)^2 - 2\left(y + \frac{1}{7}y\right)^2$$

$$\text{or } I'_{x'} = 85.2 - \frac{112}{49}y^2 = 85.2 - 2.29y^2$$

$$\text{Percent reduction } n = \frac{I_x - I'_{x'}}{I_x} (100\%)$$

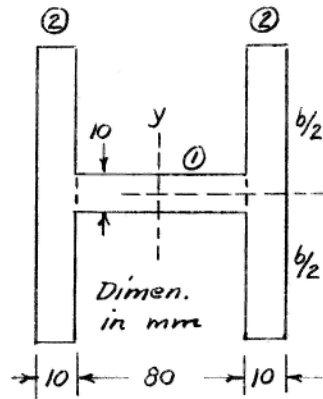
$$= \frac{(100\%)85.3 - (85.2 - 2.29y^2)}{85.3} = \frac{0.1953 + 2.68y^2}{85.3}$$

(in percent)

$$\text{For } y = 2 \text{ in.}, \quad n = 10.91\%$$



► A/55



$$① I_x = \frac{1}{12}(80)(10)^3 = 0.00667(10^6) \text{ mm}^4$$

$$I_y = \frac{1}{12}(10)(80)^3 = 0.427(10^6) \text{ mm}^4$$

$$② I_x = 2 \left[ \frac{1}{12}(10)b^3 \right] = 1.667b^3$$

$$I_y = 2 \left[ \frac{1}{12}b(10)^3 \right] + (10b)(45)^2$$

$$= 0.0407(10^6)b$$

$$\text{Total } I_x = \text{Total } I_y$$

$$(0.00667)(10^6) + 1.667b^3$$

$$= (0.427 + 0.0407)(10^6)$$

$$\text{or } b^3 - 0.0244(10^6)b - 0.252(10^6) = 0$$

Solve by cubic formula;  $\left[ \frac{0.252(10^6)}{2} \right]^2 < \left[ \frac{0.0244(10^6)}{3} \right]^3$  so 3 real roots

$$\cos u = \frac{q}{p\sqrt{p}} \text{ where } q = \frac{252(10^3)}{2} = 126(10^3), p = \frac{24.4(10^3)}{3} = 8.13(10^3)$$

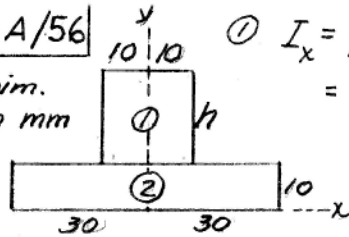
$$\cos u = 126(10^3) / [8.13(10^3)90.2] = 0.1718, u = 80.11^\circ$$

$$b_1 = 2\sqrt{p} \cos \frac{u}{3} = 2(90.2)(0.8933) = 161.1 \text{ mm or } \underline{b = 161.1 \text{ mm}}$$

$$b_2 = 2\sqrt{p} \cos \left( \frac{u}{3} + 120^\circ \right) = (-), b_3 = 2\sqrt{p} \cos \left( \frac{u}{3} + 240^\circ \right) = (-)$$

► A/56

Dim.  
in mm



$$\begin{aligned} \textcircled{1} \quad I_x &= \frac{1}{12} (20) h^3 + 20h \left(\frac{h}{2} + 10\right)^2 \\ &= 20 \left(\frac{h^3}{3} + 10h^2 + 100h\right) \text{ mm}^4 \\ I_y &= \frac{1}{12} h (20)^3 = \frac{2000h}{3} \text{ mm}^4 \end{aligned}$$

$$\textcircled{2} \quad I_x = \frac{1}{3} (60)(10)^3 = 20000 \text{ mm}^4$$

$$I_y = \frac{1}{12} (10)(60)^3 = 180000 \text{ mm}^4$$

Thus for equal  $I_x$  &  $I_y$  totals

$$20 \left(\frac{h^3}{3} + 10h^2 + 100h\right) + 20000 = \frac{2000h}{3} + 180000$$

$$\text{or } h^3 + 30h^2 + 200h = 24000$$

Substitute  $h = u - 10$  & get  $u^3 = 100u + 24000$  & solve by

Appen. B/4-4: Let  $p = 100/3$ ,  $q = 12000$  Case II  $q^2 - p^3 = (+)$

$$(q^2 - p^3)^{1/2} = \sqrt{144(10^6) - 10^6/27} = 11.99846 (10^3)$$

$$u_1 = (12000 + 11998)^{1/3} + (12000 - 11998.46)^{1/3} = 28.84 + 1.16 = 30.00$$

$$h = 30.00 - 10 = \underline{20.0 \text{ mm}}$$

A/57 |  $A = (30)(60) = 1800 \text{ mm}^2$  for each  
 $\bar{I}_{xy} = 0$  for each, so  $I_{xy} = 0 + A d_x d_y$

(a)  $I_{xy} = (40)(50)(1800) = 360 (10^4) \text{ mm}^4$

(b)  $I_{xy} = (40)(-50)(1800) = -360 (10^4) \text{ mm}^4$

(c)  $I_{xy} = (-40)(-50)(1800) = 360 (10^4) \text{ mm}^4$

(d)  $I_{xy} = (-40)(50)(1800) = \underline{-360 (10^4) \text{ mm}^4}$

A/58

$$\begin{aligned} I_{xy} &= -(3)(3) [ (-3.5)(3.5) + (-3.5)(-3.5) + (3.5)(-3.5) ] \\ &= \underline{110.2 \text{ in.}^4} \end{aligned}$$

A/59

$$I_x = \frac{1}{12} (400) (200)^3 - 3 \left[ \frac{1}{4} \pi (30^4) + \pi (30)^2 (50)^2 \right]$$
$$= \underline{2.44 (10^8) \text{ mm}^4}$$

$$I_y = \frac{1}{12} (200) (400)^3 - 3 \left[ \frac{1}{4} \pi (30^4) + \pi (30)^2 (100)^2 \right]$$
$$= \underline{9.80 (10^8) \text{ mm}^4}$$

$$I_{xy} = -\pi (30)^2 \left[ (100)(50) + (-100)(50) + (-100)(-50) \right]$$
$$= \underline{-14.14 (10^6) \text{ mm}^4}$$

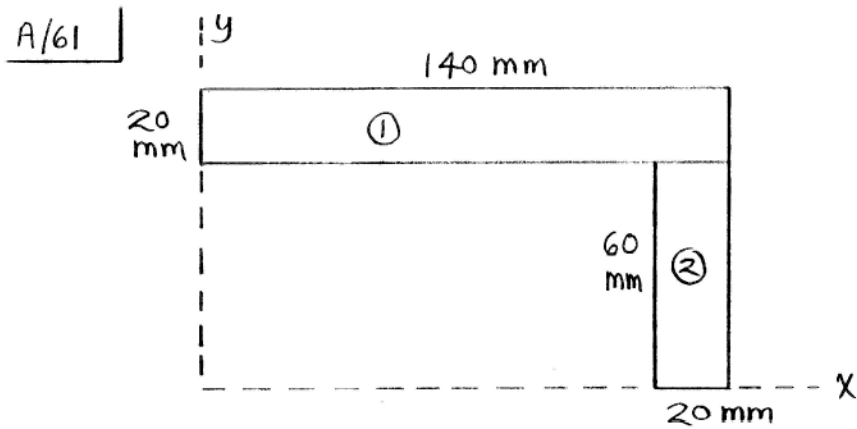
A/60

$$(a) I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50) \\ = \underline{9.60(10^6) \text{ mm}^4}$$

$$(b) I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2) \\ = \underline{-4.71(10^6) \text{ mm}^4}$$

$$(c) I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50) \\ = \underline{9.60(10^6) \text{ mm}^4}$$

$$(d) I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)\left(-40 - \frac{4(25)}{3\pi}\right) \\ \times (\pi \cdot 25^2)/2 \\ = \underline{-2.98(10^6) \text{ mm}^4}$$

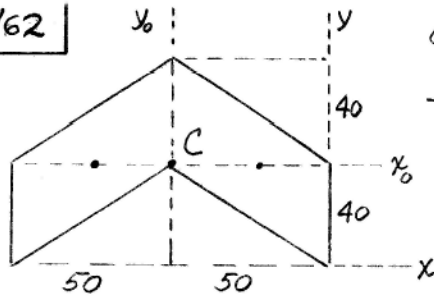


$$I_{xy_1} = 20(140)(70)(70) = 13.72(10^6) \text{ mm}^4$$

$$I_{xy_2} = 60(20)(130)(30) = 4.68(10^6) \text{ mm}^4$$

$$\text{Total: } \underline{I_{xy} = 18.40(10^6) \text{ mm}^4}$$

A/62



Dimen. in mm

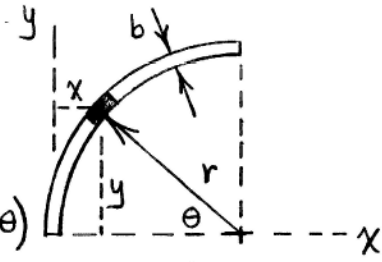
Centroid is at C

$$\begin{aligned} I_{xy} &= I_{x_0y_0} + d_x d_y A \\ &= 0 + (40)(-50)(40 \times 100) \\ &= \underline{-8(10^6) \text{ mm}^4} \end{aligned}$$



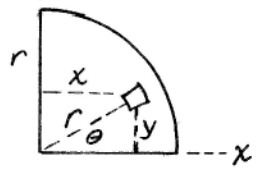
A/63

$$\begin{aligned} I_{xy} &= \int xy \, dA \\ &= \int_0^{\pi/2} (r - r \cos \theta) r \sin \theta (br \, d\theta) \\ &= br^3 \int_0^{\pi/2} (\sin \theta - \sin \theta \cos \theta) \, d\theta \\ &= br^3 \left[ -\cos \theta + \frac{1}{4} \cos 2\theta \right]_0^{\pi/2} \\ &= br^3 \left[ 0 - \frac{1}{4} + 1 - \frac{1}{4} \right] = \underline{br^3/2} \end{aligned}$$



$$\frac{A/64}{y} \quad I_{xy} = \int xy dA = \int_0^{\pi/2} \int_0^r (r \cos \theta)(r \sin \theta) r dr d\theta$$

$$= \int_0^{\pi/2} \frac{\sin 2\theta}{2} \frac{r^4}{4} d\theta = \frac{r^4}{16} (-\cos 2\theta) \Big|_0^{\pi/2}$$

$$= \frac{r^4}{16} (1 - [-1]) = \underline{r^4/8}$$


$$\bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{r^4}{8} - \frac{4r}{3\pi} \left( \frac{4r}{3\pi} \right) \frac{\pi r^2}{4} = \frac{r^4}{8} \left( 1 - \frac{32}{9\pi} \right)$$

$$= \underline{-0.01647 r^4}$$

A/65

(1) By direct integration

For elemental strip,

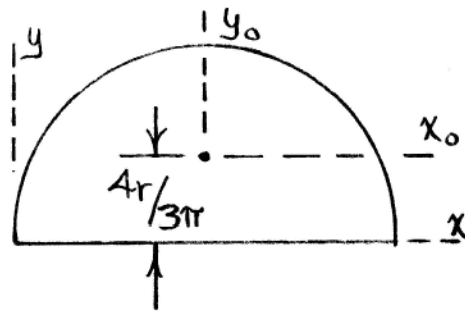
$$dI_{xy} = x \frac{y}{2} dA = \frac{xy}{2} y dx$$

$$= \frac{x}{2} [r^2 - (x-r)^2] dx$$

$$I_{xy} = \frac{1}{2} \int_0^{2r} (xr^2 - x^3 + 2rx^2 - r^2x) dx$$

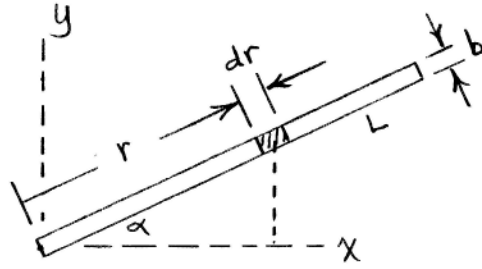
$$= \frac{1}{2} \left[ \frac{x^2 r^2}{2} - \frac{x^4}{4} + \frac{2rx^3}{3} - \frac{r^2 x^2}{2} \right]_0^{2r} = \underline{\underline{\frac{2}{3} r^4}}$$

(2) By axis transfer



$$I_{xy} = I_{x_0 y_0} + A d_x d_y = 0 + \frac{\pi r^2}{2} (r) \left( \frac{4r}{3\pi} \right) = \underline{\underline{\frac{2}{3} r^4}}$$

A/66 | Strip 1:



$$I_{xy_1} = \int xy \, dA = \int_0^L (r \cos \alpha)(r \sin \alpha) b \, dr$$
$$= b \cos \alpha \sin \alpha \left. \frac{r^3}{3} \right|_0^L = \frac{bL^3}{3} \cos \alpha \sin \alpha$$

$$\text{or } I_{xy_1} = \frac{bL^3}{6} \sin 2\alpha$$

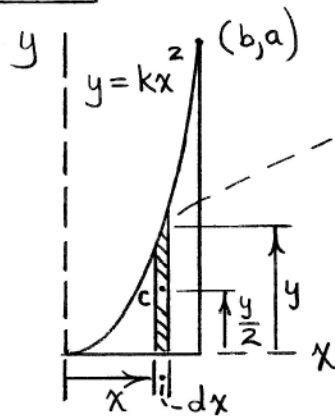
$$\text{By inspection, } I_{xy_2} = -I_{xy_1} = -\frac{bL^3}{6} \sin 2\alpha$$

$$I_{xy_3} = \frac{bL^3}{6} \sin [2(90^\circ - \alpha)] = \frac{bL^3}{6} \sin 2\alpha$$

$$I_{xy_4} = -I_{xy_3} = -\frac{bL^3}{6} \sin 2\alpha$$

$\sum_{i=1}^4 I_{xy_i} = 0$ , which must be the case due to symmetry about the y-axis.

A/67



$$y = kx^2 : a = kb^2, k = a/b^2$$

$$\Rightarrow y = \frac{a}{b^2} x^2$$

$$dA = y dx = \frac{a}{b^2} x^2 dx$$

$$d\bar{I}_{xy} = 0$$

$$dI_{xy} = d\bar{I}_{xy} + dA(x)\left(\frac{y}{2}\right)$$

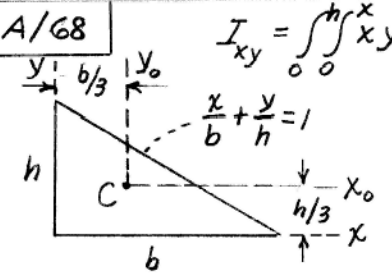
$$= 0 + y dx (x)\left(\frac{y}{2}\right)$$

$$= \frac{1}{2} \frac{a^2}{b^4} x^5 dx$$

$$I_{xy} = \int dI_{xy} = \int_0^b \frac{1}{2} \frac{a^2}{b^4} x^5 dx$$

$$= \frac{1}{2} \frac{a^2}{b^4} \frac{x^6}{6} \Big|_0^b = \underline{\underline{\frac{1}{12} a^2 b^2}}$$

$A/68$



$$I_{xy} = \int_0^h \int_0^{b(1-y/h)} xy \, dx \, dy = \int_0^h y \left( \frac{x^2}{2} \right) \Big|_0^{b(1-y/h)} dy$$

$$= \int_0^h \frac{b^2}{2} \left( y - \frac{2y^2}{h} + \frac{y^3}{h^2} \right) dy$$

$$= \frac{b^2}{2} \left[ \frac{h^2}{2} - \frac{2h^2}{3} + \frac{h^2}{4} \right] = \frac{b^2 h^2}{24}$$

$$I_{x_0 y_0} = \bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{b^2 h^2}{24} - \frac{bh}{2} \left( \frac{h}{3} \right) \left( \frac{b}{3} \right) = -\frac{b^2 h^2}{72}$$

A/69

First:  $I_{xy} = \int_0^b \int_0^y xy \, dy \, dx$

$$= \int_0^b \left[ \frac{xy^2}{2} \right]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$$

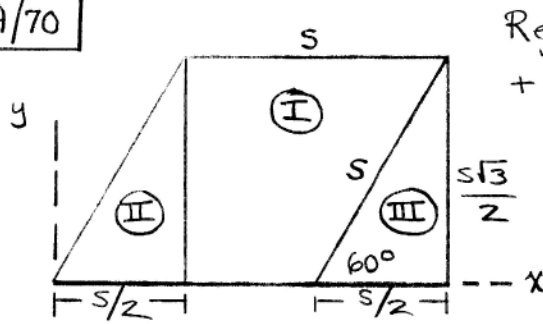
$$= \frac{h^2}{2b^2} \frac{b^4}{4} = \frac{b^2 h^2}{8}$$

Second:  $dI_{xy} = dI_{x_0 y_0} + d_x d_y (dA)$

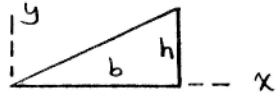
$$= 0 + \frac{y}{2} x (y dx) = \frac{h^2}{2b^2} x^3 dx$$

$$I_{xy} = \frac{h^2}{2b^2} \int_0^b x^3 dx = \frac{h^2}{2b^2} \frac{b^4}{4} = \frac{b^2 h^2}{8}$$

A/70



Regard as I (rectangle)  
+ II - III (triangles)



Use result  $I_{xy} = \frac{b^2 h^2}{8}$  from  
Prob. A/69.

$$\bar{I}_{xy} = I_{xy} - A d_x d_y = \frac{b^2 h^2}{8} - \frac{bh}{2} \left(\frac{2b}{3}\right) \left(\frac{h}{3}\right) = \frac{1}{72} b^2 h^2$$

$$I_I = \bar{I}_{xy} + A d_x d_y = 0 + s \frac{s\sqrt{3}}{2} (s) \left(\frac{s\sqrt{3}}{4}\right) = \frac{3}{8} s^4$$

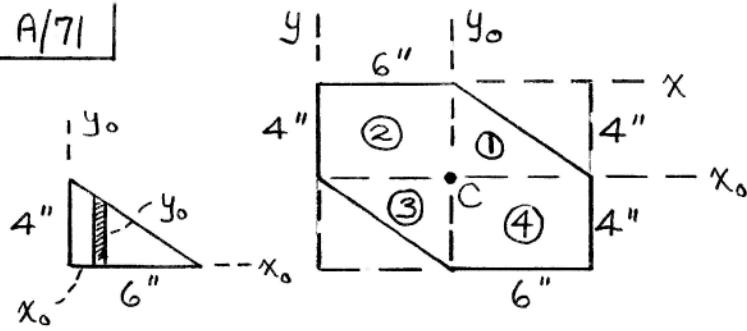
$$I_{II} = \frac{1}{8} \left(\frac{s}{2}\right)^2 \left(\frac{s\sqrt{3}}{2}\right)^2 = \frac{3}{128} s^4$$

$$\begin{aligned} I_{III} &= \bar{I}_{xy} + A d_x d_y = \frac{1}{72} \left(\frac{s}{2}\right)^2 \left(\frac{s\sqrt{3}}{2}\right)^2 \\ &\quad + \frac{1}{2} \left(\frac{s}{2}\right) \left(\frac{s\sqrt{3}}{2}\right) \left[s + \frac{2}{3} \frac{s}{2}\right] \left[\frac{1}{3} \frac{s\sqrt{3}}{2}\right] \\ &= \frac{11}{128} s^4 \end{aligned}$$

$$\begin{aligned} I_{xy} &= I_I + I_{II} - I_{III} = s^4 \left[ \frac{3}{8} + \frac{3}{128} - \frac{11}{128} \right] \\ &= \underline{\underline{\frac{5}{16} s^4}} \end{aligned}$$



A/71



$$\begin{aligned} \text{Part 1: } I_{x_0 y_0} &= \int_0^6 x_0 \frac{y_0}{2} (y_0 dx_0), \quad y_0 = 4 - \frac{2}{3} x_0 \\ &= \frac{1}{2} \int_0^6 x_0 \left( 16 - \frac{16}{3} x_0 + \frac{4}{9} x_0^2 \right) dx_0 \\ &= \frac{1}{2} \left[ 8x_0^2 - \frac{16}{9} x_0^3 + \frac{4}{9} x_0^4 \right]_0^6 = 24 \text{ in.}^4 \end{aligned}$$

$$\text{Part 3: } I_{x_0 y_0} = 24 \text{ in.}^4$$

$$\text{Part 2: } I_{x_0 y_0} = 4(6)(-3)(+2) = -144 \text{ in.}^4$$

$$\text{Part 4: } I_{x_0 y_0} = -144 \text{ in.}^4$$

$$\text{Combined: } I_{x_0 y_0} = 2(24) + 2(-144) = -240 \text{ in.}^4$$

$$\text{Combined area} = 2(4)(6) + 2\left(\frac{1}{2}\right)(4)(6) = 72 \text{ in.}^4$$

$$\begin{aligned} \text{So } I_{xy} &= I_{x_0 y_0} + A d_x d_y = -240 + 72(+6)(-4) \\ &= \underline{\underline{-1968 \text{ in.}^4}} \end{aligned}$$

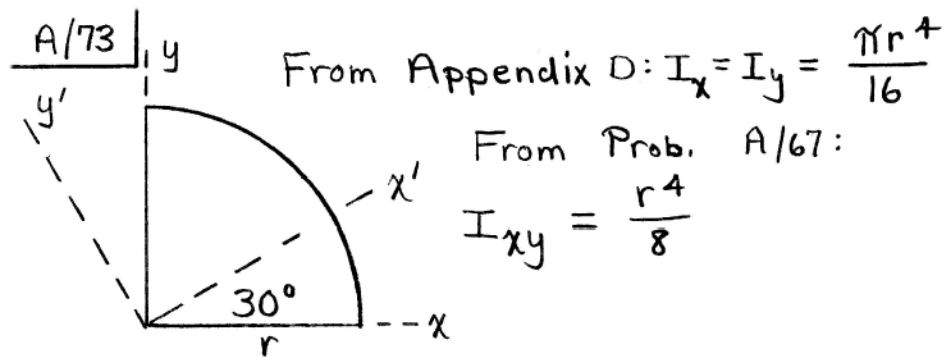
$$\frac{A}{72} \left| I_{xy} = \bar{I}_{xy} + d_x d_y A, \quad I_{x'y'} = \bar{I}_{x'y'} + d_{x'} d_{y'} A \right.$$

$$I_{xy} - I_{x'y'} = (d_x d_y - d_{x'} d_{y'}) A$$

where  $d_x = 20 \text{ mm}$ ,  $d_y = 30 \text{ mm}$ ,  $d_{x'} = -40 \text{ mm}$ ,  $d_{y'} = 80 \text{ mm}$

$$(80 - [-420]) 10^5 = [(20)(30) - (-40)(80)] A$$

$$A = \frac{500(10^5)}{3800} = \underline{1.316(10^4) \text{ mm}^2}$$



$$\text{Eq. A/9: } I_{x'} = \frac{\pi r^4}{16} + 0 - \frac{r^4}{8} \sin 60^\circ = \frac{r^4}{16} [\pi - \sqrt{3}]$$

$$I_{y'} = \frac{\pi r^4}{16} - 0 + \frac{r^4}{8} \sin 60^\circ = \frac{r^4}{16} [\pi + \sqrt{3}]$$

Eq. A/9a :

$$I_{x'y'} = 0 + \frac{r^4}{8} \cos 60^\circ = \frac{r^4}{16}$$

$$\underline{A/74} \quad I_x = \frac{1}{3} b (b^3) = \frac{1}{3} b^4; \quad I_y = \frac{1}{3} b^4$$

$$I_{xy} = 0 + \frac{b}{2} \frac{b}{2} b^2 = \frac{1}{4} b^4$$

With  $\theta = 30^\circ$ , Eqs. A/9 & A/9a give

$$I_{x'} = \frac{b^4}{3} + 0 - \frac{1}{4} b^4 \sin 60^\circ = \left(\frac{1}{3} - \frac{\sqrt{3}}{8}\right) b^4 = \underline{0.1168 b^4}$$

$$I_{y'} = \frac{b^4}{3} + 0 + \frac{1}{4} b^4 \sin 60^\circ = \left(\frac{1}{3} + \frac{\sqrt{3}}{8}\right) b^4 = \underline{0.5498 b^4}$$

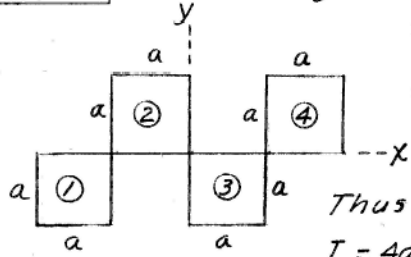
$$I_{x'y'} = 0 + \frac{b^4}{4} \frac{1}{2} = \frac{b^4}{8} = \underline{0.1250 b^4}$$

A/75 ①  $I_x = a^4/3, I_y = a^4/2 + a^2(3a/2)^2 = 7a^4/3; I_{xy} = +3a^4/4$

②  $I_x = a^4/3, I_y = a^4/3, I_{xy} = -a^4/4$

③ same as ②

④ same as ①



Thus for composite area

$I_x = 4a^4/3, I_y = 16a^4/3, I_{xy} = +a^4$

From Eq. A/11,

$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = (\frac{10}{3} + \sqrt{5})a^4 = 5.57a^4$

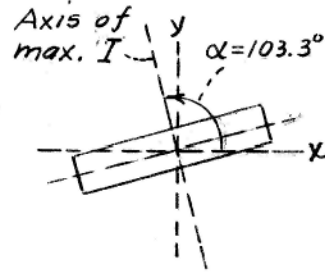
$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = (\frac{10}{3} - \sqrt{5})a^4 = 1.097a^4$

From Eq. A/10

$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2a^4}{(16/3 - 4/3)a^4} = +\frac{1}{2}$

$2\alpha = 26.6^\circ \text{ or } 206.6^\circ$

$\alpha = 13.3^\circ \text{ or } 103.3^\circ$



A/76 | Multiply Eqs. A/11 together and get

$$\begin{aligned} I_{\max} I_{\min} &= \left( \frac{I_x + I_y}{2} \right)^2 - \left( \frac{\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}}{2} \right)^2 \\ &= \frac{1}{4} (I_x^2 + 2I_x I_y + I_y^2 - I_x^2 + 2I_x I_y - I_y^2 - 4I_{xy}^2) \\ &= I_x I_y - I_{xy}^2, \text{ so } \underline{I_{xy} = \sqrt{I_x I_y - I_{\max} I_{\min}}} \end{aligned}$$

A/77

For the triangle,

$$I_x = \frac{1}{12}(2a)(a^3) = \frac{a^4}{6}$$

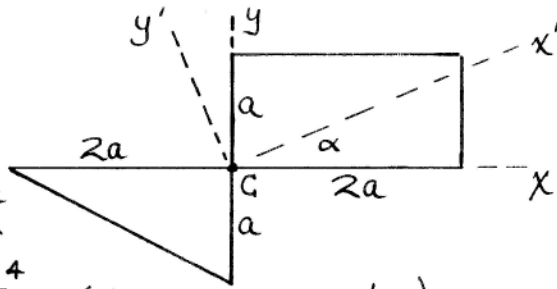
$$I_y = \frac{1}{12}(a)(2a)^3 = \frac{2a^4}{3}$$

$$I_{xy} = \frac{1}{24}(2a)^2 a^2 = \frac{a^4}{6} \quad (\text{from Prob. A/68})$$

For the rectangle,  $I_x = \frac{1}{3}(2a^2)a^2 = \frac{2a^4}{3}$

$$I_y = \frac{1}{3}(2a^2)(2a)^2 = \frac{8a^4}{3}, \quad I_{xy} = 2a^2\left(\frac{a}{2}\right)(a) = a^4$$

$$\text{Totals: } I_x = \frac{5a^4}{6}, \quad I_y = \frac{10a^4}{3}, \quad I_{xy} = \frac{7a^4}{6}$$



Eqs. A/11:

$$I_{\max} = \frac{\frac{5}{6} + \frac{10}{3}}{2} a^4 + \frac{1}{2} \sqrt{\left(\frac{5}{6} - \frac{10}{3}\right)^2 a^8 + 4\left(\frac{7}{6}\right)^2 a^8} = \underline{3.79a^4}$$

$$I_{\min} = \frac{\frac{5}{6} + \frac{10}{3}}{2} a^4 - \frac{1}{2} \sqrt{\left(\frac{5}{6} - \frac{10}{3}\right)^2 a^8 + 4\left(\frac{7}{6}\right)^2 a^8} = \underline{0.373a^4}$$

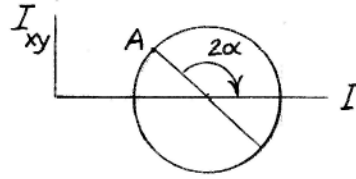
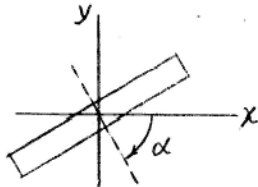
Eq. A/10:

$$\tan 2\alpha = \frac{2\left(\frac{7}{6}a^4\right)}{\frac{10}{3}a^4 - \frac{5}{6}a^4} = \frac{14}{15}, \quad 2\alpha = 43.0^\circ \text{ or } 223^\circ$$

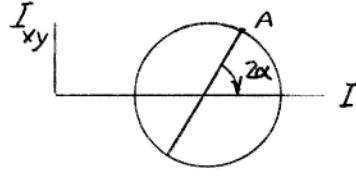
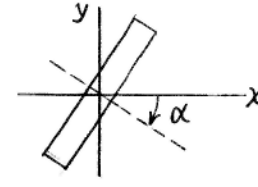
$\alpha = 21.5^\circ \text{ or } 111.5^\circ$   
 $\alpha = 111.5^\circ$  for axis of  $I_{\max}$

A/78

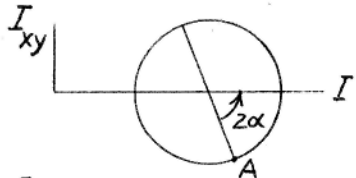
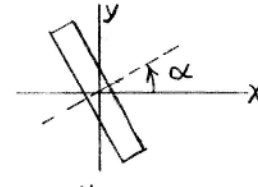
(a)  $I_x < I_y, I_{xy} (+)$



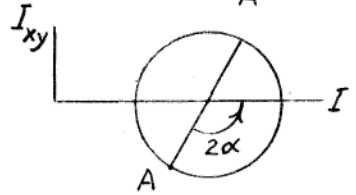
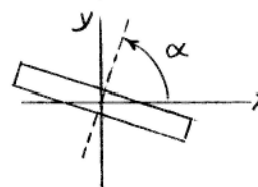
(b)  $I_x > I_y, I_{xy} (+)$



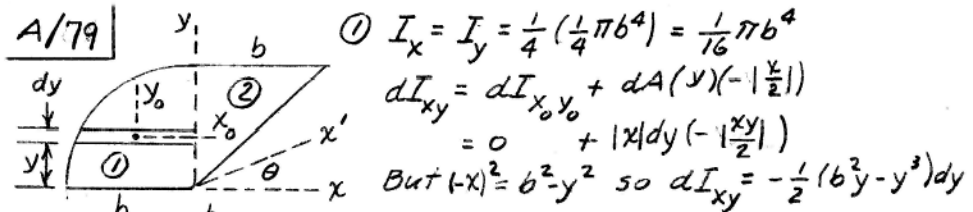
(c)  $I_x > I_y, I_{xy} (-)$



(d)  $I_x < I_y, I_{xy} (-)$







①  $I_x = I_y = \frac{1}{4} (\frac{1}{4} \pi b^4) = \frac{1}{16} \pi b^4$   
 $dI_{xy} = dI_{x_0 y_0} + dA(y)(-\frac{y}{2})$   
 $= 0 + |x|dy(-\frac{y}{2})$   
 But  $(-x)^2 = b^2 - y^2$  so  $dI_{xy} = -\frac{1}{2} (b^2 y - y^3) dy$

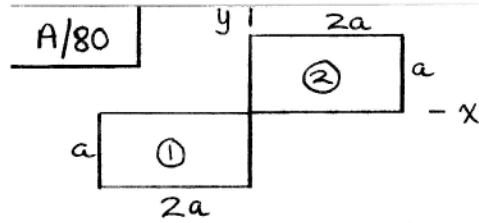
$I_{xy} = -\frac{1}{2} \int_0^b (b^2 y - y^3) dy = -b^4/8$

② From Sample Prob. A/2,  $I_x = \frac{1}{4} b^4, I_y = \frac{1}{12} b^4, I_{xy} = \frac{1}{8} b^4$  (Prob. A/69)

For total,  $I_x = (\frac{\pi}{16} + \frac{1}{4}) b^4, I_y = (\frac{\pi}{16} + \frac{1}{12}) b^4, I_{xy} = 0$

$I_x = 0.446 b^4, I_y = 0.280 b^4$

$x = \text{axis of max. moment of inertia}$   
 $y = \text{" " min. " " " } \text{ since } I_{xy} = 0$



$$\textcircled{1} I_x = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4, \quad I_y = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$$

$$I_{xy} = (2a^2)(a)\left(\frac{a}{2}\right) = a^4$$

$$\textcircled{2} I_x = \frac{2}{3}a^4, \quad I_y = \frac{8}{3}a^4, \quad I_{xy} = a^4$$

$$\text{Eq. A/11: } I_{\min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= \frac{10}{3}a^4 - \frac{1}{2} \sqrt{(-4a^4)^2 + 16a^8} = 0.505a^4$$

$$I_{\max} = \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= \frac{10}{3}a^4 + \frac{1}{2} \sqrt{(-4a^4)^2 + 16a^8} = 6.16a^4$$

$$\text{Eq. A/10: } \tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{\left(\frac{16}{3} - \frac{4}{3}\right)a^4}$$

$$2\alpha = 45^\circ \text{ or } 225^\circ$$

$$\alpha = 22.5^\circ \text{ for } I_{\min}$$

$$\text{or } \alpha = \underline{112.5^\circ} \text{ for } I_{\max}$$

A/81 | From figure,  $I_{xy}$  is (-)

Add Eqs. A/11 & get  $I_{max} + I_{min} = I_x + I_y$

$$\text{so } I_x + I_y = (12 + 2)10^6 = 14(10^6) \text{ mm}^4$$

From the 1<sup>st</sup> of Eqs. A/11,

$$(I_x - I_y)^2 = [2I_{max} - (I_x + I_y)]^2 - 4I_{xy}^2$$

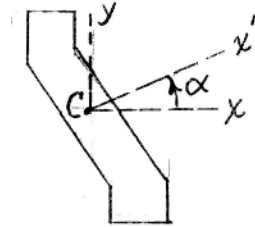
$$= [2(12) - 14]^2 10^{12} - 4(-4)^2 10^{12} = 36(10^{12}) \text{ mm}^8$$

$$\left. \begin{array}{l} I_x - I_y = 6(10^6) \text{ mm}^4 \\ I_x + I_y = 14(10^6) \text{ mm}^4 \end{array} \right\} \text{add \& get } \frac{I_x = 10(10^6) \text{ mm}^4}{\& I_y = 4(10^6) \text{ mm}^4}$$

From Eq. A/10,

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(-4)(10^6)}{-6(10^6)} = 4/3$$

$$2\alpha = 53.13^\circ, \quad \underline{\alpha = 26.6^\circ}$$





A/83

$$I_x = \frac{1}{12}bh^3 = \frac{1}{12}4(8^3) = 170.7 \text{ in.}^4$$

$$I_y = \frac{1}{12}8(4^3) = 42.7 \text{ in.}^4$$

$$I_{xy} = \frac{b^2h^2}{24} = \frac{1}{24}(4^2)(8^2) = 42.7 \text{ in.}^4$$

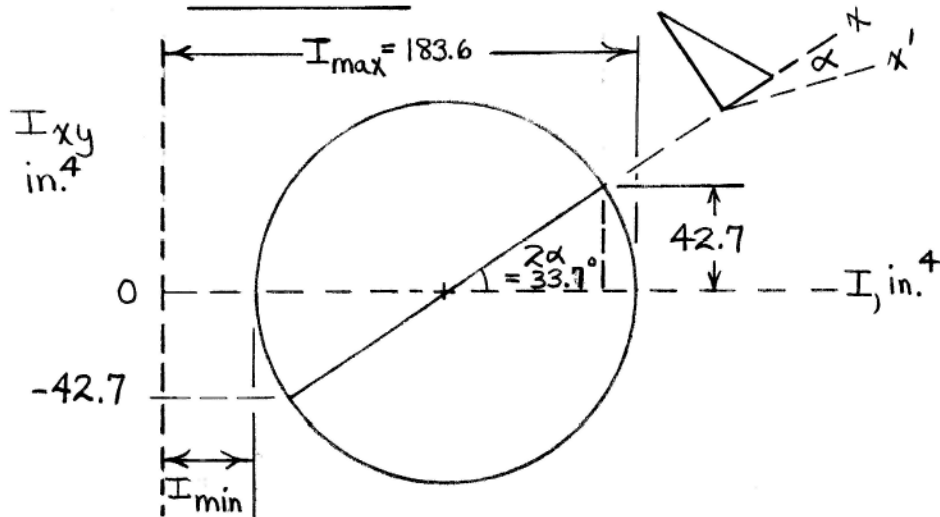
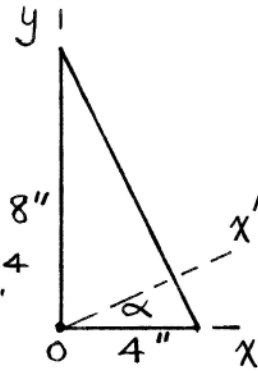
Eq. A/11:

$$I_{\max} = \frac{170.7 + 42.7}{2} + \frac{1}{2}\sqrt{(170.7 - 42.7)^2 + 4(42.7)^2}$$

$$= 106.7 + 76.9 = 183.6 \text{ in.}^4$$

Eq. A/10:  $\tan 2\theta_{cr} = \tan 2\alpha = \frac{2(42.7)}{42.7 - 170.7} = -0.667$

$$\alpha = -16.85^\circ$$



A/84 | ①  $I_x = \frac{1}{3}(10)(80)^3 = 1.707(10^6) \text{ mm}^4$ ,  $I_y = \frac{1}{3}(80)(10)^3 = 0.0267(10^6) \text{ mm}^4$

y | ②  $I_x = \frac{1}{3}(50)(10)^3 = 0.0167(10^6) \text{ mm}^4$   
 $I_y = \frac{1}{12}(10)(50)^3 + 10(50)(35)^2 = 0.7167(10^6) \text{ mm}^4$

①  $I_{xy} = 0 + 80(10)(40)(5) = 0.1600(10^6) \text{ mm}^4$

②  $I_{xy} = 0 + 50(10)(35)(5) = 0.0875(10^6) \text{ mm}^4$

Totals:  $I_x = 1.723(10^6) \text{ mm}^4$

$I_y = 0.743(10^6) \text{ mm}^4$

$I_{xy} = 0.248(10^6) \text{ mm}^4$

From Eqs. A/11

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}, \quad I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{max} = \left[ \frac{1.723 + 0.743}{2} + \frac{1}{2} \sqrt{(1.723 - 0.743)^2 + 4(0.248)^2} \right] 10^6 = 1.782(10^6) \text{ mm}^4$$

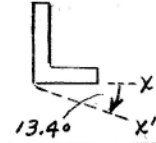
$$I_{min} = \left[ \frac{1.723 + 0.743}{2} - \frac{1}{2} \sqrt{(1.723 - 0.743)^2 + 4(0.248)^2} \right] 10^6 = 0.684(10^6) \text{ mm}^4$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(0.248)}{0.743 - 1.723} = -0.5051$$

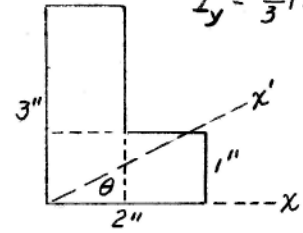
$$2\alpha = -26.8^\circ$$

$$\alpha = -13.4^\circ$$



\* A/85  $I_x = \frac{1}{3}(1)(3^3) + \frac{1}{3}(1)(1^3) = 28/3 = 9.333 \text{ in.}^4$

$I_y = \frac{1}{3}(2)(1^3) + \frac{1}{3}(1)(2^3) = 10/3 = 3.333 \text{ in.}^4$



$I_{xy} = (3)(1)(1.5)(0.5) + (1^2)(0.5)(1.5) = 3 \text{ in.}^4$

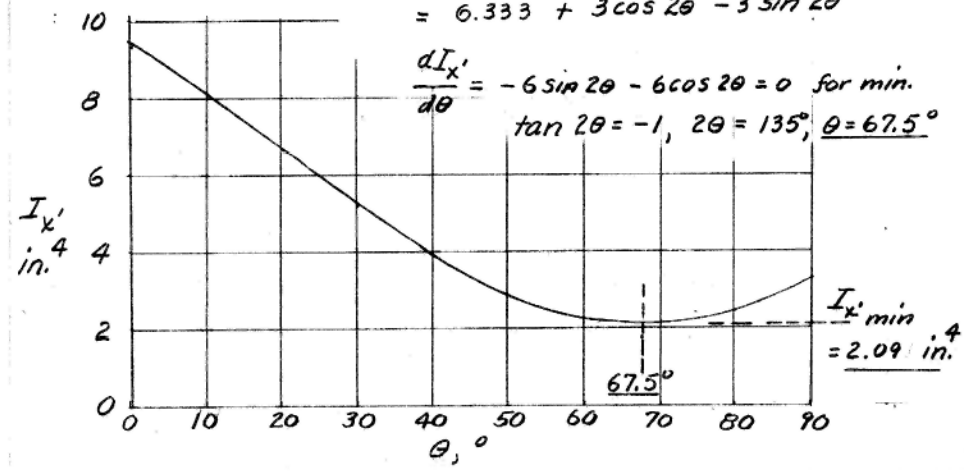
Eq. A/9

$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$

$= 6.333 + 3 \cos 2\theta - 3 \sin 2\theta$

$\frac{dI_{x'}}{d\theta} = -6 \sin 2\theta - 6 \cos 2\theta = 0$  for min.

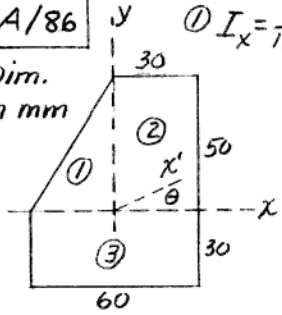
$\tan 2\theta = -1, 2\theta = 135^\circ, \theta = 67.5^\circ$



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\*A/86

Dim.  
in mm



$$① I_x = \frac{1}{12}(30)(50)^3 = \frac{125}{4}(10^4) \text{ mm}^4$$

$$I_y = \frac{1}{12}(50)(30)^3 = \frac{45}{4}(10^4) \text{ mm}^4$$

$$I_{xy} = -\frac{(30)^2(50)^2}{24} = -\frac{75}{8}(10^4) \text{ mm}^4 \quad (\text{Prob. A/49})$$

$$② I_x = \frac{1}{3}(30)(50)^3 = 125(10^4) \text{ mm}^4$$

$$I_y = \frac{1}{3}(50)(30)^3 = 45(10^4) \text{ mm}^4$$

$$I_{xy} = (30 \times 50) \frac{50}{2} \frac{30}{2} = \frac{225}{4}(10^4) \text{ mm}^4$$

$$③ I_x = \frac{1}{3}(60)(30)^3 = 54(10^4) \text{ mm}^4, \quad I_y = \frac{1}{12}(30)(60)^3 = 54(10^4) \text{ mm}^4, \quad I_{xy} = 0$$

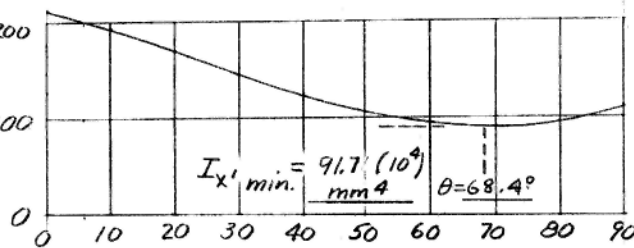
$$\text{Totals } I_x = 210.25(10^4) \text{ mm}^4, \quad I_y = 110.25(10^4) \text{ mm}^4, \quad I_{xy} = 46.875(10^4) \text{ mm}^4$$

Eg. A/9

$$I_{x'} = [160.25 + 50 \cos 2\theta - 46.875 \sin 2\theta] 10^4$$

Compute & plot

$I_{x'} (10^{-4})$   
mm<sup>4</sup>

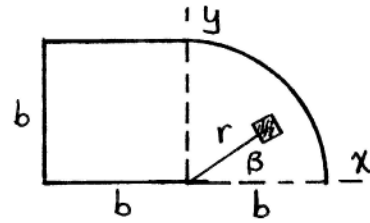




\* A/87

$$I_x = \frac{1}{3} b^4 + \frac{1}{16} \pi b^4 = 0.530 b^4$$

$$I_y = \frac{1}{3} b^4 + \frac{1}{16} \pi b^4 = 0.530 b^4$$



$$\text{Quarter circle: } I_{xy} = \int_0^{\pi/2} \int_0^b (r \cos \beta)(r \sin \beta) r dr d\beta$$
$$= \frac{r^4}{4} \Big|_0^b \times \left(-\frac{1}{4} \cos 2\beta\right) \Big|_0^{\pi/2} = \frac{b^4}{4} \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{b^4}{8}$$

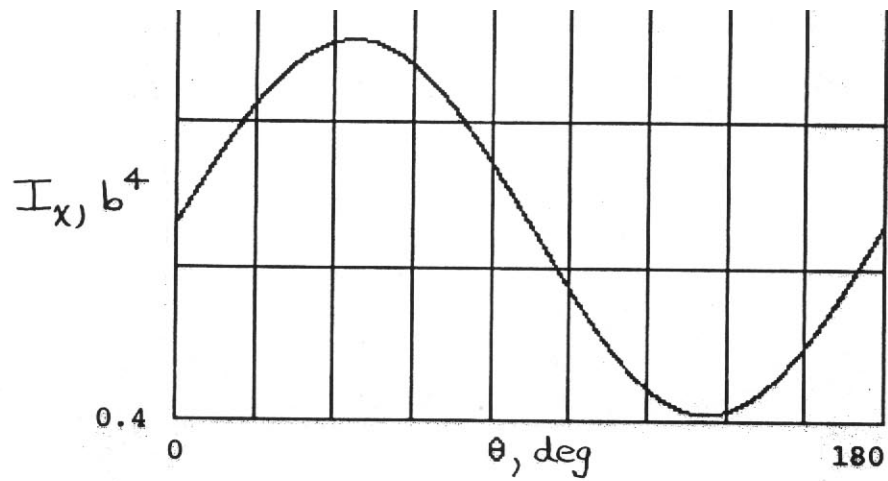
$$\text{Square: } I_{xy} = b^2 \left(-\frac{b}{2}\right) \left(\frac{b}{2}\right) = -\frac{b^4}{4} = -0.25 b^4$$

$$\text{Combined: } I_{xy} = \frac{b^4}{8} - \frac{b^4}{4} = -\frac{b^4}{8} = -0.125 b^4$$

$$\text{Eq. A/9: } I_{x'} = \frac{2(0.530 b^4)}{2} + 0 - (-0.125 b^4) \sin 2\theta$$
$$= (0.530 + 0.125 \sin 2\theta) b^4$$

For critical angle  $\theta = \alpha$ , Eq. A/10 gives

$$\tan 2\alpha = \frac{2(0.530 b^4)}{0}, \quad 2\alpha = \frac{\pi}{2}, \quad \alpha = \frac{\pi}{4}$$



$$I_{\max} = 0.655 b^4 \text{ @ } \theta = 45^\circ$$

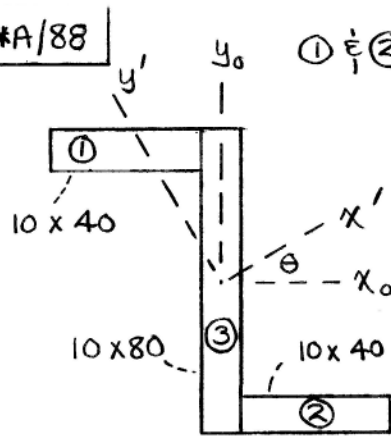
$$I_{\min} = 0.405 b^4 \text{ @ } \theta = 135^\circ$$

Eqs. A/II:

$$\begin{aligned} I_{\max} &= 0.530 b^4 + \frac{1}{2} \sqrt{0^2 + 4(-0.125 b^4)^2} \\ &= 0.655 b^4 \quad \checkmark \end{aligned}$$

$$\begin{aligned} I_{\min} &= 0.530 b^4 - \frac{1}{2} \sqrt{0^2 + 4(-0.125 b^4)^2} \\ &= 0.405 b^4 \quad \checkmark \end{aligned}$$

\*A/88



$$\textcircled{1} \text{ \& } \textcircled{2}: I_{x_0 y_0} = \bar{I}_{xy} + d_x d_y A$$

$$= 0 + (35)(-25)(10 \times 40)$$

$$= -0.350(10^6) \text{ mm}^4$$

$$\textcircled{3}: I_{x_0 y_0} = 0$$

$$\textcircled{1} \text{ \& } \textcircled{2}: I_{x_0} = \frac{1}{12}(40)(10^3)$$

$$+ (10 \times 40) 35^2 = 0.493(10^6) \text{ mm}^4$$

(Dim. in mm)

$$I_{y_0} = \frac{1}{12}(10)(40)^3 + (10 \times 40) 25^2 = 0.303(10^6) \text{ mm}^4$$

$$\textcircled{3}: I_{x_0} = \frac{1}{12}(10)(80)^3 = 0.427(10^6) \text{ mm}^4$$

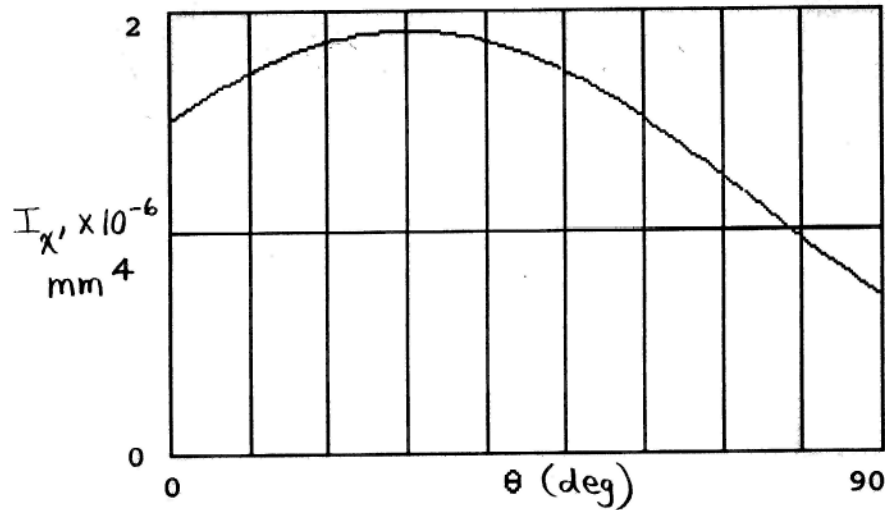
$$I_{y_0} = \frac{1}{12}(80)(10)^3 = 0.00667(10^6) \text{ mm}^4$$

$$\text{Totals: } \begin{cases} I_{x_0} = 1.413(10^6) \text{ mm}^4 \\ I_{y_0} = 0.613(10^6) \text{ mm}^4 \\ I_{x_0 y_0} = -0.700(10^6) \text{ mm}^4 \end{cases}$$

From Eq. A/9:

$$I_{x'} = \left[ \frac{1.413 + 0.613}{2} + \frac{1.413 - 0.613}{2} \cos 2\theta + 0.700 \sin 2\theta \right] 10^6$$

$$= [1.013 + 0.4 \cos 2\theta + 0.7 \sin 2\theta] 10^6$$



$$\begin{aligned}
 \text{Eq. (A/11): } I_{\max} &= \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2} \\
 &= \frac{1.413 + 0.613}{2} 10^6 + \frac{1}{2} \sqrt{(1.413 - 0.613)^2 (10^6)^2 + 4(-0.7 \times 10^6)^2} \\
 &= \underline{1.820 (10^6) \text{ mm}^4}
 \end{aligned}$$

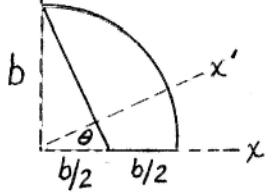
$$\begin{aligned}
 \text{Eq. (A/10): } \tan 2\alpha &= \frac{2I_{xy}}{I_y - I_x} \\
 &= \frac{2(-0.7)}{0.613 - 1.413}
 \end{aligned}$$

$$\Rightarrow \alpha = \underline{30.1^\circ}, 120.1^\circ$$

(Values from Eqs. A/10 & A/11 agree with plot.)

\*A/89 Quarter circle:  $I_x = I_y = \frac{1}{16}\pi r^4$

Use horiz strip & get  $dI_{xy} = 0 + (x dy) y \frac{x}{2} = \frac{b^2 y - y^3}{2} dy$



$$I_{xy} = \frac{1}{2} \int_0^b (b^2 y - y^3) dy = \frac{b^4}{8}$$

Triangle:  $I_x = \frac{1}{12} \frac{b}{2} b^3 = \frac{b^4}{24}$ ;  $I_y = \frac{1}{12} b \left(\frac{b}{2}\right)^3 = \frac{b^4}{96}$

$$I_{xy} = \frac{1}{24} \left(\frac{b}{2}\right)^2 b^2 = \frac{b^4}{96} \quad (\text{Prob A/49})$$

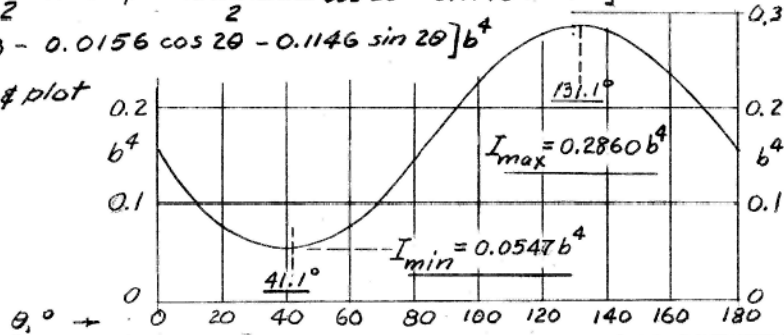
Composite:  $I_x = \left(\frac{\pi}{16} - \frac{1}{24}\right)b^4$ ,  $I_y = \left(\frac{\pi}{16} - \frac{1}{96}\right)b^4$ ,  $I_{xy} = \left(\frac{1}{8} - \frac{1}{96}\right)b^4$   
 $= 0.1547b^4$ ,  $= 0.1859b^4$ ,  $= 0.1146b^4$

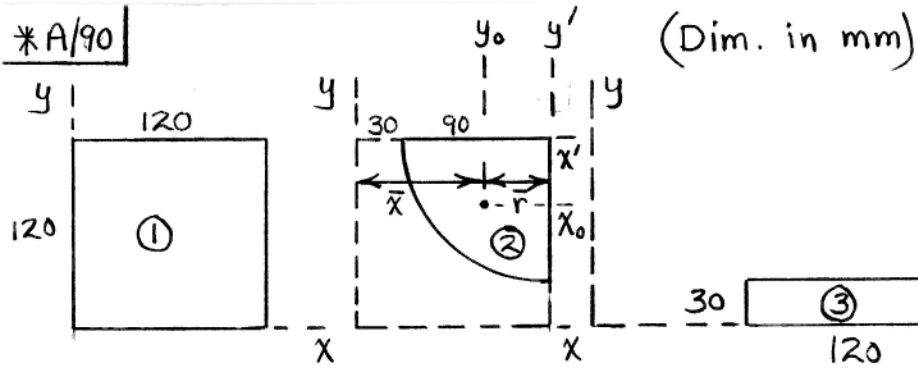
From Eq. A/9

$$I_{x'} = \left[ \frac{0.1547 + 0.1859}{2} + \frac{0.1547 - 0.1859}{2} \cos 2\theta - 0.1146 \sin 2\theta \right] b^4$$

$$= [0.1703 - 0.0156 \cos 2\theta - 0.1146 \sin 2\theta] b^4$$

Calculate & plot





$$\textcircled{1} I_x = I_y = \frac{1}{3} 120^4 = 69.1(10^6) \text{ mm}^4, A = 120^2 = 1.44(10^4) \text{ mm}^2$$

$$I_{xy} = 120^2 (60)(60) = 51.8 \text{ mm}^4$$

$$\textcircled{2} \bar{r} = \frac{4r}{3\pi} = \frac{4(90)}{3\pi} = 38.2 \text{ mm}, \bar{x} = 120 - \bar{r} = 81.8 \text{ mm}$$

$$A = \frac{1}{4} \pi (90^2) = 0.636(10^4) \text{ mm}^2$$

$$I_{x'} = I_{y'} = \frac{1}{4} \left( \frac{1}{4} \pi \times 90^4 \right) = 12.88(10^6) \text{ mm}^4$$

$$I_x = I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2$$

$$= [12.88 + 0.00636(81.8^2 - 38.2^2)] 10^6 = 46.2(10^6) \text{ mm}^4$$

$$I_{xy} = I_{x_0 y_0} + A\bar{x}\bar{y} = I_{x' y'} - A\bar{r}\bar{r} + A\bar{x}\bar{y}$$

$$\text{where } I_{x' y'} = \frac{r^4}{8} = \frac{90^4}{8} = 8.20(10^6) \text{ mm}^4 \left\{ \begin{array}{l} \text{See Sol. to} \\ \text{Prob. A/65} \end{array} \right.$$

$$\therefore I_{xy} = [8.20 + 0.00636(81.8^2 - 38.2^2)] 10^6 = 41.5(10^6) \text{ mm}^4$$

$$\textcircled{3} I_x = \frac{1}{3} (120) (30)^3 = 1.08 (10^6) \text{ mm}^4$$

$$I_y = \frac{1}{2} (30) (120)^3 + 30(120)(180^2) = 121.0 (10^6) \text{ mm}^4$$

$$I_{xy} = 30(120)(180)(15) = 9.72 (10^6) \text{ mm}^4$$

$$\text{Combined: } \begin{cases} I_x = (69.1 - 46.2 + 1.08) 10^6 = 24.0 (10^6) \text{ mm}^4 \\ I_y = (69.1 - 46.2 + 121.0) 10^6 = 143.9 (10^6) \text{ mm}^4 \\ I_{xy} = (51.8 - 41.5 + 9.72) 10^6 = 20.1 (10^6) \text{ mm}^4 \end{cases}$$

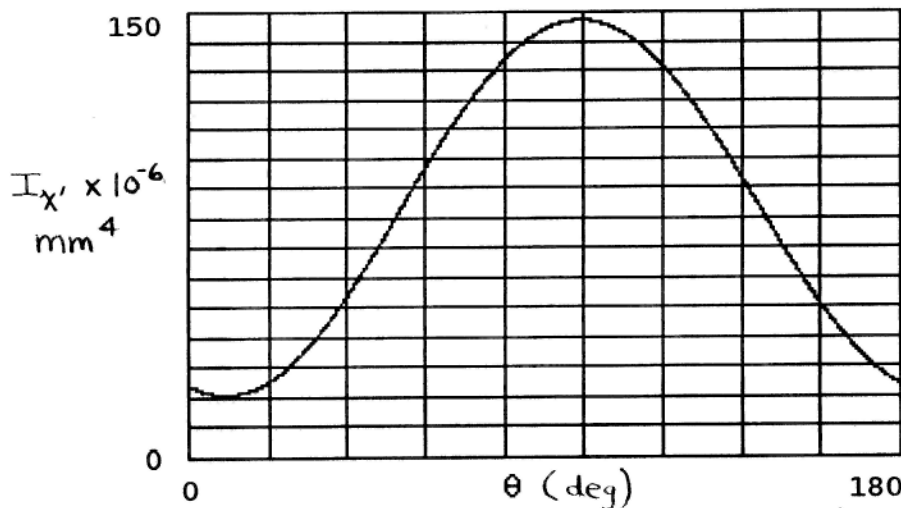
$$A/9: I_{x'} = \left\{ \frac{24.0 + 143.9}{2} + \frac{24.0 - 143.9}{2} \cos 2\theta - 20.1 \sin 2\theta \right\} \times 10^6$$

$$I_{x'} (10^{-6}) = 84.0 - 59.9 \cos 2\theta - 20.1 \sin 2\theta \text{ mm}^4$$

$$A/10: \tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.1)}{143.9 - 24.0} = 0.335$$

$$2\alpha = 18.51^\circ, \alpha = 9.26^\circ \text{ (minimum I)}$$

$$\alpha = 9.26 + 90 = 99.3^\circ \text{ (maximum I)}$$



$$I_{\min} = 20.8 (10^6) \text{ mm}^4 \text{ @ } \theta = 9.26^\circ$$

$$I_{\max} = 147.2 (10^6) \text{ mm}^4 \text{ @ } \theta = 99.3^\circ$$